SURVEYING & LEVELLING

[PART II]

"A Text-Book on Surveying & Levelling" for Engineering Students and Practising Engineers

BY

Late T. P. KANETKAR,

B.A. (HONS.), B.E., A.M.I.E. (IND.), B.E.S.,

Associate Professor of Civil Engineering,

Shri Govindram Seksaria,

Technological Institute, Indore

Lecturer in Civil Engineering, College of Engineering, Poona.

AND

Prof **S. V. KULKARNI,**

B.E. (HONS.), M.SC. (ENG.), A.M.I. STRUCT. E. (LON.), A.M.I.E. (INDIA)

Professor of Civil Engineering,

College of Engineering, Poona 5

Price Rs. 12/-

A. V. G. Prakashan, Poona 2.

Edition *First* 1945 *Second* (*Revised and Enlarged*) 1950, *Third* (*Revised and Enlarged*) 1953, *Fourth* (*Revised and Enlarged*) 1955, *Fifth* (*Revised and Enlarged*) 1957, *Sixth*, 1958, *Seventh* 1959, *Eighth*, (*Revised*) 1959, *Ninth*, (*Revised and Enlarged*) 1960, *Tenth* 1962, *Eleventh* 1963, *Twelfth* 1964, *Thirteenth*, 1965, *Fourteenth*, 1966, **Fifteenth—Oct. 1967.** (*In M K S*) *Sixteenth — May 1969*

Printed by D D Gangal, Lokasangraha Press, 1786, Sadashiv Peth, Poona 2 Published by Dr. G S Koshe, for A V G Prakashan, 1786, Sadashiv Peth, Poona 2

PREFACE TO THE FIFTEENTH EDITION

According to the Standards of Weights and Measures Act (India) 1956, the metric system has become the only recognized system of weights and measures in India from 1966 With this change, taking over, there was a constant demand from students, teachers and practising engineers to have our books on Surveying and Levelling revised to suit the present day requirements Part I of Surveying and Levelling revised and rewritten in the metric system, was published in 1965 In the revision of Part II for this edition, metric units have been adopted throughout Numerous examples are added to make students more conversant with the use of the metric system and its relationship with the F P S system Standards of materials and specifications for instruments as laid down by I S I have been adopted wherever available Elsewhere either rationalized values have been used or the practice current on the Continent has been adopted

The author expresses his indebtedness to numerous I S I publications and German text books on the subject which were found of immense help in the compilation of this volume He would like to make grateful mention of many of his students, teachers in the Engineering Colleges and Polytechnics, and engineer friends in the field who have made very useful suggestions from time to time Suggestions for further improvement will continue to be welcome

Poona,
4 Oct, 1967

S. V. Kulkarni

FOREWORD

With the large number of major and minor irrigation and power projects contemplated under the Five Year Plans, the time has now come to recognise the need for a new outlook in the training of young engineers. The speedy execution of such projects now demands greater familiarity on the part of the average engineer with more specialised techniques and subtle methods of approach than were customary in the past. Project authorities can even vouch from their experience that time schedules of several projects could have been substantially advanced if it were possible to undertake the survey and mapping by their own organisations instead of depending on specialist departments. While the comprehensive syllabus in surveying currently adopted by nearly all Indian Universities is, therefore, a matter for gratification, the need for the students to acquire a degree of familiarity with the advanced methods cannot be too strongly emphasized. There is room to think that opportunities for such familiarity, which should lead to proficiency, are still to be desired on a wider scale in many Institutions.

A suitable book which can be used for intensive study by the engineering student is one part of the improvement plan. Such a book embodying advanced techniques with appropriate emphasis on methods of application and written in a language easily understood, plays an important part. It assumes added importance in the context of a proposed change of medium of instruction in order that the transition from one medium to another is brought about with the minimum of effort on the part of the teacher and the taught. Shri Kanetkar's books on Surveying, which are among the few published in this country, while meeting the specific requirements of the comprehensive syllabus now in use, go a long way, as a possible basic publication to be adapted to regional languages when such need arises.

Numerical procedures have a special significance in Surveying no less than in other branches of engineering. Indeed

certain special methods of approach are more easily illustrated by an aptly chosen problem than pages of description Viewed from this angle, many of the problems in the book may well be regarded as a part of the subject matter rather than a mere illustration of a particular procedure The diversity of the problems presented in the book, both solved and unsolved, should suggest to the student the possibility of almost unlimited presentations and combinations and the need to tackle them not infrequently, on his own initiative The success of the book lies in leading the student, step by step, to a point where he can tackle any problem likely to be met with in the field with confidence and ability

The author's continuous efforts to revise the book and enlarge its scope notwithstanding the warm reception enjoyed by it even when it was first published in 1950, deserve the highest praise The Institution which he has served with distinction for over a quarter of a century can well feel proud of the high traditions established by him It may, it is hoped prove an inspiration to others

College of Engineering
Poona—5
26-7-1955

L T Aminbhavi,
Professor of Civil Engineering
& Head of the Civil Engineering
Department

PREFACE TO THE NINTH EDITION

This latest edition is thoroughly revised and brought up to date by Prof S V Kulkarni, B E (HONS) M SC (ENG), A M I STRUCT E (LON), A M I E (INDIA) Associate Professor of Civil Engineering, College of Engineering, Poona 5 He has a long experience in teaching this subject and has taken pains in revising the book, making it most suitable to the needs of the students concerned

September 1960 — *Manager,* A V G PUBLICATION, POONA 2

PUBLISHER S NOTE

We regret to announce the untimely death of Prof T P Kanetkar on the 23rd June 1957 To continue to publish his admirable book on Surveying and Levelling is really a fitting memorial to his soul We are doing our best to do this

We are very much thankful to Shri S V Kulkarni, B E (Hons), A M I Struct E (Lond), A M I E (Ind) Lecturer in Civil Engineering College of Engineering, Poona, for his kind suggestions

23rd May 1958 — *Manager* A V GRIHA PUBLICATION, POONA 2

PREFACE TO THE FIFTH EDITION

In undertaking the revision of this book for a fifth edition the subject matter is revised thoroughly with further addition of a few typical problems and diagrams

The author gratefully acknowledges his indebtedness to the authors of a number of standard books and text books on the subject which he has found of immense use in the preparation of this edition The author also wishes to express his thanks to the authorities of the Universities of Karnatak, Poona, Bombay and Gujrat for their kind permission to reproduce questions from their examination papers

Poona April 1957. — T P. KANETKAR

PREFACE TO THE FOURTH EDITION

In meeting the call for a new edition the opportunity has been taken to revise the subject matter thoroughly A number of additions are made, the more important ones are · Theory of Anallatic lens, the description and use of Beaman stadia arc, Direct reading tacheometers, Sun dial, and Solar attachment the methods of determining the Meridian, and Longitude, and the method of setting out a Parallel of Latitude The section on Astronomy has been enlarged, and the problems with answers from the examination papers of the Universities of Poona, Bombay, Gujrat and Karnatak are added at the end of the text for solution by the student It is hoped that the additions will add to the general usefulness of the book both to the student and the practising engineer

The author gratefully acknowledges his indebtedness to the authors of a number of standard books and text books on the subject which he has found of immense use in the preparation of this edition The author also wishes to express his thanks to the authorities of the Universities of Karnatak, Poona, Bombay, and Gujrat for their kind permission to reproduce questions from their examination papers

Poona
August, 1955

T P. KANETKAR

CONTENTS

CHAPTER I

TRAVERSE SURVEY

Omitted Measurements

A SURVEY line may be represented on plan by two rectangular co-ordinates, if its length and bearing be known, the axes of co ordinates being a North and South line, and an East and West line Distance measured parallel to the former is called *Latitude*, while that measured parallel to the latter is called *Departure*. The known length and bearing of a line are together referred to as the *Course* of the line.

The trigonometrical relations of the course with its latitude and departure are as follows : (Fig 1).

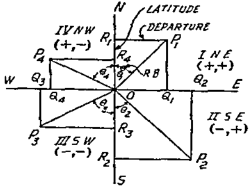


Fig 1

Latitude (L) = length × cosine reduced bearing.

Departure (D) = length × sine reduced bearing.

Latitude is positive when measured North or upwards, and negative when measured South or downwards. Similarly, departure is positive when measured East or to the right, and negative when measured West or to the left North latitudes are called 'Northings', and south latitudes 'Southings'. Similarly, east departures are known as 'Eastings', and west departures as 'Westings'.

Hence we have :

$$\begin{aligned} \text{Northing} &= \text{North latitude} = +L \\ \text{Southing} &= \text{South latitude} = -L \\ \text{Easting} &= \text{East departure} = +D \\ \text{Westing} &= \text{West departure} = -D \end{aligned}$$

The reduced or quadrantal bearing of a line determine the signs of its latitude and departure, the first letter (N or S) of a bearing giving the sign of the latitude, and the last one (E or W), the sign of the departure If the bearing of a line is given as W C B, the following table should be referred to, to determine the signs of the latitude and departure of the line

W C B	Quadrant	Sign of	
		Latitude	Departure
Between 0° and 90°	I N E	+	+
„ 90° and 180°	II S E	–	+
„ 180° and 270°	III S W	–	–
„ 270° and 360°	IV N W	+	–

A closed traverse may be said to be completely surveyed when the length and bearing of each of its sides are known The bearings of the sides may either be observed in the field or computed from the observed bearing of any one side and the included or deflection angles of the polygon

As a rule, the bearings and lengths of the sides of a closed traverse are determined by field observations in order to have a check on the field work But if, due to obstacles, it is not possible to determine them by direct observations, e g the length and bearing of a line joining two points, which are not intervisible owing to an intervening obstruction *such as a building*, or the centre line of a tunnel whose ends are not intervisible, or if, from accident, omissions occur in the field notes, the principles of latitudes and departures may be employed to determine the omitted measurements, provided they are not

more than two in number The problem is indeterminate if more than two quantities are omitted The sides, of which the parts (two bearings, two lengths, or one bearing and one length) are missing, are called the *affected* sides The affected sides may be adjoining or separated In the process of calculating the missing quantities, it must be assumed that all the field measurements are precise Consequently, there are no means of balancing the work, and all errors propagated throughout the survey are thrown into the computed values of the omitted data

The solution of the problem of omitted measurements is based upon the fact that in a closed traverse, the algebraic sum of the latitudes (ΣL) and that of the departures (ΣD) are each equal to zero

If l_1, l_2, etc , be the lengths, and θ_1, θ_2, etc , the bearings of the lines, then

$$l_1 \cos \theta_1 + l_2 \cos \theta_2 + \quad = 0 \qquad (1)$$

$$l_1 \sin \theta_1 + l_2 \sin \theta_2 + \quad = 0 \qquad (2)$$

The solution of these two simultaneous equations give the required values of the two unknown elements However this method is not convenient as it necessarily involves large numbers and may lead to confusion The following alternative method is preferable

The common cases of omitted measurements which occur in practice are

(1) (a) Bearing of one side is wanting
  (b) Length of one side is wanting
  (c) Length and bearing of one side are wanting

(2) Length of one side and bearing of another side are missing

(3) Lengths of two sides are omitted

(4) Bearings of two sides are wanting

In the first case only one side is affected by the omission while in each of the other cases (2 to 4) two sides are affected

by the omission The following trigonometric relations of the course of a line with its latitude and departure should be used in computing the unknown quantities

(i) Latitude = Length × cosine reduced bearing
Departure = Length × sine reduced bearing

(ii) Tangent reduced bearing $= \dfrac{\text{departure}}{\text{latitude}}$

or reduced bearing $= \tan^{-1} \dfrac{\text{departure}}{\text{latitude}}$

(iii) (a) Length $= \sqrt{(\text{Latitude})^2 + (\text{departure})^2}$

(b) Length = latitude × sec reduced bearing

(c) Length = departure × cosec reduced bearing

Use (iii) b or (iii) c according as the latitude or departure is greater so as always to calculate from the greater of the known quantities

Case 1 Bearing, or Length, or Length and Bearing of One side Wanting —In Fig 2, let the length, or bearing or both of the line CD be wanting To determine the missing parts,

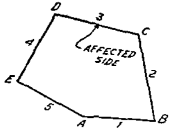


Fig 2

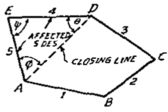


Fig 3

(i) Compute with correct signs the latitudes and departures of the known sides DE, EA, AB, and BC

(ii) Find the algebraic sum of the latitudes (ΣL) and that of the departures (ΣD) Subtract algebraically each of these sums (ΣL and ΣD) from zero in order to obtain the latitude and departure of the omitted or affected side Then

Latitude of CD = − ΣL
Departure of CD = − ΣD

(iii) Knowing the latitude and departure of CD, calculate its bearing and length from the trigonometric relations (ii) and (iii) with due regard to sign

Case 2 Length of One Side and Bearing of Another Side Missing —In Fig 3, let the length of DE and the bearing of EA be missing

(i) Ignoring the affected sides DE and EA close the polygon formed by the known sides AB BC, and CD by the closing line DA

(ii) Compute the length and bearing of the closing line DA as in case 1

(iii) Determine the angle (θ) between the closing line DA and the line DE of known bearing from their known bearings

(iv) Solve the triangle DEA formed by the closing line and the two affected sides

In the triangle DEA, the lengths of the sides DA and EA, and the angle ADE (θ) are known The angle DEA (ψ) and the length of DE may be calculated by using the *Sine rule*

$$\sin\psi = \frac{DA}{EA}\sin\theta, \quad \angle EAD = \phi = 180° - (\theta + \psi);$$

$$DE = EA\frac{\sin\phi}{\sin\theta} = DA\frac{\sin\phi}{\sin\psi}$$

(v) Determine the bearing of EA from the known bearing of DE, and the calculated value of the angle (ψ) and check the result by finding it from the calculated values of the bearing of DA and the angle ϕ

Alternative method —In order to simplify the computations the side of unknown length may be assumed to be the reference meridian (a north and south line) The bearings of the other sides should be calculated with reference to this meridian and the latitudes and departures of the known sides should then be calculated

This artifice eliminates one unknown quantity, viz. the departure of the side assumed as a meridian, since its value is zero The algebraic sum of the departures then gives the departure of the other affected side Knowing the length of this side, its bearing with respect to the assumed meridian and its latitude may be calculated The latitude of the side assumed as a meridian may then be obtained by finding the algebraic sum of all the latitudes and equating it to zero. The value thus obtained gives the length of this side as its departure is zero This method is applicable when the affected sides adjoin or not

Note —This is an ambiguous case. Two values for each of the unknowns (length and bearing) are possible in this case. However, it is usually evident which of the two solutions corresponds to the survey line, if the approximate shape of the figure is known

Case 3 Lengths of Two Sides Omitted —In Fig 3, let DE and EA be the affected sides

The first two steps are the same as in case 2.

(iii) Determine the angles θ, ϕ, and ψ of the triangle DEA from the known bearings of DA, DE, and EA Check the result by adding them and observing if their sum equals 180°.

(iv) Compute the lengths of the sides DE and EA of the triangle DEA, of which all the angles and the side DA are known Applying the *Sine rule*, we have

$$\text{DE} = \text{DA} \quad \text{and} \quad \text{EA} = \text{DA}\,\frac{\sin\theta}{\sin\psi}$$

Case 4 Bearings of Two Sides Unknown - In Fig. 3, let DE and EA be the affected sides

The first two steps are the same as in case 2.

(iii) Knowing the *lengths of the sides of the triangle* DEA, calculate its area by the formula

$$\Delta = \sqrt{s\,(s-a)\;(s-b)\;(s-c)}$$

(iv) Determine the angles of the triangle DEA by equating the calculated area to half the product of any two sides into the sine of the angle between them

(v) Find the bearings of the sides DE and EA from the known bearing of the closing line DA, and the angles θ and ϕ.

When the affected sides are not adjacent, one of these sides must be shifted to a position adjacent to the other They should be omitted and the known sides shifted each parallel to itself so as to form a connected series of the sides. The rest of the procedure is exactly similar to that in cases 2 to 4

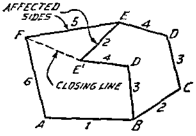


Fig 4

In Fig. 4, let BC and EF be the affected sides In order to have a connected series of the known sides, shift the known sides CD and DE parallel to themselves in a direction parallel to one of the unknowns and close the polygon by the line E'F E'E is then parallel and equal to BC. Thus a triangle FEE' is formed by the closing line E'F and the two affected sides BC and EF. It may be noted that the length and bearing of a line remain unchanged when moved parallel to itself.

Note :—(1) In solving problems of this character, it is advisable to plot the traverse to scale, showing all the conditions and the triangle that is to be solved, thereby avoiding mistakes and facilitating the work.

(2) Having obtained the values of the unknowns, the computations may be checked by finding the latitudes and departures of the affected sides and observing that the algebraic sum of the latitudes and the algebraic sum of the departures of the sides of the traverse are each equal to zero

(3) In all these cases the general direction of at least one of the affected lines must be observed.

Examples on Omitted Measurements

Example 1 :—The following lengths and bearings were recorded in running a traverse ABCDE (Fig. 2), the length and bearing of EA having been omitted :

Line.	Length in m	Bearing
AB	217 5	120° 15′
BC	318 0	62° 30′
CD	375 0	322° 24′
DE	283 5	235° 18′
EA	?	?

Calculate the length and bearing of the line EA.

In Fig 2, the line EA is the closing line of the polygon ABCDE formed by the known sides. The latitudes and departures of the known sides should be calculated in the usual way and tabulated as under .

Line	Latitude		Departure	
	+	–	+	–
AB		109 578	187·872	
BC	146 835		282 072	
CD	297 105			228 804
DE		161·397		233·076
Sum	443 940	270 975	469 944	461·880

The algebraic sum of the known latitudes

$$= \Sigma L = +443 \cdot 940 - 270 \cdot 975$$
$$= +172 \cdot 965$$

The algebraic sum of the known departures

$$= \Sigma D = + 469\ 944 - 461\ 880$$
$$= +8\ 064$$

Latitude of the closing line EA $= -\ \Sigma L = -\ 172\ 965$

Departure of ,, $= -\ \Sigma D = -\ 8\ 864$

The minus sign of the latitude denotes a south bearing and the minus sign of the departure indicates that it is west, e the line EA lies in the third (S W) quadrant Let θ be the reduced bearing of EA

Then $\tan\theta = \dfrac{\text{departure}}{\text{latitude}} = \dfrac{8\ 064}{172\ 965}$

$\theta = 2°\ 40$

Hence *R B of EA = S 2° 40 W*

W C B of EA $= 182°\ 40$

$$\text{Length of EA} = \frac{\text{lat}}{\cos\theta} = \frac{172\ 965}{\cos 2°\,40} = 173\ 151\ \text{m} = 173\ 15\text{m}$$

Check —Length of EA $= \sqrt{(172\ 965)^2 + (8\ 064)^2} = 173\ 151$ m
$= 173\ 15$ m

Example 2 —Given the following latitudes and departures of the sides of a traverse ABCDE (Fig 5) the bearing of BC and the length of CD having been omitted

No	Line	Length in m	Bearing	Latitude	Departure
1	AB	217 5	S 59° 45 E	−109 578	+187 872
2	BC	318 0	?	?	?
3	CD	?	N 37° 36 W	?	?
4	DE	283 5	S 55° 18 W	−161 307	−233 076
5	EA	173 15	S 2° 40 W	−172 989	− 8 055

Compute the bearing of BC and the length of CD

In Fig. 5, the closing line BD completes the polygon formed by the known lines 4, 5, and 1. On solving the triangle BCD

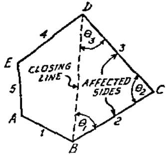


Fig 5

formed by the closing line BD and the affected sides BC and CD, the required quantities may be obtained. Adding algebraically the known latitudes, and the known departures, we get

$\Sigma L = -443.964$ and $\Sigma D = -53.259$.

$\therefore$ Latitude of the closing line BD $= -\Sigma L = +443.964$.

Departure of the closing line BD $= -\Sigma D = +53.259$.

Since the latitude and departure are both positive, BD is in the first (N E) quadrant If θ be the reduced bearing of the line BD, we have

$$\text{Tan } \theta = \frac{53.259}{443.964} \quad \text{or} \quad \theta = 6^\circ\, 50'.$$

$\therefore$ R. B. of BD = N. 6° 50′ E.

Length of BD = 443·964 sec 6° 50′ = 447·15 m.

Now R. B of DC = back bearing of CD = S. 37° 36′ E.

R B of DB = „ „ of BD = S. 6° 50′ W.

In the triangle BCD, $\angle$CDB = R. B. of DC + R.B. of DB
= 37° 36′ + 6°50′ = 44° 26′.

BD = 447 15 m ; and BC = 318 0 m.

The remaining parts may be found by applying the Sine rule.

Let the angles DBC, BCD, and CDB be denoted by θ_1, θ_2, and θ_3 respectively.

Then $\sin\ \theta_2 = \dfrac{BD}{BC}\ \sin\ \theta_3 = \dfrac{447\ 15}{318\ 0}\ \sin 44°\ 26$

or $\qquad \theta_2 = 79°55$

$$\angle DBC = \theta_1 = 180° - (\theta_2 + \theta_3)$$
$$= 180° - (79°55' + 44°26\ 30'') = 55°39$$

Now R B of BC = R B of BD + θ_1 = 6°50 + 55°39
= N 62° 29 E

Length of CD = BD $\dfrac{\sin\ \theta_1}{\sin\ \theta_2}$ = 447·15 $\dfrac{\sin 55°\ 39}{\sin 79°\ 55}$ = 374 94 m

Alternative method —

Here the side CD of unknown length is assumed to be a north and south line

The given bearings when referred to this meridian are

Bearing of CD = 0° 0
„ of DE = 55°18 + 37°36 = 92°54 S W = N 87° 6 W
„ of EA = 2°40′ + 37°36 = 40°16 S W = S 40°16 W
„ of AB = 59°45 − 37°36 = 22° 9 S E = S 22° 9 E

Then the latitudes and departures of the known sides DE EA, and AB are

Line	Latitude	Departure
DE	+ 14 34	− 283 08
EA	− 132 18	− 111 96
AB	− 201 42	+ 81 99

Now let l be the length of CD, and θ the R B of BC

Then latitude of BC = 318 0 cosθ | Departure of BC = 318 0 sinθ
„ of CD = l | „ of CD = 0

Since ABCDE is a closed traverse, ΣL and ΣD are each equal to zero

$$+ 14\ 34 - 132\ 18 - 201\ 42 + 318 \cos\theta + l = 0 \qquad (1)$$

$$- 283\ 08 - 111\ 96 + 81\ 99 + 318 \sin\theta + 0 = 0 \qquad (2)$$

Solving equation (2), we get

Departure of BC $= 318\ 0 \sin\theta = +313\ 05$

$$\text{or}\quad \theta = \sin^{-1}\frac{313\ 5}{318\ 0}$$

$$= 78°55' \quad \text{or} \quad 100°\,5'$$

Substituting the value of θ in equation (1), we have

Latitude of CD $= l = -14\ 34 + 132\ 18 + 201\ 42 + 55\ 68$

$= +374\ 94$

Length of CD $= 374\ 94$ m

Bearing of BC with respect to the assumed meridian $= 100°\ 5$ N E

„ of BC with the magnetic meridian $= 100°5' - 37°36'$
$= 62°\ 29$ N E $=$ N $62°\ 29$ E

Example 3 —Below are tabulated the measured lengths and bearings of the sides of a closed traverse ABCDE (Fig 5) together with the latitudes and departures of the known sides The lengths of BC and CD could not be measured

No	Line	Length in m	Bearing	Latitude	Departure
1	AB	217 5	S 59°45 E	—109 578	—187 872
2	BC	?	N 6°30′ E	?	?
3	CD	?	N 37°36 W	?	?
4	DE	283 50	S 55°18 W	—161 397	—233 076
5	EA	173 15	S 2°40 W	—172 989	— 8 055

Calculate the omitted measurements

As in example 2 latitude of the closing line BD $= -\Sigma L$
$= +443\ 964$

Departure of the closing line BD $= -\Sigma D = +\ 53\ 259$

Reduced bearing of the closing line BD = N $6°\ 50$ E and length of the closing line BD $= 447\ 15$ m

In Fig 5, let the angles DBC, BCD, and CDB of the $\triangle$ BCD be denoted by θ_1, θ_2, and θ_3 respectively. They may be *obtained from the known bearings* of BD, BC, and CD.

$$\angle DBC = \theta_1 = \text{R. B of BC} - \text{R. B. of BD}$$
$$= 62° 30' - 6° 50' = 55°40'.$$

$$\angle BCD = \theta_2 = 180° - \text{R B. of } CB - \text{R. B. of } CD$$
$$= 180° - 62°30' - 37°36' = 79°54'.$$

$$\angle CDB = \theta_3 = \text{R B of DC} + \text{R B. of DB}$$
$$= 37°36' + 6°50' = 44°26'.$$

Check :—$\theta_1 + \theta_2 + \theta_3 = 55°40' + 79° 54' + 44°26' = 180°$.

Knowing the length of BD and the angles θ_1, θ_2, and θ_3, the lengths *of* BC *and* CD *may be calculated by the Sine rule*

$$BC = BD \frac{\sin \theta_3}{\sin \theta_2} = 447\ 15 \frac{\sin 44°26'}{\sin 79°54'} = 318\ 015 \text{ m.}$$

$$CD = BD \frac{\sin \theta_1}{\sin \theta_2} = 447 \cdot 15 \frac{\sin 55°40'}{\sin 79°54'} = 375 \cdot 18 \text{ m.}$$

Example 4 :—Given the following observed lengths and *bearings of the sides of a closed traverse* ABCDE *(Fig 5) together* with the latitudes and departures of the known sides, the bearings of BC and CD having been omitted :

No.	Line	Length in m	Bearing.	Latitude	Departure.
1	AB	217·5	S 59° 45′ E	—109 578	—187 872
2	BC	318 0	?	?	?
3	CD	375 0	?	?	?
4	*DE*	263 5	*S* 55°18 W.	—161 397	—233 076
5	EA	173 15	S 2°40 W	—172 989	— 8 055

Find the bearings of BC and CD.

Proceeding similarly as in example 3 to obtain the length and bearing of the closing line BD, we get

Length of BD = 447·15 m and R B. of BD = N. 6°50′ E.

The area of the triangle BCD (Fig 5) should now be calculated from the formula $\triangle = \sqrt{s(s-a)(s-b)(s-c)}$ the lengths of the sides being known.

Let a = the length of the closing line BD.
b = „ of BC.
c = „ of CD
θ_1 = the angle DBC
θ_2 = „ „ BCD
θ_3 = the angle CDB
s = the semi-sum of a, b, and c
$\triangle$ = the area of the triangle BCD

Then $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(447\ 15 + 318\ 0 + 375\ 0) = 570\ 08$

$s - a = 570\ 08 - 447\ 15 = 122 \cdot 93.$

$s - b = 570\ 08 - 318 = 252\ 08$

$s - c = 570 \cdot 08 - 375 = 195\ 08.$

$\therefore\ \triangle = \sqrt{570\ 08 \times 122 \cdot 93 \times 252 \cdot 08 \times 195 \cdot 08}$

or $\log \triangle = 4 \cdot 7687$

$\triangle$ is also equal to $\frac{1}{2}ab \sin\theta_1 = \frac{1}{2}bc \sin\theta_2 = \frac{1}{2}ca \sin\theta_3$

or $\triangle = \frac{1}{2}(447\ 15 \times 318\ 0 \sin\theta_1) = \frac{1}{2}(318 \cdot 0 \times 375\ 0 \sin\theta_2)$
$= \frac{1}{2}(375\ 0 \times 447\ 15 \sin\theta_3)$

$$\therefore\quad \sin\theta_1 = \frac{2\triangle}{447\ 15 \times 318\ 0};\quad \sin\theta_2 = \frac{2\triangle}{318\ 0 \times 375\ 0};$$

$$\sin\theta_3 = \frac{2\triangle}{375\ 0 \times 447\ 15}.$$

or $\theta_1 = 55°\ 39'$, $\theta_2 = 79°\ 54'$; $\theta_3 = 44°\ 26'$

From the known bearing of BD and the angles θ_1 and θ_3, the bearings of BC and CD may be found.

R B of BC = R. B of BD $+\theta_1 = 6°\ 50' + 55°\ 39'$
$= 62°\ 29'$ N. E.
= N $62°\ 29'$ E.

R. B of DB = S. $6°\ 50'$ W

Now R B of DC $= \theta_3 -$ R. B. of DB.
$= 44°26' - 6°50' = 37°36'$ S E

$\therefore$ R. B of CD $= 37°36'$ N. W. = N. $37°\ 36'$ W.

Example 5.—The following are the measured lengths and bearings of the sides of a closed traverse ABCDE (Fig. 6) together with the latitudes and departures of the known sides, the bearing of AB and the length of CD having been omitted:

No	Line	Length in m	Bearing	Latitude.	Departure.
1	AB	217·5	?	?	?
2	BC	318·0	N 62° 30′ E	+146 835	+282 072
3	CD	?	N 37° 36′ W	?	?
4	DE	283 5	S 55° 18 W	−161 397	−233 076
5	EA	173 15	S 2° 40′ W.	−172 989	− 8 055

Calculate the missing measurements.

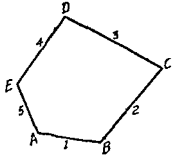


Fig 6 a

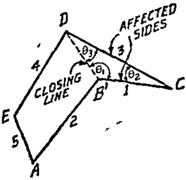


Fig 6 b

From Fig. 6a, it may be seen that the affected sides (lines 1 and 3) are not adjoining However, they may be brought into the same triangle by shifting the intervening known side (line 2) parallel to itself as shown in Fig. 6b. The closing line B′D closes the polygon formed by the known sides (lines 4, 5, and 2).

The algebraic sum of the known latitudes $= \Sigma L = -187 \cdot 551$

" " " " departures $= \Sigma D = +40 \cdot 941$

Latitude of the closing line B D $= -\Sigma L = +187\ 551$

Departure of „ „ „ $= -\Sigma D = -\ 40\ 941$

The signs of the latitude and departure being plus and minus respectively, the closing line B D lies in the fourth (N W) quadrant

Now $\tan\theta = \dfrac{40\ 941}{187\ 551}$, where θ is the reduced bearing of B'D

or $\theta = 12°\ 19$, i e R B of B'D = N 12°19′ W.

Length of B D = 187 551 sec 12°19 = 191 97 m

In the $\triangle$ DB C let the angles DB C B CD and CDB′ be denoted by θ_1, θ_2, and θ_3 respectively

Now reduced bearing of CD (line 3) = N 37° 36′ W

„ „ of DC = S 37° 36 E

Similarly „ „ of DB′ = S 12° 19 E

Hence $\angle$CDB $= \theta_3 =$ R B of DC − R B of DB

$= 37°36' - 12°19 = 25°17$

Knowing the lengths of B D and B C (line 1) and the angle θ_3, the angles θ_1 and θ_2 may be found

$$\sin\theta_2 = \frac{B\,D}{B\,C}\sin\theta_3 = \frac{191\ 97}{217\ 5}\sin 25°17' \text{ or } \theta_2 = 22°\ 9'.$$

Now $\angle$DB C $= \theta_1 = 180° - \theta_2 - \theta_3 = 180° - 22°9 - 25°17$

$= 132°\ 34$

Bearing of B C $= \theta_1 -$ R B of B D $= 132°\ 34' - 12°19'$

$= 120°\ 15$

or R B of B C (line 1) = S 59° 45 E

Length of CD (line 3) $=$ B'D $\dfrac{\sin\theta_1}{\sin\theta_2} = 639\ 9\ \dfrac{\sin 132°34'}{\sin\ 22°9}$

$= 375\ 0$ m

Example 6 —Below are given the total latitudes and departures of two stations A and B, referred to the origin of the system

Station	Total latitude.	Total departure.
A	+ 668 ·6	− 342 ·4
B	+ 820 2	+ 602 ·3

A point M is fixed by measuring a distance of 525 m from A on a bearing of N. 20° 12′ W., and a line MN 1234 m long is set out parallel to AB from M. Calculate the bearing of N from B

(i) The consecutive co-ordinates of A with respect to B may be obtained by subtracting the total latitude of B from that of A, and the total departure of B from that of A. Thus we have

Total lat of A = +668 ·6	Total dep. of A = − 342 ·4
Deduct „ „ of B = 820 ·2	Deduct „ „ of B = − 602 ·3
Latitude of BA = −151 ·6	Departure of BA = − 944 7

(ii) If θ be the reduced bearing of BA, $\tan\theta = \dfrac{944\ 7}{151 \cdot 6}$

$\therefore\ \theta = 80° 53'$ S. W. Hence, R. B. of AB = N. 80° 53′ E.

Since MN is parallel to AB, R. B. of MN = N 80° 53′E.

(iii) Now latitude of AM = 525 cos 20°12′ = + 492 ·7m.
„ of MN = 1234 cos 80°53′ = + 195 ·5 „
Departure of AM = 525 sin 20°12′ = − 181 3 „
„ of MN = 1234 sin 80°53′ = + 1218 ·0 „

(iv) Hence total latitude of N with respect to B
$= \Sigma L = -151 \cdot 6 + 492 \cdot 7 + 195 \cdot 5 = +536 \cdot 6$ m.

Total departure of N with respect to B
$= \Sigma D = -944\ 7 - 181 \cdot 3 + 1218 = +92$ m.

(v) R. B. of BN $= \tan^{-1}\dfrac{92}{536\ 6} = 9°44'$N.E. i.e. N. 9°44′ E

Example 7 :—Pegs were driven in the centre line of a railway on either side of wood To determine the distance AB, the following traverse was run from A to B along the side of wood :

Line	Length	Bearing	Line.	Length.	Bearing
AC	250 m	190°12′	DE	212 m	156°48′
CD	156 „	108°24′	EB	160 „	76°36′.

Compute the distance AB. From the traverse station D, a line DF is carried into wood on a bearing of N. 60°20′ E in order to locate an intermediate point F on AB Find the length of DF.

(i) The latitudes and departures of the lines of the traverse are:

Line.	Northing.	Southing.	Easting.	Westing.
AC		246 08		110·7
CD		49·24	148 04	
DE		194 88	83·48	
EB	37 08		155·64	
	37·08	490·10	387 16	44·28

∴ Total latitude of B with respect to A

$$= \Sigma L = -490\ 20 + 37\cdot08 = -453\cdot12.$$

" departure of B with respect to A

$$= \Sigma D = +387\cdot16 - 44\cdot28 = +342.88$$

(ii) $\text{R. B. of AB} = \tan^{-1}\dfrac{342\cdot88}{453\cdot12} = 37°\ 6'\ \text{S. E.}$

Length of $\text{AB} = 455\cdot12\ \sec\ 37°\ 6' = 568\cdot4$ m.

(iii) Now total latitude of D with respect to A

$$= \Sigma L = -246\ 08 - 49\cdot24 = -295\cdot32$$

Total departure of D with respect to A

$$= \Sigma D = -44\ 28 + 148\cdot04 = +103\cdot76$$

$\therefore$ $\text{R. B. of AD} = \tan^{-1}\dfrac{103\cdot76}{295\cdot32} = 19°\ 22'\ \text{S. E.}$

Length of $\text{AD} = 295\cdot32\ \sec\ 19°\ 22' = 313\cdot08$ m

(iv) In the triangle ADF, $\angle \text{FAD} = \text{R.B. of AB} - \text{R. B. of AD}$

$$= 37°\ 6' - 19°\ 22' = 17°\ 44'.$$

$\angle \text{DFA} = 180° -$ (sum of reduced bearings of FA and FD)

$$= 180° - (37°\ 6' + 60°\ 20') = 82°\ 34'$$

By the Sine rule, length of $\text{DF} = \dfrac{\text{AD}\sin\text{FAD}}{\sin\text{DFA}} = \dfrac{782\ 7\ \sin 17°\ 4}{\sin 82°\ 34'}$

$$= 96\cdot16 \text{ m}$$

Example 8 —From the following traverse, calculate the length of CD so that A, D, and E are in one straight line

Line	Length	Bearing	Line	Length	Bearing
AB	320 m	N 80° 30 E	CD	—	N 12° 0 W
BC	500 ,,	N 30° 15 E	DE	610 m	N 16° 45 E

(i) Latitude of AB $= 320 \cos 80°30 = +\ 52\ 81$ m

,, of BC $= 500 \cos 30°15' = +432\ \ 0$ m

Departure of AB $= 320 \sin 80°30 = +351\ \ 6$ m

,, of BC $= 500 \sin 30°15 = +251\ \ 9$ m

Total latitude of C with respect to A

$$= \Sigma L = +52\ 81 + 432\ 0 = +484\ 81 \text{ m}$$

Total departure of C with respect to A

$$= \Sigma D = +315\ 6 + 251\ 9 = +567{\cdot}5 \text{ m}$$

(ii) If θ be the reduced bearing of AC

$$\tan\theta = \frac{567\ 5}{484\ 81} \qquad \theta = 49°\ 30'$$

Hence, R B of AC = N 49°30 E and length of AC
$= 567\ 5 \operatorname{cosec} 49°\ 30 = 746\ 4$ m

(iii) Since A, D, and E lie in one straight line, the bearing of AD is the same as that of DE i e equal to N 16° 45 E

Now in the triangle DAC, $\angle$DAC = R B of AC—R B of AD
$= 49°30 - 16°45' = 32°\ 45'$

$\angle$ADC = R B of DC + R B of DA $= 12°\ 0 + 16°45' = 28°\ 45'$

By the application of the Sine rule we get

$$\text{Length of CD} = \frac{\text{AC} \sin \text{DAC}}{\sin \text{ADC}} = \frac{746\ 4 \sin 32°45}{\sin 28°45} = 839\ 5 \text{ m}$$

Partition of Land —Several problems are involved in the division of a given tract into two or more parts They may be solved by the application of methods of determining omitted measurements However, few common cases will now be considered

(1) **To Cut off a Required Area by a Line through a Given Point**.—In Fig 7, ABCDEF represents a polygon, the

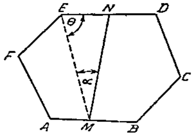


Fig 7

lengths and bearings of whose sides are known; MBCDN the required area cut off from the polygon by a line MN through a given point M on the side AB. It is required to determine the correct position of the dividing (or cut off) line MN It is here presumed that the corrected latitudes and departures of the sides of the polygon are given If the field measurements are given, the polygon may be balanced. It is assumed that the figure is drawn roughly to scale The procedure is as follows:—

(i) Calculate the area of the polygon ABCDEF by the D M D method or by the method of independent co-ordinates.

(ii) Join M to the nearest corner E of the polygon Calculate the latitude, departure, length, and bearing of EM of the closed traverse MBCDE as explained in case 1 on page 4

(iii) Compute the area of the closed traverse MBCDE by the D. M D method, and find the difference between this area and the required area This difference is represented by the triangle MNE

(iv) Determine the angle NEM (θ) from the known bearings of DE and EM. Knowing the length of EM, the angle NEM (θ), and the area of the triangle MNE, calculate the length of EN from the relation

$$\text{Area of} \triangle \text{MNE} = \tfrac{1}{2}\text{EN} \times \text{EM} \times \sin\theta \text{ or } \text{EN} = \frac{2 \times \text{area of} \triangle \text{MNE}}{\text{EM} \sin\theta}$$

(v) Knowing the lengths of EN and EM, and the angle NEM, find the angle EMN (α) and length of MN from the relations

$$\alpha = \tan^{-1} \frac{EN \sin\theta}{(EM - EN \cos\theta)}$$

$$\text{and } MN = \frac{EN \sin\theta}{\sin\alpha} = \frac{EM \sin\theta}{\sin(\theta + \alpha)}$$

$$\text{since } \frac{EN}{\sin\alpha} = \frac{EM}{\sin\{\pi - (\theta + \alpha)\}}$$

(vi) Calculate the bearing of MN from the known bearing of EM and the angle EMN (α)

(vii) Check the computations by computing the area of AMNEF, which should equal the difference between the area of ABCDEF and the required area

The line MN is established in the field by measuring its length in the required direction Both field work and computations are checked, if the point N thus established falls on the line DE, and if the measured distance EN or DN is equal to calculated distance

(2) To Cut of a Required Area by a Line Running in a Given Direction —In Fig 8, ABCDEF represents a polygon, the

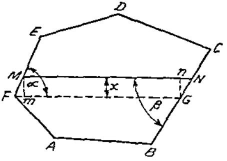


Fig 8

lengths and bearings of the sides of which are known It is to be divided into two parts, each of the required area, by a line MN running in a given direction It is required to determine the exact position of MN It is assumed that the figure is drawn roughly to scale

Procedure —(i) Draw a line FG in the given direction (parallel to MN) through the corner F nearest the dividing line MN

(ii) Calculate the area of the polygon ABCDEF by the D M D method or by the method of independent co ordinates

(iii) From the known lengths and bearings of FA and AB, and the known bearings of BG and GF of the closed traverse FABG, compute the lengths of BG and GF as explained in case 3 on page 6

(iv) Find the area of FABG by the D M D method This area will be less than the required area MFABN as shown in the figure, the difference between the two areas being represented by the trapezoid FGNM The area of FGNM must, therefore, be added to the calculated area of FABG

(v) The bearings of EF, FG, BC, and MN being known, calculate the angles EFG (α) and BNM (β)

(vi) Compute the area of the trapezoid FGNM

Area of FGNM $= \frac{1}{2} x\,(FG + MN)$ where x is the perpendicular distance between FG and MN

Now $MN = FG - x \cot \alpha + x \cot \beta$

$$\text{Area of FGNM} = \tfrac{1}{2} x \left\{ 2\,FG - x\,(\cot \alpha - \cot \beta) \right\}$$

or $$= FG \times x - \frac{x^2}{2} (\cot \alpha - \cot \beta)$$

The solution of this equation gives the value of x

(vii) Determine the lengths FM and GN from the relations $FM = x \operatorname{cosec} \alpha$ and $GN = x \operatorname{cosec} \beta$

(viii) Check the computations by finding the area of ABNMF, which should agree with its required area

The points M and N are located in the field on the lines EF and BC by measuring their calculated distances FM and CN respectively, and the line MN is then measured A complete check is obtained both on field work and computations if the measured length of MN agrees with its calculated length

If it is required to cut off a given area from an irregular tract by a line running in a given direction, the procedure is the same

as explained in the preceding case except that the area of the part cut off by the trial line is found by a planimeter, and that the strip between the trial line and the true dividing line representing the excess or deficiency of area may be considered as a trapezoid, the irregular sides of the strip being assumed to be straight ones.

(3) Given an area ABCDE. Required to divide it into two parts by a line MN perpendicular to AB and so located that the part AMNE shall contain a specified area (Fig.9)

To solve the problem, the unknown distance FM must be

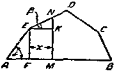


Fig. 9

determined. Draw EF and EK parallel to NM and AB respectively. Let the angles EAF and NEK be denoted by α and β, the distance FM by x, and the area specified for AMNE by Δ Then

$$AF = AE \cos\alpha \,; \quad EF = AE \sin\alpha;$$

$$AM = AF + FM = AE \cos\alpha + x$$

Area of EFMN = area of AMNE − area of EAF

or $\quad = \Delta - \frac{1}{2}\, AF \times EF = \Delta - \frac{1}{2}\, AE^2 \cos\alpha \sin\alpha$

Now area of EFMN = area of EFMK + area of EKN

$$= EF \times x + \tfrac{1}{2}\, EK \times NK = AE \sin\alpha \times x + \tfrac{1}{2}\, x^2 \tan\beta.$$

Let the area of EFMN be denoted by Δ_1. Then

$$2\,\Delta_1 = 2\, AE \sin\alpha \times x + x^2 \tan\beta$$

or $\quad x^2 + \left(\dfrac{2AE \sin\alpha}{\tan\beta}\right) x - \dfrac{2\,\Delta_1}{\tan\beta} = 0$

The solution of this equation gives the required value of x. The problem may also be solved by the application of the method of determining omitted measurements.

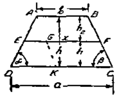


Fig 10

(4) To divide a trapezoid into two parts whose areas shall be in a given ratio, by a line parallel to the bases (Fig. 10).

In Fig. 10, ABCD is the trapezoid, which is to be divided into two parts, and EF the dividing line parallel to the bases AB and DC

Let Δ = the area of the trapezoid ABCD.
Δ_1 = the area of the part EFCD.
Δ_2 = the area of the part ABFE.
$m : n$ = the given ratio of the areas of the two parts.
b = the length of AB
a = the length of DC.
x = the length of EF.
h = the altitude of the trapezoid ABCD.
h_1 = the altitude of the part EFCD.
h_2 = the altitude of the part ABFE
α = the angle ADC.
β = the angle BCD.

Then $\dfrac{\Delta_1}{\Delta_2} = \dfrac{m}{n}$, $\Delta = \Delta_1 + \Delta_2$ $\therefore$ $\Delta_1 = \dfrac{m}{m+n}\Delta$;

$\Delta_2 = \dfrac{n}{m+n}\Delta$, $h = h_1 + h_2$; DK $= a - b$; EG $= x - b$.

From similar triangles ADK and AEG, $\dfrac{h}{h_2} = \dfrac{a-b}{x-b}$

$$\therefore \quad h_2 = \left(\frac{x-b}{a-b}\right)h \text{ and } h_1 = h - h_2 = h - \left(\frac{x-b}{a-b}\right)h$$

$$= \left(\frac{a-x}{a-b}\right)h.$$

Also, $(a - b) = h\ (\cot\alpha + \cot\beta)$

or $$h = \frac{a-b}{(\cot\alpha + \cot\beta)}.$$

Now area of the trapezoid ABCD $= \Delta = \frac{1}{2}(a + b)\,h$

$$= \tfrac{1}{2}\left(\frac{a^2 - b^2}{\cot\alpha + \cot\beta}\right).$$

" " EFCD $= \Delta_1 = \frac{1}{2}(a + x)\,h_1$

$$= \tfrac{1}{2}\left(\frac{a^2 - x^2}{\cot\alpha + \cot\beta}\right).$$

But $\quad \Delta_1 = \dfrac{m}{m+n}\,\Delta$

Substituting the values of Δ and Δ_1, we get

$$\tfrac{1}{2}\left(\frac{a^2 - x^2}{\cot \alpha + \cot \beta}\right) = \frac{m}{m+n} \times \tfrac{1}{2}\left(\frac{a^2 - b^2}{\cot \alpha + \cot \beta}\right)$$

or $a^2 - x^2 = \dfrac{m}{m+n}(a^2 - b^2)$

Whence, $x = \sqrt{\dfrac{mb^2 + na^2}{m+n}}$

The triangles AEG and ADK being similar,

$$\frac{AE}{AD} = \frac{EG}{DK} = \frac{x-b}{a-b} \qquad AE = \left(\frac{x-b}{a-b}\right) AD$$

Also $DE = AD - AE = AD - \left(\dfrac{x-b}{a-b}\right) AD = \left(\dfrac{a-x}{a-b}\right) AD$

PROBLEMS

1 Distinguish between traversing and triangulation and state under what circumstances you would adopt each

The following traverse is carried round an obstruction in a line AE

Line	Length in m	Bearing
AB	425	38° 24
BC	520	348° 0'
CD	605	300° 24
DE	430	30° 48

It is required to peg a point F midway between A and E Compute the length and bearing of CF

(Ans 243 4 m 250° 2')

2 The notes taken in the field of part of a traverse are recorded as under

Line	Length in m	Bearing
AB	405	N 12° 24 E
BC	376	N 15° 36 W
CD	530	N 20° 12' W

There is a point P which is inaccessible Its bearing from A is N 46° 48 W, and from D the bearing of P is S 40° 18 W Calculate the distance of P from A and D

(Ans PA = 963 8 m PD = 781 2 m)

3 Discuss the relative merits of the different methods of traverse survey with a theodolite What checks can be applied to a closed traverse? What do you understand by the closing error? Explain how it is adjusted

4 The following lengths and bearings were recorded in running a theodolite traverse ABCD There are obstacles which prevent direct measurement of the bearing and length of the line AD

Line	Length in m	Bearing
AB	485	341° 48
BC	1725	16° 24
CD	1050	142° 6

Calculate the length and bearing of AD

(Ans 1618 m 37° 18)

5 Given the following latitudes and departures of a traverse ABCDE, the bearings of AB and EA having been omitted

Line	Latitude	Departure	Length in ft
AB	?	?	(1970)
BC	+841 11	+336 71	
CD	+877 18	-311 74	
DE	-700 60	-728 88	
EA	?	?	(1181)

Determine the bearing of AB and EA

(Ans 110°49 254° 24)

6 The following lengths and bearings were recorded in running a theodolite traverse in the counterclockwise direction the lengths of CD and DE having been omitted

Line	Length in m	Bearing
AB	980	0° 0′
BC	675	N 25° 12 W
CD	?	S 75° 6 W
DE	?	S 56° 24 E
EA	700	N 35° 36 E

Calculate the lengths of CD and DE

(Ans. 2491 m 2746 m)

7 For the following traverse compute the length of CD so that A and E may be in one straight line.

Line	Length in m	Bearing in degrees.
AB	340	85
BC	506	32
CD	—	350
DE	622	18

(Ans. 927 4 m.)

8 The following lengths and bearings were recorded in running a theodolite traverse in the counterclockwise direction, the length of CD and bearing of DE having been omitted :

Line.	Length in m	W. C B.
AB	1970	110° 49′
BC	906	21° 49′
CD	?	340° 26
DE	1011	?
EA	1181	254° 24′

Determine the length of CD and the bearing of DE. (U. B).
(Ans 930 9 m., 226° 9′).

9. A and B are two stations whose co-ordinates are as given below .

Station.	North Co ordinate.	East Co ordinate.
A	1056·9	585·1
B	1426·5	992·7

From A is run a line AC, 154 4 m. in length, on a bearing of 132° 18′, and from C is run a line CD, of length 544 0 m, parallel to AB. Find the length and bearing of BD

(Ans. 371·76 m , 72° 12′.)

10 In order to determine the distance of an inaccessible point P from station A, a straight line BAC is run, AB and AC being 260 m and 200 m respectively The angles PBA and PCA were found to be 74° 30′ and 62° 15′ respectively Determine the distance AP.

(Ans. 581 5 m.)

(*Hint* Calculate BP from the triangle BPC. Assuming AB as the meridian, find the total latitude and departure of P with respect to A, and then calculate AP).

11. The following traverse is run round a lake :

Line.	Length in m	Bearing.	Line.	Length in m.	Bearing
AB	375	220° 26′	DE	192	15° 36′
BC	258	265° 0′	EF	180·6	47° 28′
CD	216	225° 43′	FG	274·2	78° 9′

A line KL is to be set out parallel to AG, 45 m apart, K and L being the points on the lines AB and FG respectively. Calculate the distances AK and GL. (Ans. 46·41 m ; 49·08 m.).

+ + +

CHAPTER II

ADJUSTMENT OF THE TRANSIT THEODOLITE

There are two kinds of adjustments of a surveying instrument viz (1) Temporary and (2) Permanent The temporary adjustments are those which are made at every set up of the instrument prior to taking observations while the permanent adjustments are those which establish the fixed relationships between the fundamental lines of the instrument. When once made, they remain permanent for long periods

Temporary Adjustments of Theodolite

The temporary adjustments of the theodolite are three, viz (1) Setting up the instrument, (2) Levelling up, and (3) Focussing the eyepiece and object glass (Elimination of parallax)

(1) **Setting up the Theodolite** —This includes two operations viz (a) centering the instrument over the station mark such as a tack in a station peg and (b) approximately levelling it by the tripod legs only

Centering the Instrument —For centering the instrument a plumb bob is suspended from the hook and chain beneath the instrument (i) Set up the instrument on firm ground in such a position that the plumb bob is approximately over the station point (ii) Move the legs radially and sideways so that the plumb bob is exactly over the tack and at the same time the tribrach sprang is approximately horizontal It may be noted that moving the leg radially shifts the plumb bob in the direction of the leg without seriously affecting the plate levels while moving the leg circumferentially or sidewavs tilts the instrument considerably without seriously disturbing the plumb bob Centering can be done more conveniently and rapidly by means of a centering device (e g centering plates)

(2) **Levelling the Instrument** —The instrument is levelled by means of the levelling (or foot) screws with reference to the plate bubbles To do this (i) turn the upper plate until one of the bubble tubes is parallel to the line joining any pair of levelling screws The other bubble tube will then be parallel to the line joining the third levelling screw and the mid point of the line joining the first pair (ii) Bring the bubble to the centre of its run by turning both screws simultaneously and evenly (remembering the rule right in and left out ') Similarly, bring the other bubble to its mid position by turning the third levelling screw (iv) Repeat the process until finally both bubbles are exactly centred Now rotate the instrument about its vertical axis Each bubble will now traverse provided the plate levels are in correct adjustment The vertical axis will then be truly vertical

Note —In the case of a four screw levelling head, one of the bubble tubes should be placed parallel to a pair of diagonally opposite screws The other tube will then be parallel to the other pair

(3) **Focussing the Eyepiece and Object Glass** —The object of this adjustment is to make the foci of the eyepiece and object glass coincide with the plane of cross hairs i e to eliminate parallax It is made in two steps

(a) **Focussing the Eyepiece** —The object of focussing the eyepiece is to make the cross hairs distinct and clear To do this, point the telescope towards the sky or hold a sheet of white paper in front of the object glass and move the eyepiece in and out until the cross hairs are seen quite distinctly and clearly

(b) **Focussing the Object Glass** —The object of focussing the object glass is to bring the image of the object formed by the object glass in the plane of the cross hairs Otherwise there will be an apparent movement of the image relatively to the cross hairs when the observer moves his eye the apparent movement being called *parallax* To eliminate it direct the telescope towards the object and turn the focussing screw until the image appears clear and sharp (i e in sharp focus) It must be noted that the correct position of the eyepiece depends only

upon the eyesight of the observer It is, however, necessary to use the focussing screw whenever the distance of the object from the instrument is changed

Permanent Adjustments of Theodolite

The fundamental lines of the theodolite are

(1) The vertical axis

(2) The axes of the plate levels

(3) The line of collimation (or the line of sight)

(4) The horizontal axis (also called the transverse or trunnion axis)

(5) The bubble line of the altitude (or azimuthal) level

Conditions of Adjustment —When the instrument is in perfect adjustment, the following relations should exist.

(1) The axes of the plate levels must be perpendicular to the vertical axis

(2) The line of collimation must be at right angles to the horizontal axis

(3) The horizontal axis must be perpendicular to the vertical axis

(4) The bubble line or the axis of the telescope level must be parallel to the line of collimation

(5) If the instrument has a fixed vernier for the vertical circle the vernier must read zero when the instrument is levelled (i e when the plate levels and the telescope level are centred)

(6) If the instrument is provided with a striding level, the axis of the striding level must be parallel to the horizontal axis

The permanent adjustments of the theodolite consist of the following

(1) Adjustment of the plate levels, (2) Adjustment of the line of collimation (or collimation adjustment), (3) Adjustment of the horizontal axis, (4) Adjustment of the level tube

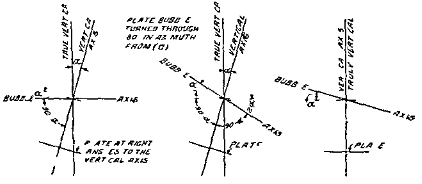
on the telescope, (5) Adjustment of the vertical index frame Since certain adjustments will upset others the adjustments must be made in the order in which they are stated

For making the adjustments, the instrument should be set up at a fairly level p ace where sights of about 100 m can be taken in either direction in the same straight line

Preliminary Adjustment —*To make the diaphragm truly erect* The object of this adjustment is to ensure that the horizontal and vertical hairs are truly horizontal and vertical This adjustment is not necessary in the ease of a modern telescone It is made as follows —

(i) Having levelled the instrument carefully, sight a distant well defined point such as the top of a spire, and with both motions clamped, rotate the telescope in azimuth by means of one of the tangent screws If the horizontal cross hair remains in contact with the point, the adjustment is correct Alternatively, move the telescope through a small vertical angle If the point travels continuously on the vertical hair the adjustment is correct. If not, loosen the diaphragm screws and rotate the diaphragm ring Repeat the test and adjustment until perfect Then carefully tighten the screws

First Adjustment —*To make the axes of the plate level perpendicular to the vertical axis* (Figs 11a 11b and 11c)



a b c

Fg 11

Necessity —If this condition exists the vertical axis will be truly vertical and the horizontal circle and the trunnion

(or horizontal) axis will both be truly horizontal when each plate bubble is in the centre of its run The trunnion axis is required to be horizontal in all work involving vertical movement of the telescope

Test —(i) Set up the instrument on firm ground Clamp the lower motion (or lower plate) and turn the upper plate until the longer plate bubble is parallel to any pair of levelling screws Bring each plate bubble to the centre of its run by means of levelling screws (Temporary adjustment)

(ii) Rotate the instrument about the vertical axis through 180° The plate bubble is again parallel to the pair of levelling screws, but reversed in direction If the bubbles remain central the axis of each plate level tube is perpendicular to the vertical axis and the vertical axis is truly vertical.

Adjustment — If not, note the deviation of the bubble (say, n divisions) Bring each bubble half way back (through $\frac{n}{2}$ divisions) by means of the two capstan headed screws at the end of the tube Bring each bubble to the centre of its run by means of the respective levelling screws

Repeat the test and adjustment until both bubbles traverse during a whole revolution of the instrument

Alternative Method —In this method the altitude bubble is used in making this adjustment to ensure greater accuracy, since it is much more sensitive than the plate bubbles

Procedure —

(a) Clamp the vertical circle at zero Revolve the instrument until the altitude bubble (fixed on the T frame or on the telescope) is parallel to the line joining any pair of levelling screws

(b) Bring the bubble to the centre of its run by turning these screws Turn the telescope through 90° and bring the bubble to the centre of its run by means of the third levelling screw Repeat until the bubble remains central in these two positions

(c) Turn the telescope through 180° in azimuth If the bubble does not remain central note the deviation (say, n divisions) of the bubble

Adjustment —(d) Correct one half of the deviation $\left(\frac{n}{2}\text{divisions}\right)$ by means of the clip screws or the vertical circle tangent screw, and the remaining half by means of the same pair of levelling screws

(e) Turn the telescope through 90° until the bubble is over the third levelling screw and bring it to the centre of its run by turning the third levelling screw only The bubble should now remain central when the telescope is turned through a complete revolution in azimuth If not, repeat the process until perfect

(f) The vertical axis is now truly vertical Bring each plate bubble to the centre of its run by means of the capstan headed screws at the end of the tube

When this adjustment is made, all the bubbles will traverse during a complete revolution of the instrument and the vertical axis will be truly vertical

It should here be noted that when the bubble is reversed end for end, the deviation of the bubble called the apparent error is twice the actual error in the axis of the level and, therefore, the correction is only half the amount of the apparent error

After the adjustment is completed, clamp the upper motion (or vernier plate) and loosen the lower motion (or lower plate). On repeating the test if it is found that the bubbles do not traverse on reversal, the outer axis is not vertical and is not, therefore, parallel to the inner axis The instrument then needs repairs if the error is large It may be noted that if the axes are not parallel, no error will be caused in the measurement of horizontal angles provided the angles are not measured by repetition, and the plate bubbles are adjusted perpendicular to the inner axis.

Second Adjustment —*To make the line of collimation coincide with the optical axis of the telescope* (To place the intersection of the cross hairs in the optical axis of the telescope) If there are two inclined hairs instead of a single vertical hair, their intersection is adjusted as for a vertical hair This adjustment is made in two steps, viz (1) adjustment of the horizontal hair, and (2) adjustment of the vertical hair

Adjustment of the Horizontal Hair—(*Fig* 12) To make the line of collimation in so far as defined by the horizontal hair coincide with the optical axis

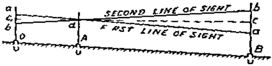


Fig 12

Necessity—The object of this adjustment is to place the horizontal hair into the plane of motion of the optical centre of the object glass (i e to bring the horizontal hair into the horizontal plane through the optical axis), the movement of the object glass being assumed along the optical axis If the horizontal hair is not in the optical axis the direction of the line of sight will change slightly when the objective is moved in and out for focussing This adjustment is necessary only when the instrument is used for measuring vertical angles or when it is used for levelling operations It is immaterial in measurements of horizontal angles

Test—(i) Drive two pegs at O and B at a distance of about 100 m apart Fix a third peg at A in line with O and B and at a distance of about 10 m from O Set up the theodolite at O and level it accurately

(ii) With the telescope direct, take readings on the staff held on A and B Let the readings be A*d* and B*a*

(iii) Transit the telescope and swing it through 180° Set the line of sight to the former staff reading A*d* on the near peg A

(iv) Again read the staff held on B If this staff reading is the same as the former staff reading (B*a*) on B, the adjustment is correct

Adjustment—If not, let the staff reading be B*b* Find the mean of the two staff readings B*a* and B*b* and call it B*c* Bring the horizontal hair to the mean reading B*c* by means of the vertical diaphragm screws Repeat till perfect

Alternative Method :—(Fig. 13). In this method the vertical angle is noted when a staff reading is taken on the distant peg instead of taking a reading on the near peg.

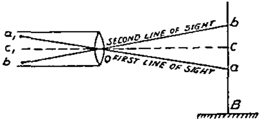


Fig 13

(i) Set up the theodolite at a convenient point and level it accurately.

(ii) With the telescope direct, take a reading on the staff held on the peg B driven at about 100 m from the instrument station (O), and note the vertical angle (α). Let the staff reading be B*a*.

(iii) Plunge (or transit) the telescope and turn through 180° in azimuth. Set the vertical vernier to the former angle (α) and again take a staff reading on B. If the staff reading agrees with the previous reading B*a*, no adjustment is necessary.

Adjustment :—(iv) If not, let the second staff reading be B*b*. Move the horizontal hair by means of the vertical diaphragm screws until the mean (Bc) of the two readings B*a* and B*b* is obtained.

Adjustment of the Vertical Hair . (Fig. 14) *To make the line of collimation perpendicular to the horizontal axis.*

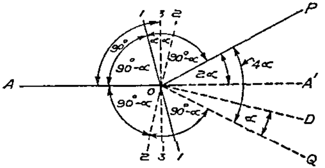


Fig. 14

1—1 : First Position of Horizontal Axis.
2—2 : Second .. .. ..
3—3 : Correct .. .. ..

Necessity — If this condition obtains the line of collimation will generate a plane when the telescope is transitted But if not it will generate a cone, the axis of which is the horizontal axis The adjustment is necessary when a line is to be prolonged either by transitting or changing the inclination of the telescope, or when a horizontal angle between two points at different elevations is to be measured

Test —(i) Set up the instrument at a convenient point O on a fairly level ground and level it carefully Fix a peg or an arrow at a point A at a distance of about 100 m from the instrument station O With both horizontal motions clamped, bisect A.

(ii) Now plunge the telescope and mark a point P in the line of sight at about 100 m from O and at about the same level as A

(iii) Unclamp the upper motion (vernier plate), swing through 180°, and again bisect A (with the telescope reversed) Clamp the upper motion

(iv) Transit the telescope If the point P is again bisected by the cross hairs the adjustment is correct

Adjustment — If P is not now on the line of sight, mark a point Q in the line of sight opposite P

Mark a point D at one-fourth of the distance from Q to P ($QD = \frac{1}{4} QP$) Move the diaphragm by means of the horizontal diaphragm screws until the vertical hair is on the point D

Repeat the process until the adjustment is perfect.

It will here be noticed that the apparent error PQ is four times the real error QD, since the telescope is transitted twice

The points A and P are taken at the same level in order to avoid the error due to the horizontal axis not being perpendicular to the vertical axis The distances OA and OP are equalised so that focussing need not be done when a foresight is taken A board or a levelling staff placed horizontally may be used for marking the points P and Q

Note —(i) If the line of collimation is perpendicular to the horizontal axis (i e $\angle AO3$ is exactly 90°, on taking a back-sight on A and transitting the telescope, the line of collimation will generate a plane and strike a point A′ which is in the prolongation of AO But if it is out of adjustment by an amount α, the angle AO1 is $90° - \alpha$ When the telescope is transitted, the line of collimation will generate the surface of a cone and strike a point P instead of A′ It is evident from the figure that $\angle 1OP$ is $90° - \alpha$ and consequently, $\angle POA' = 2\alpha$ On again backsighting on A with the telescope reversed, and plunging the telescope, the line of collimation will strike a point Q, $\angle QOA'$ being 2α Thus the apparent error ($\angle POQ$) $= 4\alpha$ As two reversals of the telescope are involved in the test, the real error is α, and, therefore, a point D is marked at one fourth the distance QP

(ii) In order to move the diaphragm (cross hair ring), one screw should be loosened and the opposite screw tightened. The cross hair ring moves towards the tightened screw By loosening the upper screw and tightening the lower screw, the cross-hair ring is drawn downward and vice versa Similarly, if the screw on the right hand side of the telescope is loosened and the opposite screw tightened, the cross hair ring is drawn to the left and vice versa

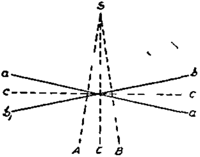


Fig 15

$a\ a_1$	First	position of Horizontal Axis
$b\ b_1$	Second	,, ,, ,,
$c\ c_1$	Correct	,, ,, ,,

Third Adjustment —*To make the horizontal axis perpendicular to the vertical axis* (Fig 15)

Necessity —By means of the adjustment of the vertical hair, we ensure that the line of sight will revolve in a plane perpendicular to the horizontal axis The object of this adjustment is to make this plane *vertical* when the instrument is levelled (i e the vertical axis has been made truly vertical) By means of the second and third adjustments, we ensure that the line of sight will revolve in a vertical plane The adjustment becomes essential in all work necessitating motions of the telescope in altitude

Test —(i) Set up the theodolite at about 10 m from a high building or other object on which there is a well-defined point at a considerable altitude such as a flag pole, lightning conductor and level it very carefully Let S be such a point.

(ii) Sight the point S, and with both horizontal motions clamped depress the telescope and mark a point A on the wall near its base in the line of sight

(iii) Unclamp, plunge the telescope and swing it through 180° With the telescope, inverted, again sight on the point S Depress the telescope If the line of sight now strikes the point A previously marked the adjustment is correct

Adjustment —If not, mark another point B in the line of sight on the wall at the same level as A. Mark a point C midway between A and B Sight on the point C and clamp the upper motion Raise the telescope The line of sight will not now strike the point S Raise or lower the adjustable end of the trunnion (horizontal) axis by means of the screws near the top of the standard or A frame until the line of sight passes through the point S Repeat the test and correction until perfect

Instead of marking the points on wall an ordinary levelling staff may be placed horizontally near the base of the wall and the readings on the scale noted each time when the telescope is depressed

It may be noted that the high end of the horizontal axis and the point set are always on the same side of the vertical plane passing through the high object.

This method is known as the *Spire Test*

Alternative Method —The adjustment may be made with the help of a striding level in the following manner —

The striding level should be tested to ascertain if it is in adjustment

Test —(i) Set up the instrument and level it carefully

(ii) Remove the caps which cover the ends of the trunnion axis

(iii) Place the striding level on the ends of the trunnion (horizontal) axis and bring the bubble exactly to the centre of its run by the levelling screws

(iv) Reverse the striding level end for end, leaving the instrument undisturbed If the bubble traverses the level is in adjustment

Adjustment —If not, note the deviation of the bubble Bring the bubble half way back (half its deviation) by means of the capstan headed screws on one of the legs of the striding level and the remaining half by the levelling screws Repeat the process until the adjustment is perfect

Test for the Third Adjustment —(i) Having adjusted the striding level place it in position

(ii) Centre the bubble of the striding level exactly by means of the levelling screws Gently lift the striding level and reverse the bearing trunnions by turning the head of the instrument through 180° in azimuth Replace the striding level, the legs now resting on different pivots to those upon which they rested before If the bubble remains central, the adjustment is correct

Adjustment —If not, note the deviation of the bubble Correct half the deviation by means of the capstan headed screws near the top of the standard, which raise or lower one end of the horizontal axis and the other half by means of the levelling screws Repeat the operation until the test is satisfied

In some instruments no means are provided for making this adjustment The condition is permanently established by the maker

Fourth Adjustment :—*To make the axis of the telescope level parallel to the line of collimation* (Figs. 16 and 17)

Necessity —With this adjustment, the lines of collimation becomes horizontal when the telescope bubble is brought in the centre The adjustment is a necessity when the theodolite is to be used as a level or when vertical angles are to be measured

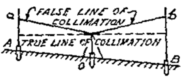


Fig. 16

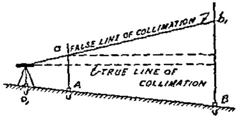


Fig 17

Test — The procedure of testing is the same as in the "two peg" adjustment of the dumpy level

(i) Drive two pegs A and B on a fairly level ground say, 100 m apart Set up the instrument at O exactly midway between A and B Clamp the vertical circle and bring the telescope bubble exactly to the centre of its run by means of the tangent screw of the vertical circle

(ii) With the bubble exactly central, take readings on the staff held on A and B, and find the difference between these readings, which gives the true difference of level between A and B

(iii) Shift the instrument and set it up at O_1 on the line BA produced, at about 10 m from A. Level it accurately.

(iv) With the bubble exactly central, read the staff first on A and then on B and find the difference between the two readings. If this difference agrees with the first (true) difference, the adjustment is correct.

Adjustment —(v) If not, calculate the correct staff readings on A and B Bring the horizontal hair exactly to the correct reading on B by means of the tangent screw of the vertical circle. Bring the bubble exactly to the centre of its run by means of the level tube nuts (capstan headed screws attaching the level tube to the telescope)

(vi) Sight the staff on the near peg and note whether the calculated correct reading is obtained Repeat the process until the test is satisfied

Alternative two peg method —Many surveyors prefer this method The procedure is exactly similar to that in the above method except for the following —

(1) The vertical vernier is set at zero and the telescope bubble is brought to the centre of its run by means of clip screws prior to taking staff readings on A and B (2) The horizontal hair is brought exactly to the calculated reading on the far peg B by means of the clip screws Since the vernier has been clamped at zero, there will be no index error

Fifth Adjustment —*To make the vertical circle or arc read zero when the line of collimation is horizontal (when the telescope bubble is centred)*

Necessity —The adjustment is carried out for convenience only. If the index error i e the reading on the vertical circle when the telescope bubble is in the centre is noted down and corresponding correction applied to the observed reading no error will be introduced But as there is likely to be some confusion between + and — signs of the correction it is desirable that the index error is removed wherever possible The index error is eliminated when the vertical angle between two objects is determined as a difference between two readings

Test —(i) Having centred the plate bubbles bring the telescope bubble exactly to the centre of its run by means of the vertical tangent screw as in first adjustment, and read the vernier of the vertical circle

Adjustment —(ii) If the vernier does not read zero, loosen it and move it until it reads zero by means of the screws which hold

it to the standard If the vernier is not adjustable, note the angular error and its sign This angular error is called the "*index error*" and is applied as a correction to the observed values of vertical angles

In a transit theodolite, the vernier can be clamped at zero, and the telescope is then brought into a horizontal position by means of the clipping screws There should, therefore, be no index error in the case of a transit instrument

In the case of a theodolite having an altitude level attached to the vernier arm (index arm) the fourth and fifth adjustments can be combined into one adjustment so that the line of collimation is horizontal when the altitude bubble is centred and the reading of the vertical circle is zero

There are two types of the instrument (1) one in which the clamp and tangent screw of the vertical circle are on the same side of the telescope as the clip screw, and (2) the other in which the clamp and tangent screw of the vertical circle are placed on one side of the telescope, and the vertical circle and the clip screw on the other side This type of instrument is usually packed as one piece In the case of the former when the clip screw is turned, the pointing of the telescope is altered, but the vertical circle readings remain unchanged, since it rotates the vertical circle and the verniers together, while in the case of the latter turning the clip screw moves the vernier, and changes the reading of the vertical circle, and tilts the bubble tube on the vernier arm but does not change the pointing of the telescope (the vertical circle and the telescope remaining unchanged)

Procedure for test—(i) Set up the instrument and level it carefully with reference to the plate levels (ii) Bring the altitude (azimuthal) bubble mounted on the index arm to the centre of its run by means of the clip screw Set the vertical vernier exactly to read zero by means of the clamp and slow motion screw of the vertical circle

(iii) Take a reading on a staff held at a distance of about 100 m from the instrument station

(iv) Change face (i e transit and swing through 180°) and again clamp the vernier exactly at zero Level the instrument if necessary

(v) Again sight the staff held on the same point and note the reading If this reading is the same as the first reading the adjustment is correct

(vi) Adjustment —(*First type of the instrument*) If not find the mean of the two readings and by turning the clip screw bring the horizontal hair exactly on to the mean reading thus setting the line of collimation truly horizontal Then bring the altitude bubble to the middle of its run by means of the level tube nuts (i e capstan screws fixing it to the index arm)

Repeat the test and adjustment until the adjustment is perfect

Adjustment —(*Second type of the instrument*) Bring the horizontal hair on to the mean reading by turning the vertical circle tangent screw Set the vernier index to zero by turning the clip screw

Then bring the bubble of the altitude level to the centre of its run by means of the capstan screws attaching it to the vernier arm Repeat the test and adjustment until all error is eliminated

In the case of an instrument fitted with two levels one on the index arm and the other on the telescope the adjustment should be made by reference to one of the bubbles Having adjusted that bubble the bubble of the other level tube is centred by means of the level tube nuts

Relative Importance of the Adjustments —The first adjustment is important in the measurement of *horizontal* and *vertical* angles The vertical axis must be truly vertical It may be remembered that the error due to the vertical axis not being truly vertical cannot be eliminated by taking face left and face right observations The adjustment should therefore be tested frequently Adjustment of the vertical hair and the third adjustment are very important in the measurement of horizontal angles or in prolonging a straight line By taking double face observations the errors of the second and third adjustments may be eliminated Adjustment of the horizontal hair and the fourth and fifth adjustments are of utmost importance only in

the measurement of vertical angles or in the levelling operations done with the theodolite Face left and face right observations should be taken to eliminate the errors of these adjustments

---

PROBLEMS

1 Give a list of the permanent adjustments of a transit theodolite and state the object of each of the adjustments Describe how you would make the trunnion axis perpendicular to the vertical axis (U B)

2 Give a list of permanent adjustments of a transit theodolite Explain clearly how you would test a theodolite to discover if the horizontal axis and the line of sight were perpendicular to each other If adjustment of the line of sight be found necessary describe how you would carry it out (U B)

3 Give a list of temporary and permanent adjustments of a transit theodolite A given line is prolonged with a theodolite but it is found that the point lie on a curve What is the source of the error? Describe how you would test and adjust the instrument

4. Describe with the aid of neat sketches how you would set the plate level at right angles to the vertical axis

5 You are asked to measure vertical angles correctly with a transit theodolite Explain clearly with sketches how you would test the instrument and, if necessary adjust it

6 Explain the adjustment for making the axis of the spirit level over T frame of the vertical circle perpendicular to the vertical axis of the theodolite (G U)

7 Mention the permanent adjustments of a transit theodolite and explain the object of each of these adjustments (U P)

\+ \+ \+

CHAPTER III

TRIGONOMETRICAL LEVELLING

Trigonometrical Levelling is a branch of levelling in which the relative elevations of different stations are determined from the observed vertical angles and known horizontal or geodetic distance The vertical angles may be measured by means of a theodolite, and the horizontal distances may be either measured or computed

Various cases will now be considered

Curvature and Refraction —The effect of curvature is to make the objects appear lower than they really are and that of refraction is to make them appear higher than they really are The effect of refraction is in opposite direction to that of curvature and is taken as one seventh of that of curvature The combined effect of curvature and refraction is, therefore, to cause the objects appear lower than they really are The correction for curvature and refraction is applied in two ways

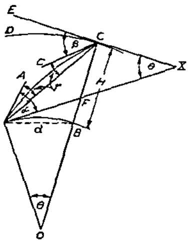


Fig 18

(1) The apparent difference of elevation of two stations is first calculated in the usual way and then corrected by applying algebraically the combined correction in linear measure

(2) The observed vertical angle is corrected by applying the angular correction algebraically before calculating the required difference of elevation

In Fig 18, let A and C = the two stations whose difference of level is desired

AB = the level line passing through A

CD = „ „ „ „ C

AF = the horizontal line at A (tangential to AB)

CE = the horizontal line at C (tangential to CD)

H = the true difference of elevation of A and C

$\angle$A AF = α = the angle of elevation observed at A.

$\angle$C CE = β = the angle of depression observed at C

d = the horizontal distance in m between A and C

θ = the angle subtended by the horizontal distance AB at the centre of the earth

$\angle$ A'AC = $\angle$ AC C = r = the angle of refraction

m = the coefficient of refraction

R = the radius of the earth in m (6381000 m)

It may be noted that on account of refraction the observer at A does not sight along the true line AC, but sights in the direction of AA , which is tangential to the curved line of sight AaC, since the signal at C is apparently seen in that direction Therefore, the angle actually observed at A with a transit is the angle A AF, while the true angle is CAF Similarly, the angle observed at C is the angle C CE and the true angle is the angle ACE Hence the correction for refraction is subtractive in the case of the angle of elevation (plus angle) and additive for the angle of depression (minus angle)

Corrected angle at A = $\angle$CAF = $\angle$A AF − $\angle$A AC

$= \alpha - r$

„ „ at B = $\angle$ACE = $\angle$C CE + $\angle$C CA

$= \beta + r$

The angle of refraction (r) is usually expressed in terms of the central angle (θ) The coefficient of refraction (m) is the ratio of the angle of refraction and the central angle so that

$$m = \frac{r}{\theta} \text{ or } r = m\theta$$

The correction for curvature is additive for the angle of elevation, and subtractive for the angle of depression.

Now we shall consider the two cases which occur in practice.

Case I :—One angle will be an angle of elevation and the other an angle of depression. This happens when the difference in elevation of the two stations is great and the distance between them is comparatively small

Case II ·—However, when the distance between the two stations is great and their difference in elevation is small, both angles will be angles of depression.

Refraction · Case I ·—When one angle is an angle of elevation and the other an angle of depression : The angle of refraction or refraction error (r) may be obtained as follows : (Fig 18)

The exterior angle of the $\triangle ACX = \angle ACE = \angle CAF + \angle AXC$.

Now $\angle ACE = \beta + r$, $\angle CAF = \alpha - r$; and $AXC = \theta$

$$\therefore \quad \beta + r = \alpha - r + \theta ; \text{ i e. } r = \frac{\theta + \alpha - \beta}{2} = \frac{\theta}{2} - \frac{(\beta - \alpha)}{2} \quad \cdots \quad (1)$$

Case II ·—When both angles are angles of depression Changing the sign of α in (1), we get

$$r = \frac{\theta}{2} - \frac{(\beta + \alpha)}{2} \quad \cdots \quad (2)$$

in which $\theta = \dfrac{d}{R}$ radians $= \dfrac{d}{R \sin 1''}$ seconds.

It is assumed that the refraction error is the same at both the stations

Correction for Curvature and Refraction :—(Fig 18)

(1) The angular correction for curvature $= \angle FAB$

$$= \tfrac{1}{2} \angle AOB = \frac{\theta}{2} = \frac{d}{2R} \text{ radians} = \frac{d}{2R \sin 1''} \text{ seconds}$$

The corresponding linear correction $= FB = \dfrac{d^2}{2R}$ m,

d and R being expressed in metres

(ii) The angular correction for refraction $= \angle A\,AC = C\,CA$

$$= r - m\theta = \frac{md}{R \sin 1''} \text{ seconds}$$

The value of m may be taken as 0 07 for sights over land and 0 08 for sights over sea R sin 1″ = 30 88 m to 30 94 m

$$\text{The corresponding linear correction} = \frac{1}{7} \text{ of } \frac{d^2}{2R} = \frac{0\ 14\ d^2}{2R}$$

$$= \frac{2md^2}{2R} \text{ m}$$

(iii) The combined angular correction

$$= (\text{curvature} - \text{refraction}) = \left\{ \frac{d}{2R \sin 1''} - \frac{md}{R \sin 1''} \right\}$$

$$= \left\{ \frac{(1 - 2m)\,d}{2R \sin 1''} \right\} \text{ seconds} \qquad (3)$$

$$\text{The combined linear correction} = \frac{d^2}{2R} - \frac{2md^2}{2R}$$

$$= \frac{(1 - 2m)\,d^2}{2R} \text{ m} \qquad (4)$$

The combined correction is additive in the case of an angle of elevation and subtractive in the case of an angle of depression

Distance between Two Stations —The given distance may be horizontal or geodetic By geodetic distance is meant the distance reduced to mean sea level The required horizontal distance may be computed from the given geodetic distance by the formula

$$d = \frac{(R + h_A) \times l}{R} \qquad (5)$$

where l = the geodetic distance R = the mean radius of the earth h_A = the elevation of station A.

44574

Axis-signal Correction :—The axis-signal correction, also called the eye and object correction, requires to be applied to the observed vertical angles when the height of the signal at one station is not the same as that of the instrument at the other station

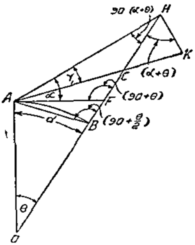


Fig 19.

Referring to Fig 19, let

s_1 = the height of signal at A.
s_2 = „ „ „ at C.
h_1 = „ of instrument at A.
h_2 = „ „ „ at C.
d = the horizontal distance between the stations A and C.
CH = the difference between the height of signal at C and the height of inst. at A $= (s_2 - h_1)$.

γ_1 = the axis-signal correction to the vertical angle observed at A.

γ_2 = „ „ „ „ vertical angle observed at C.

The correction may be found from the formula

$$\gamma = \frac{(\text{height of signal} - \text{height of inst.})}{(\text{horizontal distance})\ \sin 1''} \text{seconds (approximate)} \quad (6)$$

$$\therefore \angle HAC = \gamma_1 = \frac{(s_2 - h_1)}{d \sin 1''} \text{ seconds}; \quad \gamma_2 = \frac{(s_1 - h_2)}{d \sin 1''} \text{ seconds.}$$

This formula gives sufficiently accurate results when the vertical angle is small and the difference between the height of signal and that of the instrument is also small If, however, the vertical angle is large, the angle HAC must be taken at its correct value. It may be shown that

$$\text{Tan HAC} = \frac{(s_2 - h_1) \cos^2 (\alpha + \theta)}{d \cos \frac{\theta}{2}} \text{ (exact)} \quad . \quad . \quad . \quad (7)$$

θ is usually small (a few minutes) and may, therefore, be ignored

$$\left.\begin{aligned} \text{Then } \tan \text{HAC} = \tan \gamma_1 &= \frac{(s_2 - h_1)\cos^2 \propto}{d} \\ \text{Similarly,} \qquad \tan \gamma_2 &= \frac{(s_1 - h_2)\cos^2\beta}{d} \end{aligned}\right\} \quad \ldots \quad (8)$$

The correction is minus to + angle and plus to − angle

The formula may be derived as follows : After drawing HK perpendicular to AH, meeting AC produced in K, it will be seen from Fig 19 that $\angle AHO = 180° - \angle HAO - \angle AOH = 180° - (90° + \propto) - \theta = 90° - (\propto + \theta)$ and $\angle CHK = 90° - \angle AHO = \propto + \theta$

Now $HK = CH \cos(\propto + \theta)$ very nearly In the triangle ABH

$$AH = AB \frac{\left(\sin 90° + \frac{\theta}{2}\right)}{\sin\{90 - (\propto + \theta)\}} = AB \frac{\cos\frac{\theta}{2}}{\cos(\propto + \theta)}.$$

$$\text{Now} \quad \tan \text{HAC} = \frac{HK}{AH} = \frac{CH\cos^2(\propto + \theta)}{AB \cos\frac{\theta}{2}}.$$

But $\quad CH = s_2 - h_1$ and $AB = d$

$$\therefore \text{Tan HAC} = \frac{(s_2 - h_1)\cos^2(\propto + \theta)}{d \cos\frac{\theta}{2}}.$$

There are two methods by which the difference of elevation of two stations may be determined

First Method By Single Observation —The method is used when it is not possible to occupy both the stations, one of them being inaccessible In such a case, the vertical angle is observed at one station and the observed angle is then corrected for the curvature and refraction effects, assuming the value of the coefficient of refraction to be 0 07 Since refraction is

very uncertain, the results obtained by this metho d a renot so accurate as those obtained by the second method.

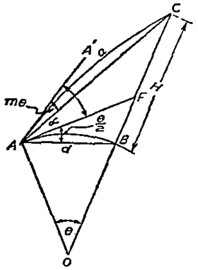


Fig 20

In Fig 20, let A and C

= the two stations, the difference of level of which is required.

$\propto$ = the angle of elevation observed at A.

d = the horizontal distance between A and C.

= arc AB = chord AB = AF.

H = the difference of level of A and C

Neglecting the correction for curvature and refraction, and the axis-signal correction, the formula for H may be derived as follows.

Case I : When the distance is very great —

In the $\triangle$ AOC, $\angle$ AOC $= \theta$; $\angle$CAO $= 90° + \propto$.

$\therefore \quad \angle \text{ACO} = 180° - \theta - (90° + \propto) = 90° - (\propto + \theta)$.

In the $\triangle$ ACF, $\angle$ CAF $= \propto$; $\angle$ACF $= 90° - (\propto + \theta)$.

$\therefore \quad \angle \text{AFC} = 180° - \propto - 90° + \propto + \theta = 90° + \theta$.

By the Sine rule, we get

The apparent height (CF)

$$= \frac{\text{AF} \sin \text{CAF}}{\sin \text{ACF}} = \frac{\text{AF} \sin \propto}{\sin\{90° - (\propto + \theta)\}} = \frac{\text{AF} \sin \propto}{\cos(\propto + \theta)}.$$

But AF may be taken equal to d without appreciable error

$$\therefore \quad \text{CF} = d \frac{\sin \propto}{\cos(\propto + \theta)} \text{ (exact)} \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

where $\theta = \dfrac{d}{\text{R} \sin 1''}$ seconds.

Case II.—*When the distance is comparatively short and the angle α is fairly small*, θ may be neglected. In other words, we assume the angle AFC to be a right angle. The above formula may, therefore, be written as

$$CF = d \tan \alpha \text{ (approximate)} \quad \ldots \quad \ldots \quad \ldots \quad (9a)$$

To determine the value of H, the axis-signal correction and the corrections for curvature and refraction must be applied to the apparent height thus found

Case I —(a) (Very great distances) —When the observed angle is an angle of elevation (+ angle). Applying the axis signal correction to the observed angle α, we have

Corrected angle $\alpha_1 = \alpha - \dfrac{s-h}{d \sin 1''}$, the *correction being*

minus in the case of an angle of elevation (+ angle). The angle thus obtained is further corrected for curvature and refraction. Thus we get the true value of the observed angle α.

Then in the $\triangle$ ABC, $\angle CAB = \alpha_1 - m\theta + \dfrac{\theta}{2}$;

$$\angle ABC = 90° + \frac{\theta}{2}.$$

$$\therefore \angle ACB = 180° - \left(90° + \frac{\theta}{2}\right) - \left(\alpha_1 - m\theta + \frac{\theta}{2}\right)$$

$$= 90° - (\alpha_1 - m\theta + \theta).$$

$$\text{Now } H = CB = AB \frac{\sin CAB}{\sin ACB} = AB \frac{\sin\left(\alpha_1 - m\theta + \frac{\theta}{2}\right)}{\sin\{90° - (\alpha_1 - m\theta + \theta)\}}$$

$$= AB \frac{\sin\left(\alpha_1 - m\theta + \frac{\theta}{2}\right)}{\cos(\alpha_1 - m\theta + \theta)}.$$

Substituting the values of θ and AB, we have

$$H = d\,\frac{\sin\left\{ \alpha_1 + (1-2m)\,\dfrac{d}{2R\sin 1''} \right\}}{\cos\left\{ \alpha_1 + (1-m)\,\dfrac{d}{R\sin 1''} \right\}} \text{ (exact)} \qquad (10)$$

Case I —(b) (Very great distances) —*When the observed angle is an angle of depression* (— angle), we proceed as follows

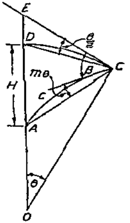


Fig 21

Applying the axis signal correction to the observed angle β and denoting the corrected angle by β_1, we have

$$\beta_1 = \beta + \frac{(s-h)}{d\sin 1''},$$

the correction being plus in the case of an angle of depression (— angle) β_1 is further corrected for curvature and refraction, thus obtaining the true value of the observed angle β

Thus in Fig 21, the true angle

$$= ACD = \beta_1 + m\theta - \frac{\theta}{2}$$

$\angle ACC = 90° - \theta$, and $\angle CAD = \theta + 90° - (\beta_1 + m\theta)$
$= 90° - (\beta_1 + m\theta - \theta)$

Then from the $\triangle$ ACD, $AD = H = CD\,\dfrac{\sin ACD}{\sin CAD}$

$$= CD\,\frac{\sin\left(\beta_1 + m\theta - \dfrac{\theta}{2}\right)}{[\sin 90° - (\beta_1 + m\theta - \theta)]}.$$

Substituting the values of θ and CD, we get

$$H = d\,\frac{\sin\left[\beta_1 - (1-2m)\,\dfrac{d}{2R\sin 1''} \right]}{\cos\left[\beta_1 - (1-m)\,\dfrac{d}{R\sin 1''} \right]} \text{ (exact)} \qquad (11)$$

Case II (a) :—(Great distances): —*When the observed angle is an angle of elevation* · Assuming the angle ABC (Fig 20) to be a right angle, $\left(\frac{\theta}{2}\text{ being ignored}\right)$,

$$CB = AB \tan CAB$$

$$= AB \tan\left(\alpha_1 - m\theta + \frac{\theta}{2}\right).$$

Substituting the values of θ and AB, we have

$$H = d \tan\left\{\alpha_1 + (1-2m)\frac{d}{2\,R \sin 1''}\right\} \text{(approximate)} \quad \text{.. (12)}$$

Case II (b).—(Great distances) —*When the observed angle is an angle of depression* · When $\frac{\theta}{2}$ is neglected, i. e. $\angle ADC$ taken as a right angle (Fig 21), we have

$$H = AD = CD \tan ACD = CD \tan\left(\beta_1 + m\theta - \frac{\theta}{2}\right)$$

Substituting the values of θ and CD, we get

$$H = d \tan\left\{\beta_1 - (1-2m)\frac{d}{2R \sin 1''}\right\} \text{(approximate)} \quad \text{.. (13)}$$

To avoid confusion, computation work should methodically be done in the following steps :

(1) Find the axis-signal correction from formula (6) and apply it algebraically to the observed vertical angle, due attention being paid to the sign of the correction. Thus we get the value of α_1 or β_1.

(2) Obtain the values of $(1-2m)\frac{d}{2R \sin 1''}$ and $(1-m)\frac{d}{R \sin 1''}$, and add them algebraically to the calculated value of α_1 or β_1, thus obtaining the value of the observed angle corrected for the difference in height of the signal and the instrument, and for curvature and refraction.

(3) When the distance (d) is very great, substitute this value of the corrected angle in formula (10) or (11) according as the observed angle is + angle or — angle, thus determining the value of the difference of elevation (H) of the two stations A and O.

(4) When the distance is great, add the value of

$(1-2m)\dfrac{d}{2R\sin 1''}$ algebraically to the calculated value of α_1

or β_1 and substitute the value thus obtained in formula (12) or (13), thus obtaining the value of H

Approximate Method —In this method the apparent height CF is calculated from formula (9a) and is then corrected by applying the corrections for height of instrument, height of signal, curvature, and refraction in linear measure Thus we have

(a) *When the observed angle is an angle of elevation*

$$H = d\tan\alpha + \text{height of inst} - \text{height of signal}$$
$$+ (\text{curvature} - \text{refraction})$$

or $$H = d\tan\alpha + h - s + (1-2m)\frac{d^2}{2R} \qquad (14)$$

(b) *When the observed angle is an angle of depression* ·

$$H = d\tan\beta - \text{height of inst} + \text{height of signal}$$
$$- (\text{curvature} - \text{refraction})$$

or $$H = d\tan\beta - h + s - (1-2m)\frac{d^2}{2R} \qquad (15)$$

Second Method By Reciprocal Observations —In this method the vertical angle to each station is observed from the other station, and the refraction effect is assumed to be the same at each station In order to completely eliminate the refraction effect, simultaneous observations should be

taken whenever possible It is not, however, usually possible to measure the vertical angles simultaneously. They should therefore, be measured at the time when the refraction effect is minimum and on different days. Since refraction is less variable between 10 A M and 3 P. M., the vertical angles should be measured during these hours The results obtained by this method are more accurate than those obtained by the first method.

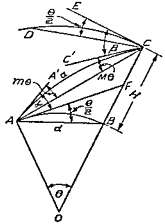


Fig. 22

In Fig. 22, let A and C be the stations whose difference in elevation is required.

d = the horizontal distance in m between A and C.
AB = the level line passing through A.
CD = " " " C.
AF = the horizontal line at A (tangential to AB).
CE = the horizontal line at C (tangential to CD).
AaC = the curved line of sight.
AA′ = the line tangential to AaC at A.
CC′ = " " " " " at C.
$\angle$A′AF = the angle of elevation observed at A.
$\angle$ C′CE = the angle of depression observed at C.
$\angle$A′AC = the angle of refraction (r) at A.
$\angle$C′CA = the angle of refraction (r) at C.
$\angle$AOC = the central angle (θ).
H = the difference of elevation of A and C.

We will now derive the formula for H on the assumption that observations are made from ground level to ground level, i e. upon a signal of the same height above ground as that of the instrument. Correcting the observed angles α and β for curvature and refraction, we have

The corrected angle at A = CAB = the observed angle A'AF + total correction for curvature and refraction, the sign of the correction being positive in the case of an angle of elevation

Now the correction for curvature = FAB = $\frac{\theta}{2}$ = ECD

" " refraction = A'AC = r = $m\theta$ = C'CA.

$$\text{Combined correction} = \left(\frac{\theta}{2} - m\theta\right)$$

Hence the corrected angle at A = CAB

$= \alpha + \left(\frac{\theta}{2} - m\theta\right) = \alpha - m\theta + \frac{\theta}{2}$, the sign of the correction being positive in the case of an angle of elevation

Similarly, the corrected angle at C = ACD = $\beta - \left(\frac{\theta}{2} - m\theta\right)$

$= \beta + m\theta - \frac{\theta}{2}$, the sign of the correction being negative in the case of an angle of depression Since chords AB and CD are parallel, CAB = ACD

$$\alpha - m\theta + \frac{\theta}{2} = \beta + m\theta - \frac{\theta}{2}, \text{ and each} = \frac{\alpha + \beta}{2}$$

Now in the $\triangle$ACB, AB = d, CAB = $\alpha - m\theta + \frac{\theta}{2}$

ABC = $90° + \frac{\theta}{2}$, ACB = $90° - (\beta + m\theta)$

By the Sine rule, we get

$$CB = AB\frac{\sin CAB}{\sin ACB} = d\,\frac{\sin\left(\alpha - m\theta + \frac{\theta}{2}\right)}{\sin\{90° - (\beta + m\theta)\}}$$

$$= d\,\frac{\sin\left(\alpha - m\theta + \frac{\theta}{2}\right)}{\cos(\beta + m\theta)}$$

But $\beta + m\theta = \frac{\alpha + \beta}{2} + \frac{\theta}{2}$; CB=H; and $\alpha - m\theta + \frac{\theta}{2} = \frac{\alpha + \beta}{2}$

$$H = d \frac{\sin\left(\frac{\alpha + \beta}{2}\right)}{\cos\left(\frac{\alpha + \beta}{2} + \frac{\theta}{2}\right)} \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

When the distance d is not very great, $\frac{\theta}{2}$ being very small, may be neglected Then $H = d \tan\left(\frac{\alpha + \beta}{2}\right) \quad \ldots \quad \ldots \quad (17)$

From these, the formulae for the case when both α and β are angles of depression may be deduced by writing $-\alpha$ for α (changing the sign of α)

$\therefore$ For very great distances, $H = d \frac{\sin\left(\frac{\beta - \alpha}{2}\right)}{\cos\left(\frac{\beta - \alpha}{2} + \frac{\theta}{2}\right)} \quad (18)$

For great distances, $H = d \tan\left(\frac{\beta - \alpha}{2}\right) \quad \ldots \quad (19)$

The following procedure may be adopted in making the computations.

(i) If the geodetic distance between the two stations be given, find the corresponding horizontal distance from formula (5)

(ii) Find the axis-signal corrections for the observed angles α and β from formula (6) or (8) Apply them algebraically to the angles α and β, and denote the angles thus corrected by α_1 and β_1

(*Note* —The sign of the correction is minus to plus angle and plus to minus angle)

(iii) Find the semi sum of the corrected angles $\left(\frac{\alpha_1 + \beta_1}{2}\right)$

and substitute it in formula (16) or (17) to obtain the true difference of level of the two given points.

(iv) When both the observed angles are angles of depression, find half the difference of the corrected angles $\left(\frac{\beta_1 - \alpha_1}{2}\right)$, and substitute it in formula (18) or (19)

Examples on Trigonometrical Levelling

Example 1 :—Determine the difference in elevation between two stations A and B and the elevation of B from the following data :—

Observed angle of elevation at A	$= 3° 46' 30''$.
Height of instrument at A	$= 1{\cdot}56$ m
Height of signal at A	$= 3\ 84$ m.
Horizontal distance between A and B	$= 2347\ 68$ m.
Reduced level of A	$= 526{\cdot}750$.

(a) *By approximate method* —Here the corrections for height of instrument, height of signal, curvature, and refraction are applied in linear measure

Apparent height of the top of signal above the instrument axis at A $= h = d \tan \alpha = 2347{\cdot}68 \tan 3° 46' 30''$;

$$\log h = 2\ 1900613 \quad \therefore \quad h = 154\ 903 \text{ m}$$

The correction for curvature and refraction

$$= 0{\cdot}0673 \times \left(\frac{2347{\cdot}68}{1000}\right)^2$$

$$= 0\ 371 \text{ m}$$

Hence the difference in elevation of the stations A and B $= d \tan \alpha + ht.$ of inst $- ht$ of signal $+$ correction for curvature and refraction

$$= 154{\cdot}903 + 1{\cdot}560 - 3\ 840 + 0{\cdot}371 = 152{\cdot}994 \text{ m.}$$

R. L. of B $=$ R. L of A $+$ H $= 526{\cdot}750 + 152{\cdot}994$

$$= 679\ 744.$$

(*b*) *Alternative method* —Correcting the observed angle for the difference between the height of signal and that of instrument, and for curvature and refraction in angular measure and denoting it by α_0, we get

(i) Axis signal correction $= \gamma = \dfrac{s-h}{d \sin 1''}$ seconds

$$= \frac{3\ 840 - 1\ 560}{2347\ 68 \sin 1''}$$

$$= 200\ 317 \text{ seconds}$$

The sign of the correction is minus, since the observed angle is + angle.

(ii) Correction for curvature and refraction

$$= C_{cr},$$

$$= \frac{(1-2m)d}{2R \sin 1''} = \frac{(1-0\ 14)\ 2347\ 68}{2 \times 6371000 \sin 1''}$$

$$= 32\ 683$$

Corrected angle of elevation (α_0)

$$= 3^\circ\ 46\ 30'' - 200''\ 319 + 32''\ 683$$

$$= 3^\circ\ 43\ 42''\ 91$$

Hence the difference in elevation of the stations A and B

$$= H = d \tan \alpha_0$$

$$= 2347\ 68 \times \tan 3^\circ\ 43\ \ 42''\ 366$$

Log H $= 2\ 1837237$ H $= 152\ 659$

R L of B $= 526\ 75\ 0 + 152\ 659 = 679\ 409$

(*c*) *By rigorous method* —(i) Angle subtended at the centre of the earth by the distance AB $= \theta = \dfrac{d}{R \sin 1''}$ seconds

$$= \frac{2347\ 68}{6371000 \sin 1''} = 76\ 007 \text{ seconds}$$

(ii) Axis signal correction $= \gamma' = \dfrac{(s-h) \cos^2 \alpha}{d \tan 1''}$

$$= \frac{(3\ 84 - 1\ 56) \cos^2 3^\circ\ 46\ 30''}{2347\ 68 \tan 1''} = 199\ 438 \text{ seconds } (-ve)$$

(iii) Correction for refraction $= m\theta$

$= 0\ 0''(76\ 007)$

$= 5\ 321$ seconds $(-ve)$

(iv) Corrected observed angle $= \alpha_0$

$= 3°\ 46\ 30'' - 199\ ''438 - 5''\ 321$

$= 3°\ 43\ 5''\ 24$

(v) Find the angle of depression β_0 from the relation

$$\beta_0 = \alpha_0 + \theta$$

$$\beta_0 = 3°\ 43\ \ 5\ ''24 + 76''\ 01 = 3°\,44\ 21''\ 25$$

$$\text{Hence} = \frac{\alpha_0 + \beta_0}{2} = \tfrac{1}{2}\ (3°43\ 5''\ 24 + 3°44\ 21''\ 25)$$

$$= 3°43\ 43''\ 25$$

(vi) Difference in elevation $H = \dfrac{d \sin \frac{1}{2}(\alpha_0 + \beta_0)}{\cos \beta_0}$

$$= \frac{2347\ 68 \times \sin 3°43\ 43''\ 25}{\cos 3°44\ 21''\ 25}$$

$$= 152\ 848$$

R L of B $= 526\ 750 + 152\ 848 = 679\ 598$

Example 2 —The following data refer to the elevations of the ground stations of a triangle ABC in a trigonometrical survey

(i) Vertical angle from A to B $= +1°20\ 20''$; Vertical angle from B to A $= -1°12\ 24''$; Distance AB $= 4777\ 8$ m	Weight 2	Height of signal at A $= 4\ 92$ m; Height of inst at A $= 1\ 50$ m; Height of signal at B $= 4\ 44$ m; Height of inst at A $= 1\ 47$ m
(ii) Vertical angle from B to C $= -\ 49\ \ 24''$; Vertical angle from C to B $= +\ 55\ \ 12''$; Distance BC $= 4068\ 2$ m	Weight 2	Height of signal at B $= 4\ 71$ m; Height of inst at B $= 1\ 44$ m; Height of signal at C $= 5\ 52$ m; Height of inst at C $= 1\ 41$ m
(iii) Vertical angle from C to A $= -\ 47\ \ 12''$; Distance CA $= 3187\ 5$ m		Height of signal at A $= 3\ 96$ m; Height of inst at C $= 1\ 44$ m

Find the elevations of B and C and adjust them to close, given that the elevation of A is 1600 550.

(i) (a) Axis signal correction — (— to + angle and + to — angle)

Inst at A : $\gamma_1 = \dfrac{s_2 - h_1}{d \sin 1''} = \dfrac{4\ 44 - 1 \cdot 50}{47''77\ \ 1} = 126'' \cdot 92 \quad (-ve).$

Inst at B · $\gamma_2 = \dfrac{s_1 - h_2}{d \sin 1''} = \dfrac{4\ 92 - 1 \cdot 47}{4777\ 8\ \sin} = 148'' \cdot 94 \quad (+ve)$

(b) Correcting the observed angles for axis-signal :—

$\alpha_1 = 1°\ 20'\ 20'' - 126''\ 92 = 1°\ 18'13'' \cdot 08$

$\beta_1 = 1°\ 12'\ 24'' + 148''\ 94 = 1°\ 14'\ 52'' \cdot 94$

$\dfrac{\alpha_1 + \beta_1}{2} = 1°\ 16'\ 33'' \cdot 01.$

(c) Difference of level between A and B ·—

$H_1 = d \tan \tfrac{1}{2} (\alpha_1 + \beta_1) = 4777\ 8 \tan 1°\ 16'\ 33'' \cdot 01$

$= 106\ 404$ m

(ii) (a) Axis-signal correction ·—

Inst at B $\gamma_\circ = \dfrac{5\ 52 - 1\ 44}{4068 \sin 1''} = 206'' \cdot 87 \quad (+ve).$

Inst at C $\gamma_1 = \dfrac{15\ 7 - 4\ 7}{4068 \sin 1''} = 167'' \cdot 32 \quad (-ve)$

(b) Correcting the observed angles for axis-signal :—

$\alpha_1 = 55\ 12'' - 167''\ 32 = 52'\ 24'' \cdot 68$

$\beta_1 = 49'\ 24'' + 206''\ 87 = 52'\ 50'' \cdot 87$

$\tfrac{1}{2} (\alpha_1 + \beta_1) = 52'\ 37'' \cdot 78$

(c) Difference of level B and C —

$H_2 = 4068 \tan 52'\ 37\ \ 78 = -\ 62\ 280$m

(iii) Axis signal Correction —

Inst at c $\gamma = \dfrac{(3\ 96 - 1\ 44)}{3187\ 5} = 163''\ 07 + ve$

(b) Correction for curvature and refraction :—

The correction is necessary, since only one vertical angle was observed

$$C_{cr} = \frac{(1-2m)\,d}{2R \sin 1'} = \frac{(1-0\ 14)\ 3187\ 5}{2 \times 3\ 89} = 44{\cdot}37\ (-ve)$$

R sin 1" being taken as 30 89

(c) Correcting the observed angle for axis signal and for curvature and refraction

$$\beta_0 = 47'\,12" + 2'\,43"\ 07 - 44 \quad 37$$
$$= 49'\ 10''\ 70$$

(d) Difference of level between C and A :—

$$H_3 = 3187{\cdot}5\ \tan\ 49'\,10"\ 70$$
$$= 45\ 588\ \text{m}$$

Assuming the elevation of A as zero,

$$\text{Closing error} = +106\ 404 - 62\ 280 - 45\ 588$$
$$= 1\ 464\ \text{m}$$

This error should be distributed among the calculated differences of level inversely as the weights of observations, i e as $\frac{1}{2} : \frac{1}{2} . 1$

$$\text{Correction to } H_1 = \tfrac{1}{4}(1\ 464) = 0\ 366$$
$$\text{,, \quad ,, } H_2 = \tfrac{1}{4}(1\ 464) = 0\ 366$$
$$\text{,, \quad ,, } H_3 = \tfrac{1}{2}(1\ 464) = 0\ 732$$

Hence the corrected differences of level are :

$$H_1 = +106\ 404 + 0\ 366 = \ 106\ 770$$
$$H_2 = -\ 62\ 280 + 0{\cdot}366 = -61\ 914$$
$$H_3 = -\ 45\ 588 + 0{\cdot}732 = -44\ 856$$
$$\textit{check} \text{ —sum } = \overline{000\ 00}$$

Elevation of B = 1600 550 + 106 770
= 1707 320

Elevation of C = 1600 550 − 61 914
= 1645 406

(Elevation of A = 1645 406 − 44·856
= 1600·550)

Example 3 —*To determine the mean elevation of a station O interpolated in a triangulation system the following observations were made*

Inst station.	Ht of inst	Station observed	Distance in. m	Height of signal	Vertical Angle	Remarks.
O	1 53	D	3684	5 58	+1° 1 20″	R sin 1″ = 30 88 m.
O	1 53	E	4698	4 11	−52 50″	m 0 07
O	1 53	F	5028 6	4 92	−34 10″	log sin 1″ = $\bar{6}$ 685575

Find the mean elevation of station O, given that the elevations of D E and F are 293 58 157 725, and 179 355 respectively

(i) Axis signal correction —By $\gamma = \dfrac{s-h}{d \sin 1''}$ seconds

OD $\gamma_1 = \dfrac{(5\ 58 - 1\ 53)}{3684 \sin 1''} = 226''\ 76 = 3\ \ 46''\ 76\ (-ve)$

OE $\gamma_2 = \dfrac{(4\ 11 - 1\ 53)}{4698 \sin 1''} = 113''\ 27 = 1\ \ 53''\ 27\ (+ve)$

OF $\gamma_3 = \dfrac{(4\ 92 - 1\ 53)}{5028\ 6 \sin 1''} = 139''\ 05 = 2\ \ 19''\ 05\ (+ve)$

(ii) Correction for curvature and refraction —

$C_{cr} = \dfrac{(1-2m)\,d}{2\,R \sin 1''}$,

OD $C_{cr_1} = \dfrac{(1 - 0\ 14)\ 3684}{2 \times 30\ 88} = 51''\ 30$

OE $C_{cr_2} = \dfrac{(1 - 0\ 14)\ 4698}{2 \times 30\ 88} = 65''\ 42$

OF $C_{cr_3} = \dfrac{(1 - 0\ 14)\ 5028\ 6}{2 \times 30\ 88} = 71''\ 15$

(iii) Correcting the observed angles for axis signal, and for curvature and refraction,

O to D : $\alpha_1 = 1°1'20'' - 3'46''{\cdot}76 + 51''{\cdot}30 = 58'24''{\cdot}54.$
O to E $\alpha_2 = 52'50'' + 1'53''{\cdot}27 - 1'5''{\cdot}42 = 53'37''{\cdot}85.$
O to F : $\alpha_3 = 34'10'' + 2'19''05 - 1'11''{\cdot}15 = 35'17''{\cdot}90.$

(iv) Difference of level of the stations :—

O and D : $HD = 3684 \tan 58'24''54 = +62{\cdot}587.$
O and E : $HE = 4698 \tan 53'37''{\cdot}85 = -73{\cdot}326.$
O and F $HF = 5028 \tan 35'17''{\cdot}90 = -51\ 633$

Elevation of O in the first case = 293 580 − 62 587 = 230 993.
Elevation of O in the second case = 157 725 + 73·326 = 231 051.
Elevation of O in the third case = 179 355 + 51 633 = 230·988.

Elevation of O = 231 017 mean = $\frac{693\ 032}{3}$

= 231 017

PROBLEMS

1 Correct the observed altitude for the height of signal, and refraction from the following data

Observed altitude = + 3° 12 48″ Height of signal = 4 343 m
Height of instrument = 1 585 m Horizontal distance = 3787 14 m

(Ans 2 30″ 237, 8 179″, 3° 10 9 584″)

2 Find the difference of level of the points A and B and the reduced level of B from the following data

Horizontal distance between A and B = 5625 389 m
Angle of depression from A to B = 1° 29 34″
Height of signal at B = 3 886 m
Height of instrument at A = 1 497 m
Co efficient of refraction = 0 07
R sin 1″ = 30 876 m R L of A = 1265 850

(Ans 145 213 m 1120 637)

3 Find the difference in level between two points A and B and the refraction correction from the following data

Horizontal distance between A and B = 6382 384 m
Angle of elevation of B at A = 1° 50 20″
Angle of depression of A at B = 1° 51 10″
Height of signal at A = 4 145 m
Height of signal at B = 3 597 m
Height of instrument at A = 1 463 m
Height of instrument at B = 1 554 m

(Ans. 222 03 m 15″ 35)

4. Two stations A and B are situated at a distance apart of 2896 819 m. The depression angle of B at A is 5′ 10″ and the depression angle of A at B is 7′ 48″ The heights of signal at A and B are respectively 3 90 and 3 22 m and the heights of instrument at A and B 1·463 m and 1·554 m. respectively Calculate the difference of level of A and B and the refraction at the time of observation (Ans 11·339 m. 1′ 5″ 12)

5 Determine the reduced level of B from the following data

Horizontal distance between A and B	= 3489·96 m.
Height of instrument at A	= 1·433 m
Height of instrument at B	= 1·463 m
Height of signal at A	= 4·572 m
Height of signal at B	= 3·962 m.
Angle of elevation of B at A	= 1° 52′ 4″.
Angle of depression of A at B	= 1° 48′ 20″.
Reduced level of A	= 950·75
R sin 1″	= 30·937 m

(Ans. 1062·952.)

6 Two stations A and B are situated at a distance apart of 3541·776 m. The following observations were recorded.

Height of signal at A	= 1 517.
Height of instrument at A	= 1·463 m
Angle of elevation from A to B	= 2° 12′ 20″.
Reduced level of A	= 1420·50 m.
Height of signal at B	= 3 938 m
Height of instrument at B	= 1 512 m.
Angle of depression from B to A	= 2° 2′ 30″.

Determine the reduced level of B

(Ans. 1552·97.)

7 Two stations A and B are 3791 712 ft. apart The following observations were recorded

Height of instrument at A	= 1 463 m.
Height of signal at A	= 5 09 m
Height of instrument at B	= 1 494 m
Height of signal at B	= 4 511 m,
Vertical angle from A to B	= + 1° 54 30″.
Vertical angle from B to A	= − 1° 50′ 25″
Reduced level of A	= 1275·60 m.

Find the reduced level of B

(Ans 1399·955)

+ + +

CHAPTER IV

TACHEOMETRIC SURVEYING

Tacheometry is a branch of angular surveying in which the horizontal and vertical distances of points are obtained by instrumental observations chaining being thus entirely eliminated The method is most rapid though less accurate The accuracy of tachcometry is less than that of chaining, but it is far more rapid in rough and difficult country where ordinary levelling is tedious, and chaining is inaccurate, difficult, and slow When obstacles such as steep and broken ground, deep ravines, stretches of water or swamps are met with, tacheometry is best adapted from the point of view of speed and accuracy The primary object of tacheometry is the preparation of contoured maps or plans It is extensively used on hydrographic surveys, location surveys for roads, railways, reservoirs, etc It can be used for checking more precise measurements It is well adapted for locating contours and filling in detail in topographic surveys

Instruments —The instruments usually employed in tacheometry are (1) a tacheometer and (2) a levelling or stadia rod A tacheometer, in a general sense, is a transit theodolite having a stadia telescope, i e a telescope fitted with a stadia diaphragm, i e a telescope equipped with two horizontal hairs called stadia hairs in addition to the regular cross-hairs The additional hairs are equidistant from the central one and are also known as stadia lines, webs, wires, or points The types of stadia diaphragm commonly used are shown in Fig 23 The kinds of telescopes used in stadia

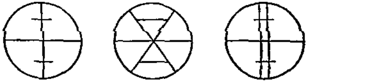


Fig 23

surveying are (i) the external focussing telescope, (ii) the internal focussing telescope, and (iii) the external focussing

anallatic telescope (i e telescope fitted with an anallatic lens) The term tacheometer is restricted to a transit theodolite provided with an anallatic telescope The essential characteristics of a tacheometer are (i) The value of the constant $\frac{f}{i}$ should be 100 (ii) the telescope should be fitted with an anallatic lens (iii) the telescope should be powerful the magnification being 20 to 30 diameters, (iv) the aperture of the objective should be 35 to 45 mm in diameter in order to have a sufficiently bright image, and (v) the magnifying power of the eyepiece should be greater to render staff graduations clearer at a long distance

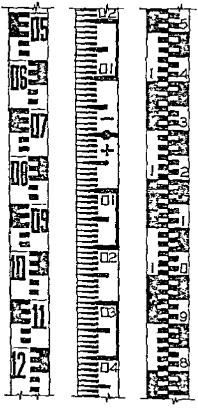


Fig 24 Fig 25 Fig 26

Stadia rod —The stadia rod is usually of one piece but for ease of transport it may be folding or telescopic It is 5 cm to 15 cm wide and 3 to 4 m long It is graduated in metres decimetres and centimetres The pattern of graduations should be bold and simple Some patterns of graduation are shown in Figs 24 25 & 26

For short sights upto 100 m an ordinary transit equipped with a stadia telescope (or stadia theodolite) and a levelling staff may be used The stadia theodolite is an instrument of low precision the accuracy of a single measurement being about 1 in 500 However it is most suitable for filling in topographic detail But for long sights say, more 300 m a tacheometer and stadia rod are required.

Tacheometric method —The various tacheometric methods may be classified as

Tacheometry
- Stadia method
  - Fixed hair
  - Movable hair
- Tangential method

The principle underlying these methods is as follows — If C and D be two points, and if a transit is set up at C, the horizontal distance of D from C and the elevation of D with respect to the instrument axis at C can be obtained from (1) the vertical angle to D from C, and (2) the angle subtended at C by a known distance on the staff held at D This principle is utilised in different ways in the above methods and consequently, the methods of observation and reduction are different

I (a) Fixed Hair Method —The interval between the stadia hairs being fixed

In this method the stadia hair interval is fixed When a staff is sighted through the telescope, a certain length of the staff (staff intercept) is intercepted by the stadia lines and from this value of the staff intercept, the distance from the instrument to the staff station may be determined It may be noted that the staff intercept varies with the distance at which the staff is held In the case of inclined sights the staff may be held vertically or normal to the line of sight This method of tacheometry is in most common use

(b) Movable Hair Method —The interval between the stadia hairs being variable

In this method the stadia lines are not fixed, but can be moved by means of micrometer screws The staff is provided with two vanes or targets fixed at a known distance apart, usually 3 m The variable stadia interval is measured, and from this value the required horizontal distance may be computed This method is now rarely used

II. Tangential Method :—In this method a staff fitted with two targets or vanes at a fixed distance apart, usually 3 m is held at a station and the vertical angles to the two vanes are observed with a theodolite. This method is used when the telescope of the instrument is not equipped with a stadia diaphragm

For the complete location of a point with respect to the instrument station (i e its horizontal distance and elevation) the following observations are required —

(1) The bearing of the line joining the instrument station to the point

(2) The vertical angle an angle of elevation (+ angle) or an angle of depression (− angle) as recorded on the vertical circle of the transit

(3) The staff readings of the bottom, middle, and top wires

Principle of Stadia Method —The principle of the stadia method is as follows

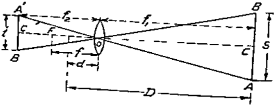


Fig 27

In Fig. 27, let O = the optical centre of the object glass

A', C', and B' = the top, axial, and bottom hairs or lines

B, C, and A = the points on the staff cut by the three lines

B'A' = i = the interval between the stadia lines or hairs, (B'A' is the length of the image of BA)

BA = S = the staff intercept (the difference of the stadia hair readings)

f = the focal length of the object glass, i e the distance from the optical centre (O) to the principal focus (F) of the lens

f_1 = the horizontal distance from the optical centre (O) to the staff.

f_2 = the horizontal distance from the optical centre (O) to the image of the staff, f_1 and f_2 being called the conjugate focal lengths of the lens

d = the horizontal distance from the optical centre (O) to the vertical axis of the tacheometer

D = the horizontal distance from the vertical axis of the instrument to the staff

$\triangle s$ AOB and A'OB' being similar, $\dfrac{AB}{A'B'} = \dfrac{OC}{OC} = \dfrac{f_1}{f_2}$

or $\qquad \dfrac{S}{i} = \dfrac{f_1}{f_2}$

By the formula for lenses, $\dfrac{1}{f} = \dfrac{1}{f_1} + \dfrac{1}{f_2}$

i. e. $\qquad \dfrac{f_1}{f} - 1 = \dfrac{f_1}{f_2} = \dfrac{S}{i}$

or $\qquad f_1 = \dfrac{f}{i} S + f$

The distance from the vertical axis of the instrument to the staff $= f_1 + d$

$$D = f_1 + d = \frac{f}{i} S + (f + d) \qquad (1)$$

The formula is to be used in stadia measurements when the line of sight is horizontal and the staff held vertically, i e perpendicular to the line of sight The quantities $\dfrac{f}{i}$ and $f + d$ are called the constants of the instrument, their values being usually marked on a card attached to the inside of the box by the maker The constant $\dfrac{f}{i}$ is called the *constant multiplier or multi-*

plying constant and its value is usually 100 (in some telescopes it is made equal to 50 or 200), while the constant $f + d$ is called the *additive constant* its value varying from 30 to 60 cm in the case of an external focussing telescope In the case of an internal focussing telescope, $f + d$ has a value of a few cm (8 to 20 cm) and is, therefore, often ignored To make the value of the additive constant exactly equal to zero, an additional convex lens, known as the *anallatic* lens, is provided in the telescope of a tacheometer between the object glass and the eyepiece at a fixed distance from the former By this arrangement calculation of heights and distances for inclined sights is very much simplified

Alternative Proof —In Fig 28

let O = the optical centre of the objective

p and q = the top and bottom stadia wires

P and Q = the points on the staff cut by the two wires

F = the principal focus of the objective.

f_1 = the horizontal distance from the optical centre (O) to the staff

f_2 = the horizontal distance from the optical centre (O) to the image of the staff

f = the focal length of the objective.

d = the horizontal distance from the optical centre (O) to the vertical axis of the instrument.

D = the horizontal distance from the vertical axis of the instrument to the staff

H = the horizontal distance of the staff from the principal focus (F)

S = QP = the staff intercept.

i = qp = the interval between the stadia wires

Since the rays of light passing through the principal focus of the objective emerge parallel to the axis of the telescope, we have

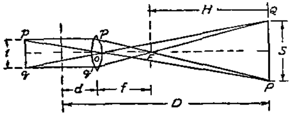


Fig 28

$qp = q'p' = i$

Since the triangles PFQ and p' Fq' are similar we get

$$\frac{QP}{q'p'} = \frac{H}{f} = \frac{S}{i} \quad \text{or} \quad H = \frac{f}{i} S$$

$$D = H + f + d = \frac{f}{i} S + (f + d) \qquad (1)$$

Determination of the Instrument or Tacheometric Constants —Two methods are available for determining the values of the constants $\frac{f}{i}$ and $f + d$ of a given instrument

First Method —In this method the value of $(f + d)$ is obtained by direct measurement and that of $\frac{f}{i}$ by computation, as the stadia hair interval (i) is too small (not exceeding 2 mm to 3 mm) to be measured very accurately

Procedure —(i) Sight any far distant object and focus it properly

(ii) Measure accurately the distance along the top of the telescope between the object glass and the plane of the cross hairs (diaphragm screws) with a rule, the measured distance being equal to the focal length (f) of the objective

(iii) Measure the distance (d) from the object glass to the vertical axis of the instrument.

(iv) Measure several lengths D_1, D_2, D_3, etc along AB from the instrument position A and obtain the staff intercepts S_1, S_2, S_3, etc at each of these lengths

(v) Knowing $f+d$, determine the several values of $\frac{f}{i}$ from formula (1)

(vi) The mean of the several values gives the required value of the constant $\frac{f}{i}$ Calculation work is simplified, if the instrument is placed at a distance of $f+d$ beyond the beginning (A) of the line

There are two types of an external focussing telescope, viz (i) one in which the object glass is moved in focussing, in which case the value of d is variable for different lengths of sights, being slightly greater for short sights than for long sights, and (ii) the other in which the eyepiece and diaphragm are moved in focussing, in which case the value of d is constant

However, the variation in the value of d is negligible, since it is few millimetres The value of d is measured when the telescope is focussed for an average length of sight

Note —The additive constant of an internal focussing telescope cannot be determined in this way One has to rely upon the figure supplied by the maker

Second Method —In this method the values of the constants $\frac{f}{i}$ and $f+d$ are obtained by computation

Procedure —(i) Measure a line OA, about 240 m long, on a fairly level ground with a steel tape, and fix pegs along it at intervals of, say, 30 m

(ii) Set up the instrument at O and obtain the staff intercepts by taking stadia readings on the staff held truly vertical on each of the pegs

On substituting the values of D and S in the formula (1) we get a number of equations which, when solved in pairs

determine the several values of the constants $\frac{f}{i}$ and $f+d$, their mean values being adopted for the values of the constants. Thus, let D_1, D_2, D_3 etc = the distances measured from the instrument, and S_1, S_2, S_3, etc = the corresponding staff intercepts Then we have

$$D_1 = \frac{f}{i} S_1 + (f+d), D_2 = \frac{f}{i} S_2 + (f+d), D_3 = \frac{f}{i} S_3 + (f+d),$$

etc

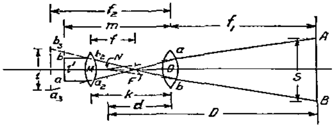


Fig 29

Anallatic Lens —The object of providing an additional convex lens, called an anallatic lens in the telescope is to eliminate the additive constant $(f+d)$ This can be done by bringing the apex (N) of the tacheometric angle ANB (or the vertex N of the measuring triangle ANB) (Fig 29) into exact coincidence with the centre of the instrument Fig 29 illustrates the arrangement of lenses in an anallatic telescope The anallatic lens is placed between the eyepiece and the object glass at a fixed distance from the latter It may be noted that it is provided in an external focussing telescope only and not in the internal focussing telescope which is virtually anallatic since the value of $(f+d)$ is only a few centimetres The disadvantage of the anallatic lens is the reduction in brilliancy of the image due to increased absorption of light The theory of the anallatic lens may be explained as follows —

In Fig 29, let S = the staff intercept AB

i' = the length *ba* of the image of AB when the anallatic lens is interposed (the actual stadia interval)

i = the length b_3a_3 of the image of AB when no anallatic lens was provided

O = the optical centre of the object glass

M = „ „ of the anallatic lens.

k = the distance between the optical centres of the object glass and the anallatic lens

f = the focal length of the object glass

f = the focal length of the anallatic lens

F_1 = the principal focus of the anallatic lens

N = the centre of the instrument

d = the distance from the centre of the object glass to the vertical axis of the instrument.

D = the distance from the vertical axis of the instrument to the staff

f_1 and f_2 = the conjugate focal lengths of the object glass

m = the distance from the optical centre of the object glass to the actual image ba

The rays of light emanating from A and B along AN and BN are refracted by the object glass and meet at F_1 The anallatic lens is so placed that F_1 is its principal focus Therefore these rays passing through F_1 would emerge in a direction parallel to the axis of the telescope after passing through the anallatic lens Thus the path of the ray from A is $Aa_1F_1a_2a$ and that of the ray from B is $Bb_1F_1b_2b$ Thus ba represents the actual image of the staff intercept AB

These rays would have formed the image b_3a_3 if the anallatic lens was not interposed

Now by laws of lenses $$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \text{(i)}$$

and $$\frac{1}{f} = \frac{1}{(m-k)} - \frac{1}{(f_2-k)} \qquad \text{(ii)}$$

The negative sign is used in (ii) as ba and b_3a_3 are on the same side of the anallatic lens $(m-k)$ and (f_2-k) are the conjugate *focal lengths of the anallatic lens*

$$\text{Also, } \frac{S}{i} = \frac{f_1}{f_2} \quad .. \quad \text{(iii)} \quad \text{and} \quad \frac{i}{i'} = \frac{(f_2-k)}{(m-k)} \quad .. \quad \text{(iv)}$$

By eliminating m, f_2, and i from these equations, we get

$$f_1 = \frac{ff'\,S}{i'(f+f'-k)} - \frac{f(k-f')}{(f+f'-k)}$$

$$\therefore\ D = f_1 + d = \frac{ff'}{(f+f'-k)} \cdot \frac{S}{i'} - \frac{f(k-f')}{(f+f'-k)} + d \quad . . \quad \text{(v)}$$

Now the term $d - \left\{ \dfrac{f(k-f')}{(f+f'-k)} \right\}$ should be equal to zero in order that D should be proportional to S.

$\therefore$ The distance of the anallatic lens from the object glass

$$= k = f' + \frac{fd}{(f+d)} \quad .. \quad ... \quad ... \quad .. \quad (2)$$

When this condition obtains, the vertex (N) of the measuring triangle ANB is exactly coincident with the centre of the instrument, *i. e* N is situated on the vertical axis of the instrument. By adopting suitable values of f, f', k, and i', $\dfrac{ff'}{i'(f+f'-k)}$ is made equal to 100 Hence we have D = 100

Case 1 —When the line of sight is horizontal and the staff held vertically:

Horizontal distance (D) of the staff from the vertical axis of the instrument is given by $D = \dfrac{f}{i} S + (f+d)$

Elevation (or R L.) of the staff station
= elevation of the instrument axis − axial hair reading

Elevation of the instrument axis
= elevation of the bench mark + backsight
or = elevation of the inst station + H I.

where H I = the height of instrument or instrument axis i e the vertical distance from the instrument station (top of peg) to the centre of the object glass

Inclined Sights —When the ground is rough, horizonta sights are not possible and, therefore, inclined sights must be taken In this case the staff may be held either vertical or normal to the line of collimation (or sight)

Case II —When the line of collimation (or sight) is inclined to the horizontal and the staff is held vertically (Fig 30)

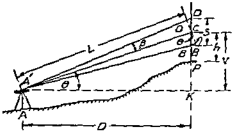


Fig 30

Let A = the instrument station
A = the position of the instrument axis
P = the staff station.
D, C, and B = the points on the staff cut by the hairs of the diaphragm
CA′K = θ = the inclination of the line of collimation A C to the horizontal
DB = S = the staff intercept
PC = h = the axial reading
A′C = L = the distance along the line of collimation from the instrument axis A′ to the point C
A′K = D = the horizontal distance from the vertical axis of the instrument to the staff station P
KC = V = the vertical distance from the instrument axis to the point C

Through C draw a line perpendicular to the line of collimation A′C, cutting A′D and A′B in D′ and B′ respectively so that D′B′ is the projection of the staff intercept DB perpendicular to A′C

It will be seen from the figure that the lines DB and D′B are perpendicular to the lines A′K and A′C respectively, and, therefore, the angles DCD′ and BCB′ are each equal to θ. Now let the angles DA′C and BA′C be each denoted by β. Then the exterior angle DD′C of the triangle D′A′C = $\angle$ A′CD′ + $\angle$D′A′C $= 90^\circ + \beta$.

Also, in the triangle CA′B′, $\angle$A′B′C + $\angle$B A′C $= 90^\circ$,

since A′CB′ is a right angle Hence $\angle$A′B′C $= 90^\circ - \beta$, i.e. $\angle$BB′C $= 90^\circ - \beta$

Since β is a very small angle $\left(\tan^{-1}\dfrac{1}{200}\right)$, it may be neglected, and the angles DD′C and BB′C may be assumed to be 90°.

$\therefore$ D′B′ = DB cos θ = S cos θ.

If the angles DD′C and BB′C be taken at their correct values, it may be shown that $D'B' = S\cos\theta - S\dfrac{\sin^2\theta}{\cos\theta}\tan^2\beta$.

Now by formula (1), $L = \dfrac{f}{i} D B' + (f + d)$

$$= \frac{f}{i} S\cos\theta + (f + d)$$

Horizontal distance A′K $= D = L\cos\theta$

$$= \frac{f}{i} S\cos^2\theta + (f + d)\cos\theta \quad \text{.. (3)}$$

Vertical distance KC $= V = L\sin\theta$

$$= \frac{f}{i} S\sin\theta\cos\theta + (f + d)\sin\theta$$

$$= \frac{f}{i} S\frac{\sin 2\theta}{2} + (f + d)\sin\theta \quad \text{.. (4)}$$

or " " " $= D\tan\theta$ ... ... (4a)

Knowing V, the elevation of the staff station P may be determined as follows

(*a*) When the observed vertical angle is an *angle of elevation* (+ angle) (Fig 29)

Elevation (or R L) of the instrument axis
= elevn of bench mark + backsight
= elevn of inst station + H I

Elevation (or R L) of the staff station P
= elevn of the inst axis + V − axial reading (h) (5)

(*b*) When the observed vertical angle is an *angle of depression* (− angle) (Fig 31)

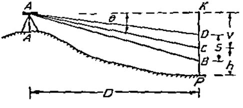


Fig 31

Elevation of the staff station P
= elevn of the inst axis − V − axial reading (h) (6)

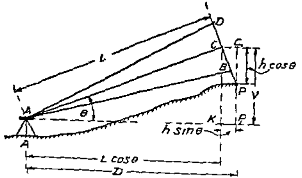


Fig 32

Case III —When the line of collimation is inclined to the horizontal and the staff is held normal to the line of collimation.

(a) When the vertical angle is an angle of elevation (+ angle) (Fig 32) —

Let AC = the line of collimation inclined at an angle θ to the horizontal

$DB = S$ = the staff intercept

$PC = h$ = the axial reading

Through C draw CC_1 horizontal meeting the vertical line through P in C_1

$CPC_1 = CAK = \theta$ so that $CC_1 = PC \sin CPC_1 = h \sin \theta$ and $PC_1 = PC \cos CPC_1 = h \cos \theta$

Now the distance along the line of collimation

$$= L = \frac{f}{i} S + (f + d)$$

The horizontal distance $D = L \cos \theta + KP_1 = L \cos \theta + h \sin \theta$ since $KP_1 = CC_1 = h \sin \theta$

$$D = \frac{f}{i} S \cos \theta + (f + d) \cos \theta + h \sin \theta \quad (7)$$

The vertical distance $V = L \sin \theta$

$$= \frac{f}{i} S \sin \theta + (f + d) \sin \theta \quad (8)$$

Elevation (or R L) of the staff station P
= elevn of the instrument axis + $V - h \cos \theta$ (9)

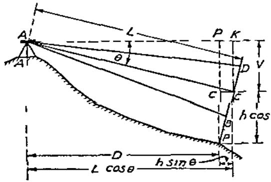


Fig 33

(b) When the vertical angle is an angle of depression ($-$angle):—KP_1 has to be subtracted from $L\cos\theta$ to obtain the horizontal distance D as is evident from the Fig. 83

$$\therefore\ D = \frac{f}{i}\ s\cos\theta + (f+d)\cos\theta - h\sin\theta \quad \ldots \quad \ldots \ (10)$$

The expression for the vertical distance V is the same as (8)

Elevation (or R. L.) of the staff station P

$=$ elevn. of the inst. axis $- V - h\cos\theta$ ... ... (11)

When θ is small, $h\sin\theta$ may be neglected and $h\cos\theta$ taken equal to h.

Subtense Method (Movable Hair Method):—In this method the instruments used are (1) a subtense theodolite and (2) a staff provided with two vanes or targets at some known distance apart, usually 3 to 6 m A third vane is fixed exactly midway for levelling purposes The subtense theodolite (Fig. 34) is equipped with a diaphragm, the axial wire of which is fixed in the optical axis of the telescope and the other two wires can be moved from the axial wire by means of two finely threaded micrometer screws so as to intercept the distance between the targets. The distance through which either wire is moved from the middle one is measured by the number of turns made by the micrometer screw, the whole turns being read on the comb (scale) seen in field of view and the fractional parts of a turn on the graduated drums of the micrometer screws, which are placed one above and one below the eyepiece Thus the distance through which the stadia wires are moved is given by the sum of the micrometer readings It may be observed that in this method the staff intercept (S) is constant and the stadia interval is variable.

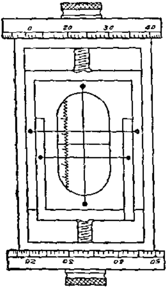


Fig 34

In observing with the instrument, the middle vane or target is first bisected with the axial wire and the micrometer screws are then simultaneously turned to move the stadia wires

When the line of sight is horizontal the horizontal distance D is given by $D = \frac{KS}{n} + (f + d)$ (12)

in which K and $(f + d)$ are the constants of the instrument, and n the sum of the micrometer readings

The value of K varies from 600 to 1000 If there is an index error e, $D = \frac{KS}{n - e} + (f + d)$ (12a)

When the line of sight is inclined the formulae 8 to 11 may be used in making the necessary calculations

Reduction of Stadia Notes —In practice, the horizontal and vertical distances are not calculated by the direct application of formulæ, since it is laborious But they are found by the use of (i) stadia tables, (ii) stadia diagrams, or (iii) stadia slide rule The reduction work is also greatly facilitated by the use of an instrument fitted with a Beaman stadia arc, or the direct reading tacheometer In stadia tables the values of $\cos^2\theta$ and $\frac{1}{2}\sin 2\theta$ for various values of θ are given in columns headed as Hor Dist, and Diff Elev for each metre of staff intercept when $\frac{f}{i} = 100$ The values of $(f + d)\cos\theta$ and $(f+d)\sin\theta$ for a few values of $f + d$ are also given for each degree of vertical angle at the bottom of the columns Thus, suppose the vertical angle is $+ 3° 20$, and staff intercept 1 75 m From the tables, the values of $\cos^2 3° 20$ and $\frac{1}{2}\sin 6° 40$ are found to be 99 66 and 5 80, and those of $(f + d)\cos 3°$ and $(f + d)\sin 3°$, 0 80 and 0 018 for $f + d = 0\ 30$ Then the horizontal distance $D = 99{\cdot}66 \times 1\ 75 + 0\ 30 = 174\ 7$ ft and the vertical distance $V = 5\ 80 \times 1\ 75 + 0\ 018 = 10\ 17$ m

The Beaman Stadia Arc (Fig 35) —It is a mechanical device fitted to the vertical circle of a theodolite or to the telescopic alidade of a plane table It enables the surveyor to reduce

rapidly an inclined stadia distance (L) to the corresponding horizontal distance (D) and the vertical component (V) (difference in elevation) without measuring vertical angles and without intricate calculations, or without the use of tables, diagrams, or stadia slide rule It consists of two scales (1) The vertical scale marked V in the figure and (2) the horizontal scale marked H The graduations on the vertical scale are figured by whole numbers in terms of $100 \times \frac{1}{2} \sin 2\theta$ When the telescope is horizontal the index I is opposite the zero graduation The horizontal scale gives the percentage corrections to be deducted from the observed stadia distance $\left(\frac{f}{i} S\right)$

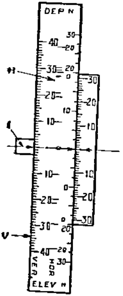


Fig 35

Method of use to determine the vertical component (V) —(1) Set up the instrument and direct the telescope to the staff (2) Read the stadia wires and find the stadia intercept (S) Then raise or depress the telescope slightly by means of the vertical tangent screw until the index I coincides exactly with the nearest graduation on the vertical scale (V) and note the reading A plus reading indicates elevation and a minus reading depression (3) Observe the reading of the central wire on the staff (4) Multiply the stadia intercept by the whole number reading This gives the vertical component (V) (5) Obtain the elevation of the staff point by the relation

Elevation of the staff point = elevation of inst axis $\pm$ V − central wire reading

Illustration —Suppose the stadia intercept = 1 450 m, the vertical scale reading = + 20 the central wire reading = 2 05 m, and the elevation of the inst. axis = 104 85 Then V = 1 45 × 20 = 29 00 m

Elevation of the staff point = 104 85 + 29 00 − 2 05 = 131 80

To determine the horizontal distance (*D*) :—(1) Read the horizontal scale simultaneously as the vertical scale is read, and note the reading by means of the same index I (2) Multiply the stadia intercept by the reading obtained in (1) This gives the correction to be subtracted from the distance (100 S) obtained from the stadia intercept. (3) Add the value of the additive constant ($f+d$) to this computed distance. The result gives the horizontal distance (D).

It may be noted that observations with the Beaman stadia arc do not include the effect of the additive constant ($f+d$).

Illustration :—Suppose the horizontal scale reading = 4 and the stadia intercept =1·45 m ; $f+d$ = 0 30

Then the correction = 4 × 1·45 = 5·80 (— ve).

∴ The horizontal distance (D)= 145 − 5·80+ 0 30 = 139 50m.

In another form of the Beaman stadia arc (Fig 36) the zero graduation of the vertical scale is marked 50 instead of zero so that when the telescope is horizontal the index I is opposite 50. Therefore, 50 must be subtracted from every reading. When the telescope is elevated, the vertical scale reading is greater than 50 (a plus reading indicating elevation), while it is less than 50 when the telescope is depressed (a minus reading indicating depression). Then the vertical component (V) = stadia intercept × (stadia arc reading — 50).

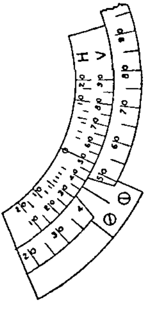


Fig 36

Direct-Reading or Auto Reduction (Self-Reducing) Tacheometers ·— These instruments are so designed that the values of the horizontal distance (D) and the vertical component (V) may be read directly on the staff without measuring vertical angles. There are three types of these instruments used in practice.

The Jeffcott Direct-Reading Tacheometer —In this instrument the diaphragm carries three points or pointers (Fig 37) by

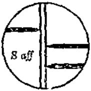


Fig 37

means of which staff readings are taken Of these three points, the middle one is fixed and the other two are movable They are actuated by a system of cams and levers, and are automatically set as the telescope is elevated or depressed The right-hand movable pointer is called the distance pointer, and the staff intercept between the fixed pointer and the distance pointer multiplied by 100 gives the horizontal distance (D), the telescope of instrument being anallatic The left-hand movable pointer is called the height pointer, and the staff intercept between the fixed pointer and the left hand pointer multiplied by 10 gives the vertical component (V) The staff readings are taken by first setting the fixed pointer at a metre or decimetre mark and then reading the other two pointers Suppose the readings are 1 65) 1 20 , 0 84 Then the horizontal distance (D) = 100 (1 65 — 1 20 = 45 m and the vertical component (V) = 10 (1 20 — 0 84) = +3 6 m. It may be noted that the left hand pointer moves upwards from the fixed pointer for angles of elevation, while it moves downwards for angles of depression

The Szepessy Direct-Reading Tacheometer —In this instrument a scale of tangents of vertical angles is engraved on a glass arc which is fixed to the vertical circle cover By means

Fig 38

of prisms the scale is brought into the field of view of the eye piece, and when the staff is sighted, the image of the staff is seen

alongside that of the scale (Fig 38) It is graduated to 0 005 and numbered at every 0 01 Thus the graduation 12 corresponds to the angle whose tangent is equal to 0 12

To read the staff, (i) bring a numbered division, say, 16, opposite the horizontal cross hair by means of the vertical circle tangent screw. Note the staff reading at this division (axial hair reading) (ii) Read the staff intercept between the short 0 005 divisions immediately above and below the numbered division. This intercept multiplied by 100 gives the horizontal distance D, *while the vertical component (V) is obtained* by multiplying the intercept by the number marking the division brought opposite the horizontal cross hair Suppose the staff *intercept is 0 72 and the number is 16 Then the horizontal* distance (D) is 72 m and the vertical component equals 0 72 × 16 = 11 52 m

The Auto-reduction Tacheometer (Hammer-Fennel) — This instrument is provided with a special auto reduction device In the field of view are seen four curves marked by the letters N, E, D, and *d* The N curve is the zero curve, the E curve is for reading distance the D curve is to be used for angles upto ± 14°, while the *d* curve is to be used for angles upto ± 47° The *curves* D *and d are marked + for angles of elevation, and* — for angles of depression The multiplying constant for the distance curve is 100 that for the height curve (D) is 10, while for the height curve (*d*), it is 20

To take a reading the zero (N) curve is made to bisect the specially marked zero point of the staff by bringing the perpendicular edge of the prism into line with the staff The staff readings are taken with the distance curve and the height curve The distance curve reading multiplied by 100 gives the horizontal distance D while the height curve reading multiplied by the corresponding multiplying constant gives the vertical component V.

Tangential Method :—The method is used when the telescope is not fitted with a stadia diaphragm. The horizontal and vertical distances of the staff station from the instrument station may be computed from observations taken to two vanes or targets on the staff at a known distance (S) apart, usually 3 m.

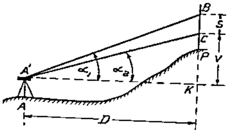


Fig. 39

Case I :—When both the observed angles are angles of elevation : (Fig. 39) Let

A = the instrument station.
A′ = the position of the instrument axis.
P = the staff station.

BA′K = α_1 = the vertical angle to the upper vane.
CA′K = α_2 = " " " the lower vane.
BC = S = the distance between the vanes.
KC = V = the vertical distance from the instrument axis to the lower vane.
A′K = D = the horizontal distance from the instrument station A to the staff station P.
PC = h = the height of the lower vane above the foot of the staff.

Then BK $= V + S =$ A′K tan BA′K $= D \tan \alpha_1$.
CK $= V =$ A′K tan CA′K $= D \tan \alpha_2$.

$\therefore$ S $= D\ (\tan \alpha_1 - \tan \alpha_2)$

or $$D = \frac{S}{(\tan \alpha_1 - \tan \alpha_2)} = \frac{S \cos \alpha_1 \cos \alpha_2}{\sin\ (\alpha_1 - \alpha_2)} \quad \dots \quad \dots \quad (13)$$

$$V = D \tan \alpha_2 = \frac{S \tan \alpha_2}{(\tan \alpha_1 - \tan \alpha_2)} = \frac{S \cos \alpha_1 \sin \alpha_2}{\sin\ (\alpha_1 - \alpha_2)} \quad (14)$$

Elevation of the staff station P
= elevn. of the instrument axis + V − h ... ... (15)

Case II —When both the observed angles are angles of depression (Fig 40)

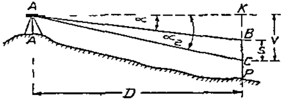


Fig 40

As before, $V = D \tan \alpha_2$ and $V - S = D \tan \alpha_1$

$$S = D(\tan \alpha_2 - \tan \alpha_1)$$

$$\text{or} \quad D = \frac{S}{(\tan \alpha_2 - \tan \alpha_1)} = \frac{S \cos \alpha_1 \cos \alpha_2}{\sin(\alpha_2 - \alpha_1)} \quad (16)$$

$$V = D \tan \alpha_2 = \frac{S \tan \alpha_2}{(\tan \alpha_2 - \tan \alpha_1)} = \frac{S \cos \alpha_1 \sin \alpha_2}{\sin(\alpha_2 - \alpha_1)} \quad (17)$$

Elevation of the staff station P
= elevn of the instrument axis $- V - h$ (18)

Case III —When one of the observed angles is an angle of elevation and the other an angle of depression (Fig 41)

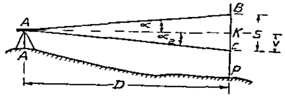


Fig 41

Let α_1 be the plus angle and α_2 the minus one

Now $V = D \tan \alpha_2$, $S - V = D \tan \alpha_1$

$$S = D(\tan \alpha_1 + \tan \alpha_2)$$

$$\text{or} \quad D = \frac{S}{(\tan \alpha_1 + \tan \alpha_2)} = \frac{S \cos \alpha_1 \cos \alpha_2}{\sin(\alpha_1 + \alpha_2)} \quad (19)$$

$$V = D \tan \alpha_2 = \frac{S \tan \alpha_2}{(\tan \alpha_1 + \tan \alpha_2)} = \frac{S \cos \alpha_1 \sin \alpha_2}{\sin(\alpha_1 + \alpha_2)} \quad (20)$$

Elevation of the staff station P
= Elevn of the instrument axis $- V - h$ (21)

The disadvantages of the tangential method are (i) two vertical angles have to be observed (ii) the instrument may be disturbed between the two observations, (iii) change in the atmospheric refraction may occur during the interval, and (iv) it lacks speed, and readings are not easily reduced

The amount of calculation work is practically the same in both the tangential and stadia methods However, the tangential method is considered inferior to the stadia method and is not in common use In general the stadia method (fixed hair method) with the staff held vertically is the most commonly used method of tacheometry

Holding the Staff (or Rod) —There are two ways of holding the staff in stadia method viz (i) vertical holding, and (ii) normal holding

(i) **Vertical Holding** —The staff must be held truly vertical In ordinary work the verticality of the staff is judged by eye, but in important work it is determined by suspending a plumb line, or by means of a folding circular bubble (Fig 42) attached to the

Fig 42

rear side of the staff and perpendicular to it so that the staff is vertical when the bubble is central Since the error due to non verticality of the staff is much more serious with large vertical angles than with small ones great care should be taken to ensure that the staff is held quite plumb when the vertical angles are large as in rough ground

(ii) **Normal Holding** —The staff is held normal i e at right angles to the line of sight its direction being judged by the staffman by sighting the instrument with the help of a pair of open sights called the staff director, or by means of a small telescope fixed at right angles to the side of the staff The advantages of normal holding are (i) the accuracy of the direction of

the staff can also be judged by the transitman who can see the staff director through the telescope, (ii) the staff can be swung in high wind, and (iii) the errors caused in the distances and elevations due to the error in the direction of the staff are less serious in the case of normal holding than those due to the same amount of error in the direction in the case of vertical holding

In general, however the method of holding the staff vertically is most commonly adopted for the following reasons

(1) Ease with which the staff can be held plumb (2) Since the formulæ for the horizontal and vertical distances are simpler, reduction of stadia notes is less laborious and greatly simplified by the use of stadia tables (3) The verticality of the staff can be ensured by the use of a plumb line or circular bubble

Reading the Staff —There are three methods of taking stadia readings (1) Sight the staff, and raise or depress the telescope by means of the vertical tangent screw until one of the stadia wires (usually the apparent lower one) strikes some convenient graduation on the staff Read the other wire The difference between the two readings gives the staff intercept Read the middle wire and the vertical angle to the nearest minute or 20 seconds By this method mistakes in making deductions are avoided and the staff can be read easily

(2) Direct the telescope to the staff and clamp it so that the wires appear to be suitably placed for reading Read all the wires The difference between the readings of the stadia wires gives the staff intercept Stadia readings may be checked by the reading of the middle wire, which is the mean of the stadia readings Measure the vertical angle

(3) Direct the telescope to the staff and clamp it in a suitable position for reading Adjust the vertical circle to the nearest 20 and note the vertical angle Read all the three wires By this "even angle" method (in which vertical angles are multiples of 20), the trouble of reading vertical angles to the nearest minute or 20" is avoided It is easier and quicker to lay off an even angle on the circle than to read a vertical angle to the nearest 1' or 20"

Limiting Length of Sight —The length of sight depends upon (i) the magnifying power of the telescope, (ii) the fineness of the stadia wires, (iii) the graduations of the staff, (iv) the atmospheric conditions, and (v) the accuracy desired With an ordinary instrument and under average atmospheric conditions, sights upto 120 m may be taken with an error not exceeding 1 in 400 With high grade instruments and under favourable atmospheric conditions, sights up to 200 m may be taken with an error less than 1 in 1600

Field Work

Tacheometric Survey —A tacheometric survey is mainly conducted for contouring and filling in detail It is conducted by running (i) a traverse open (unclosed) or closed, depending upon the area to be surveyed, and (ii) locating the required detail from the traverse stations When the area is narrow, an open traverse is run approximately along the centre line of the strip as in route surveys When it is broad, a closed traverse is run When the area is very extensive as in the case of a proposed reservoir stations are arranged so as to form triangles, and polygons with central stations, from which the topographical detail may be surveved, or alternatively, a series of traverses are run to cover the whole area Tacheometer stations should be so selected that they will command a clear view of the area to be surveyed lying within the range of observation, and that the use of large vertical angles is obviated

Equipment —(i) A tacheometer, (ii) a levelling staff or stadia rod, (iii) a tape, (iv) ranging rods, etc

Field Party —The party comprises

(1) The surveyor who superintends all operations, selects stations, directs staffmen to different staff stations, and prepares a sketch showing the positions of stations, contours, etc

(2) The instrument man who is responsible for the actual observations

(3) The recorder who assists the instrument man and records the observations in the field book.

(4) Three or more staffmen.

(5) Two or more axemen for clearing

(6) A draughtsman if the survey is to be plotted in the field

Procedure:—The survey should be conducted in the following steps —

(1) Set up the instrument over the station which has been selected by the surveyor Centre and level it accurately The instrument should be levelled first with reference to the plate levels and then with reference to the altitude level

(2) With the vertical vernier set to zero and the altitude bubble central, measure the height of instrument (H I), i e the vertical distance from the top of the peg to the centre of the object glass with a tape or stadia rod read through the object glass

(3) The instrument should be correctly oriented at the first station of a traverse If the reference meridian is a true meridian and the true bearing of some other station or reference object with reference to the first station is known the instrument is oriented by setting one of the verniers to read this bearing and turning the telescope about the outer axis until the station is bisected On the other hand if the reference meridian is a magnetic meridian it is oriented by setting one of the verniers to zero and rotating the instrument about the outer axis until the compass needle points towards north

(4) Sight the staff held on the nearest bench mark with the line of sight horizontal or inclined as the case may be Observe (i) the vertical angle, (ii) the bearing and (iii) the staff readings of the three hairs (top, axial, and bottom) If the bench mark is not nearby, flying levels may be run from the B M to establish one near the area of the survey

(5) Locate all the representative points around the station and within the range of the instrument by taking observations to those points At each point (staff station), the observations required for its complete location should include (i) the bearing, (ii) the vertical angle, and (iii) the staff readings of the three wires, the observations to the staff points (or stations) being

known as *side shots* The bearings should be taken to the nearest 5′, and the vertical angles read to the nearest 1′ Observations to the staff stations can be taken more easily and quickly, if the staff stations are placed on the radial lines through the station, the angular interval being, say, 15° or 30°

(6) Take a foresight on the second traverse station and determine (i) the bearing, (ii) the vertical angle, and (iii) the staff readings of the three wires In taking the bearing, a ranging rod should be held over the station and bisected, or the edge of the staff may be bisected

(7) Transfer the instrument to the second station Centre and level it

(8) Measure the height of the instrument (H. I) as before

(9) Sight the staff held on the first station (backsight on the first station) Observe (i) the bearing (ii) the vertical angle, and (iii) the staff readings of the three wires Since each station is sighted twice, we obtain two values for the distance and elevation of each station If they agree within the limits of accuracy, the average of the two values may be taken as the value for the distance and elevation of the station If not, work should be repeated

(10) Locate all the points around the second station and within the range of the instrument as described above

(11) Take a foresight on the third station and take the necessary observations

(12) Proceed similarly at each of the successive stations The tacheometric traverse is usually run by the *fast needle* method

Form of Field Book —The field notes may be recorded in the following form

Instrument station	Height of instrument	Staff station.	W C bearing	Vertical Angle	Stadia Hair Reading	
					Top	Bottom

Stadia Intercept	Axial Hair Reading	Horizontal Distance D	Vertical Distance V	Reduced Level		Remarks
				Instrument axis	Staff station	

Errors in Stadia Surveying —The sources of error in stadia measurements may be listed as follows —

(I) Instrumental Errors —Errors due to (i) imperfect adjustment of tacheometer To eliminate this error, the instrument should be carefully adjusted particular attention being paid to the adjustment of the altitude bubble (ii) erroneous divisions on the stadia rod To minimise this error the rod should be standardised and corrections for erroneous length applied to the observed stadia intervals In ordinary work this error is negligible (iii) incorrect value of the multiplying constant $\frac{f}{i}$ This is the most important source of error because of its cumulative effect For accurate work, its value should be tested before commencing work by comparing stadia distances with measured distances during the hours which correspond to those of field observations

(II) Errors of Manipulation and Sighting — They include errors due to

(i) Inaccurate centering and levelling of the instrument

(ii) Non verticality of the staff or rod If the rod is not held truly vertically, the errors of horizontal distance and elevation vary with the magnitude of the observed vertical angles the errors being smaller for small vertical angles and greater for large vertical angles It may be eliminated by using a plumb line or a small circular spirit level

(iii) Inaccurate estimation of the stadia intercept Accurate reading is possible only when the rod can be clearly seen For this reason focussing should be done properly so as

to eliminate parallax. To guard against this error, read all the three wires. The centre wire reading checks the stadia wire readings, since it is the mean of the readings of the top and bottom wires. When the intercept between the stadia wires cannot be read, read the intercept between the middle wire and one of the stadia wires and multiply by two. Guard against blunders in reading the staff. Do not mistake the axial hair for a stadia hair.

(III) Errors Due to Natural Causes.—They comprise errors due to

(i) Wind: accurate reading is not possible in high wind, since the rod cannot be kept steady and quite plumb.

(ii) Unequal refraction: unequal refraction is due to varying densities in different strata of air. The error due to this cause is cumulative. The density of the stratum of air within 1 m of the ground being greater than that of air above it, rays of light passing through it are bent upward and consequently, the staff intercept as observed is less than what it should be. To avoid this error, do not take during mid-day hours readings requiring the line of sight to pass within 1 m of the ground. When the sights are long, take the upper half reading.

(iii) Unequal expansion: when working in a hot sun the instrument should always be protected by an umbrella.

(iv) Bad visibility: It is caused by glare of strong light coming from the wrong direction.

Precision of Stadia Surveying.—

The error in a single horizontal distance should not exceed 1 in 500 and the error in the measurement of a single vertical distance should not be more than 0.1 m.

Average error in distance varies from 1 in 600 to 1 in 850.

Error of closure in elevation , $0.08\sqrt{M}$ to $0.25\sqrt{M}$ m in which M = the distance in kilometre.

Closing error in a stadia traverse should not be greater than $0.055\sqrt{5}$ m in which P = the perimeter of the traverse in m.

Example 1 —A tacheometer was set up at a station A and the following readings were obtained on a vertically held staff —

Station	Staff station.	Vertical Angle	Hair readings	Remarks.
P	B M	$-4° 22$	1 050 1 103 1 156	R L of B M.
	Q	$+10° 0'$	0 952 1 055 1 158	= 1958 300

The constants of the instrument were 100 and 0 1

Find the horizontal distance from P to Q and the reduced level of Q

The horizontal distance to the staff station and the vertical distance of the axial reading above or below the inst axis may be obtained by

$$D = \frac{f}{i} S \cos^2\theta + (f + d) \cos\theta$$

and $V = \frac{f}{i} S \frac{\sin 2\theta}{2} + (f + d) \sin\theta$ respectively

$$\frac{f}{i} = 100 \text{ and } f + d = 0\ 1$$

First observation —

$S_1 = 1\ 156 - 1\ 050 = 0\ 106 \qquad \theta_1 = -4°\ 22$

$$V_1 = 100 \times 0\ 106 \frac{\sin 8°44}{2} + 0\ 1 \times \sin 4°\ 22 = 812 \text{ m}$$

Second observation —

$S_2 = 1\ 158 - 0\ 952 = 0\ 206, \quad \theta_2 = +10°$

$$V_2 = 100 \times 0\ 206 \frac{\sin 20°}{2} + 0\ 1 \sin 10° = 0\ 354 \text{ m}$$

$D_2 = 100 \times 0\ 206 \cos^2 10° + 0\ 1 \cos 10° = 20\ 078$ m

Now R L of inst axis = R L of B M + backsight + V_1

= 1958 300 + 1 103 + 0 812 = 1960 215

R L of Q = 1960 215 + 0 354 − 1 055 = 1959 514

Distance PQ = 20 078 m

Example 2 —To determine the elevation of the first station A of a tacheometric survey, the following observations were made, the staff being held vertically The instrument was fitted with an anallatic lens and the value of the constant was 100

Inst station	Height of instrument	Staff station	Vertical Angle	Staff readings	Remarks.
O	1 440	B M	−5°40	1 332, 1 896 2 460	R L of B M
„	1 440	C P	+8°20	0 780, 1 263 1 746	=158 205
A	1 380	C P	−6°24	1 158 1 617 2 076	

Calculate the reduced level of A.

Since the instrument was fitted with anallatic lens, the additive constant $(f+d)=0$ The multiplying constant $\left(\frac{f}{i}\right)=100$

Vertical distance of the axial reading above or below the inst axis —

$$V_1 = 100(2\ 460 - 1\ 332)\frac{\sin 11^\circ 20}{2} = 11\ 082 \text{ m}$$

$$V_2 = 100(1\ 746 - 0\ 780)\frac{\sin 16^\circ 40'}{2} = 13\ 854 \text{ m}$$

$$V_3 = 100(2\ 076 - 1\ 158)\frac{\sin 12^\circ 48}{2} = 10\ 170 \text{ m}$$

Now R L of inst axis at O = R L of B M + axial reading + V_1
= 158 205 + 1 896 + 11 082 = 171 183

∴ R. L of C P = R L of inst axis + V_2 − axial reading
= 171 183 + 13 854 − 1 263 = 183 774

R L of inst axis at A = R L of C P + axial reading + V_3
= 183 774 + 1 617 + 10 170 = 195 561

∴ R L of A = R L of inst axis at A − *ht* of inst
= 195 561 − 1 380 = 194 181

Example 3 —The following notes refer to a line levelled tacheometrically with an anallatic tacheometer, the multiplying constant being 100

Inst. station.	Height of axis	Staff station	Vertical Angle.	Hair readings	Remarks.
P	1 50	B M	$-$ 6° 12′	0 963, 1 515, 2 067	R L of B. M.
P	1.50	Q	$+$ 7° 5′	0 819, 1 341, 1·863	=460·650
Q	1 60	R	$+$ 12° 27′	1 860, 2 445, 3·030	Staff being held vertically.

Compute the reduced levels of P, Q, and R, and the horizontal distances PQ and QR.

(i) Distance to the staff station :—By $D = \frac{f}{i} S \cos^2 \theta$.

Staff intercept $= (1\cdot863 - 0\cdot819) = 1\cdot044$; $\theta = 7° 5'$.
$PQ = 100 \times 1\cdot044 \cos^2 7° \; 5' = 102.84$ m.

Staff intercept $= 3\ 03 - 1\ 86 = 1\cdot17$; $\theta = 12° 27'$.
$QR = 100 \times 1\ 17 \cos^2 12° 27' = 111\cdot54$ m.

(ii) Vertical distance of the axial reading above or below the inst. axis :— $= \frac{f}{i} S \frac{\sin 2\theta}{2}$.

$S_1 = (2\cdot067 - 0\cdot963) = 1\ 104,\ \theta = -6° 12'$.

$V_1 = 100 \times 1\cdot104 \frac{\sin 12° 24'}{2} = 11\cdot853$ m.

$S_2 = (1\cdot863 - 0\ 819) = 1\ 044;\quad \theta = 7° 5'$.

$V_2 = 100 \times 1\cdot044 \frac{\sin 14° 10'}{2} = 12\ 777$ m

$S_3 = 3\ 03 - 1\cdot86 = 1\ 17,\quad \theta = 12° 27'$.

$V_3 = 100 \times 1\ 17 \frac{\sin 24° 54'}{2} = 24\cdot636$ m.

Whence, R. L of the inst. axis at P

$= 460\cdot650 + 1\cdot515 + 11\cdot883$ $= 474\cdot048$.

R. L. of P $= 474\cdot048 - 1\ 500$ $= 472\cdot548$.

R. L. of Q $= 474\cdot048 + 12\cdot777 - 1\cdot341$ $= 485\cdot484$.

R. L. of inst. axis at Q $= 485\cdot484 + 1\cdot500$ $= 486\cdot984$.

R. L. of R $= 486\cdot984 + 24\cdot636 - 2\cdot445$ $= 509\cdot175$.

Example 4 :—The following is the data relative to observations made on a vertically held staff with a tacheometer fitted with an anallatic lens. The constant of the instrument was 100.

Instrument station.	Ht of axis.	Staff station	W.C B	Vertical Angle	Hair readings.	Remarks
O	1 56	A	12° 25′	0° 0′	1 88, 2 25, 2 62	R L. of O
		B	60° 45	+ 15°10′	1 83, 2 15, 2 47	= 130 25

Calculate the distance AB, and the reduced levels of A and B.

(i) Distance to the staff station :—By $D = \frac{f}{i} S \cos^2 \theta$.

$OA = 100\ (2 \cdot 62 - 1 \cdot 88) \cos^2 0 \qquad = 74$ m.
$OB = 100\ (2 \cdot 47 - 1 \cdot 83) \cos^2 15° 10' = 59 \cdot 62$ m.

(ii) Vertical distance of the axial reading above the inst. axis :—

By $V = \frac{f}{i} S \frac{\sin 2\theta}{2}$.

$V_1 = 0$ and $V_2 = 100\ (2\ 47 - 1 \cdot 83) \frac{\sin 30° 20'}{2} = 16 \cdot 15$ m.

(iii) Angle subtended at O by AB :—
$\angle AOB$ = bearing of OB − bearing of OA
$= 60° 45' - 12° 25' = 48° 20'$.

(iv) Distance AB :—
In the $\triangle$ AOB, OA = 74 m ; OB = 59·62m ; and $\angle AOB = 48° 20'$.

$AB = \sqrt{74^2 + (59\ 62)^2 - 2 \times 74 \times 59 \cdot 62 \cos 48° 20'}$.
$= \sqrt{3166} = 56 \cdot 27$ m.

Now R. L. of inst. axis at O = 130·25 + 1·56 = 131·81.
∴ R. L. of A = 131·81 − 2·25 = 129·56.
and R. L of B = 129·56 + 16·15 −2·15
= 143·56

Example 5 :—To determine the distance between two points C and D, and their elevations, the following observations

were taken upon a vertically held staff from two traverse stations A and B The tacheometer was fitted with an anallatic lens, the constant of the instrument being 100

Traverse station	Height of inst	Co-ordinates of station		Staff station	Bearing	Vertical Angle	Staff readings
A	1 56	875 65	2140 45	C	340° 24	+17° 28	1 25 1 89 2 53
B	1 50	1040 35	3020 75	D	16° 12	+14° 14	1 53 2 10 2 67

Compute (*a*) the distance CD (*b*) the gradient from C to D, and (*c*) the reduced levels of C and D, given that the reduced levels of stations A and B were 1825 60 and 1828 45 respectively

(i) Horizontal distance —By $D = \frac{f}{i} S \cos^2\theta$

$S \ = 2\ 53 - 1\ 25 = 1\ 28 \text{ m},\ \theta = 17°\ 28$

$AC = 100 \times 1\ 28 \cos^2 17°\ 28 \quad = 116\ 31 \text{ m}$

$S \ = 2\ 67 - 1\ 53 = 1\ 14 \text{ m},\ \theta = 14°\ 14$

$BD = 100 \times 1\ 14 \cos^2 14°\ 14 \quad = 117\ 1 \text{ m}$

(ii) Reduced bearing —

R B of AC = 360° − 340° 24 = 19° 36 or N 19° 36 W

, of BD = 16° 12 or N 16° 12 E

(iii) Latitudes and Departures —

By L = *l* cos reduced bearing and D = *l* sin reduced bearing

Latitude of AC = 116 31 cos 19° 36 = + 109 56

Departure of AC = 116 31 sin 19° 36 = − 39 00

Latitude of BD = 107 cos 16° 12 = + 102 87

Departure of BD = 107 sin 16° 12 = + 29 88

(iv) Co ordinates of stations C and D —

Total latitude of A = 875 65	*Total departure of A* = 2140 45
Add lat of C = 109 56(+*ve*)	Deduct dep of C = 39 00 (−*ve*)
Total lat of C = 985 21	Total dep of C = 2101 45 E
Total lat of B = 1040 35 N	Total dep of D = 3020 75
Add lat. of D = 102 87 (+*ve*)	Add dep of D = 29 88 (+ *ve*)
Total lat of D = 1143 22 N	Total dep of D = 3050 63 E

(v) Length of CD :—

Difference between north co-ordinates of C and D

$= 1143 \cdot 22 - 1040 \cdot 55 = + 102 \ 87$

Difference between east co-ordinates of C and D

$= 3050 \ 63 - 2101 \cdot 45 = + 949 \cdot 18$

$\therefore$ Latitude of CD $= + 102 \cdot 87$ and departure of CD$= +949 \cdot 18$

Hence length of CD $= \sqrt{(102 \ 87)^2 + (949 \cdot 18)^2} = 954 \ 73$ m.

or length of CD $= 949 \cdot 18$ cosec α

where α = R. B of CD $= \tan^{-1} \dfrac{949 \cdot 18}{102 \cdot 87} = 83° \ 49'$.

$= 949 \cdot 18$ cosec $83° \ 49'$

$= 954 \ 75$ m.

(vi) Vertical distance of the axial reading above inst. axis—

By $V = \dfrac{f}{i} S \ \sin 2\theta$

$$V_1 = 100 \ (1 \cdot 28) \ \frac{\sin 34° \ 56'}{2} = 37 \cdot 6 \text{ m.}$$

$$V_2 = 100 \ (1 \cdot 14) \ \frac{\sin 28° \ 28'}{2} = 27 \cdot 17 \text{ m.}$$

(vii) Reduced levels of C and D :—

R. L. of inst. axis at A $= 1825 \cdot 60 + 1 \cdot 56 = 1827 \cdot 16$

R. L. of C $= 1827 \cdot 16 + 37 \cdot 60 - 1 \cdot 89$

$= 1862 \cdot 87$.

(viii) R L. of inst. axis at B $= 1828 \cdot 45 + 1 \cdot 50 = 1829 \cdot 95$

R L of D $= 1829 \cdot 95 + 27 \cdot 17 - 2 \cdot 10$

$= 1855 \cdot 02$.

(ix) Gradient of CD .—

Difference in elevation between C and D

$= 1862 \cdot 87 - 1855 \cdot 02 = - 7 \ 85$ m.

The fall is from C to D.

Gradient from C to D $= \dfrac{7 \cdot 85}{954 \ 75} = 1$ in $82 \cdot 5$ (falling).

Example 6 :—To determine the constant multiplier of a tacheometer, the following observations were taken on a staff held vertically at distances measured from the instrument.

Observations	Horizontal distance in m	Vertical Angle	Stadia readings
1	60	0° 0	0 835 1 425
2	120	1° 15	1 140 2 345
3	180	1° 40′	1 240 2 990

Find the mean value of the constant, given that the additive constant was 0 25 m

Substituting the observed values in formula

$$D = \frac{f}{i} S \cos^2\theta + (f+d) \cos\theta, \text{ we have}$$

$$60 = (1\ 425 - 0\ 835)\frac{f}{i} + 0\ 25 = 0\ 59\frac{f}{i} + 0\ 25 \qquad (1)$$

$$120 = (2{\cdot}345 - 1\ 140)\frac{f}{i}\cos^2 1^\circ 15 + 0\ 25 \cos 1^\circ 15'$$

$$= 1\ 204\ \frac{f}{i} + 0\ 25 \qquad .. \qquad (2)$$

$$180 = (2\ 988 - 1\ 260)\frac{f}{i}\cos^2 1^\circ 40 + 0\ 25 \cos 1^\circ 40'$$

$$= 1\ 748\frac{f}{i} + 0\ 25 \qquad (3)$$

On solving the equations 1 to 4, we get

$$(1)\ \frac{f}{i} = 101{\cdot}3,\quad (2)\ \frac{f}{i} = 99\ 47;\ (3)\ \frac{f}{i} = 102\ 9$$

$\therefore$ The mean of these values gives the required value of the

constant $\dfrac{f}{i} = \dfrac{(101{\cdot}3 + 99{\cdot}47 + 102\ 9)}{3} = 101\ 2.$

Example 7 —A staff was held vertically at distances of 45 m and 120 m from the centre of a theodolite fitted with stadia hairs and the staff intercepts with the telescope horizontal were 0 447 m and 1 193 m respectively The instrument was then set over a station P of R L 500 25, and the height of the instrument was 1 45 m The hair readings on a staff held vertically at stations Q were 1 20 1 93 and 2 66 m while the vertical angle was $-9^{\circ}\ 30$ Find the distance PQ and the R L. of Q

(i) The constants of the instrument by equation

$$D = \frac{f}{i}s + (f + d) \text{ are}$$

$$45 = 0\ 447\ \frac{f}{i} + (f + d)$$

$$120 = 1\ 193\ \frac{f}{i} + (f + d)$$

The solution of these two equations gives $\frac{f}{i} = 100\ 5$ and $(f + d) = 0\ 10$

(ii) Stadia intercept $= 2\ 66 - 1\ 20 = 1\ 46$ m angle of depression $= 9^{\circ}\ 30$

The horizontal distance PQ by equation

$$D = \frac{f}{i}\, S \cos^2 \theta + (f + d) \cos \theta$$

$$= 100\ 5 \times 1\ 46 \cos^2 9^{\circ}\ 30 + 0\ 1 \cos 9^{\circ}\ 30 = 142\ 17 \text{ m}$$

(iii) The vertical distance (V) of the axial hair reading below the inst. axis by equation $V = \frac{f}{i}\, S \sin \theta \cos \theta + (f + d) \sin \theta$

$$= 100\ 5 \times 1\ 46 \times 16°8 + 0\ 1 \times 1650 = 23\ 89 + 0\ 02$$

$$= 23\ 91$$

(iv) R L of inst axis $= 500\ 25 + 1\ 45 = 501\ 70$

R L. of Q $= 501\ 70 - 23\ 91 - 1\ 93 = 475\ 86$

Example 8 :—Two observations were taken upon a vertical staff by means of a theodolite, the reduced level of its trunnion axis being 160·50. In the case of the first, the angle of elevation was 4° 36′ and the staff reading 0·75. In the case of second observation, the staff reading was 3·45 and the angle of elevation 5° 48′. Calculate the reduced level of the staff station and its distance from the instrument.

Here $S = 3{\cdot}45 - 0{\cdot}75 = 2{\cdot}70$ m ; $\alpha_1 = 5° 48'$; $\alpha_2 = 4° 36'$

(i) Horizontal distance :—By $D = \dfrac{S}{(\tan \alpha_1 - \tan \alpha_2)}$.

$$D = \frac{2.70}{(\tan 5° 48' - \tan 4° 36')} = 127{\cdot}95 \text{ m.}$$

(ii) Vertical distance of the smaller reading above the inst. axis :—By $V = D \tan \alpha_2$.

$V = 127{\cdot}95 \tan 4° 36' = 10{\cdot}30$ m.

(iii) Elevation of staff station :—

R. L. of inst. axis $= 160{\cdot}95$

∴ R. L. of the staff station $= 160{\cdot}95 + 10{\cdot}30 - 0{\cdot}75 = 170.50$

Example 9 :—The stadia intercept read by means of a tacheometer on a vertically held staff was 1·266 and the angle of elevation 7° 42′. The constants of the instrument were 100 and 0.3. Find the total number of turns registered on a movable hair instrument at the same station, if the intercept on the staff held on the same point was 1·65, the angle of elevation being 7° 36′. The constants of this instrument were 1000 and 0·45.

(i) The horizontal distance from the inst. station to the staff station is

$$D = \frac{f}{i} S \cos^2 \theta + (f + d) \cos \theta$$

$$= 100 \times 1{\cdot}266 \cos^2 \theta + 0{\cdot}3 \cos 7° 42' = 124{\cdot}65 \text{ m.}$$

(ii) In the second case, the same distance is obtained by formula

$$D = \frac{KS}{n} \cos^2 \theta + (f + d) \cos \theta.$$

$K = 1000$; $f + d = 0{\cdot}45$;

$$\therefore \quad D = \frac{1000 \times 1 \cdot 65}{n} \cos^2 7^\circ 36' + 0 \cdot 45 \cos 7^\circ 36' = 124 \cdot 65$$

Hence $\quad n = \dfrac{1650 \cos^2 7^\circ 36'}{(124 \cdot 65 - 0 \cdot 48)} = 13 \cdot 05.$

Example 10 :—The following notes refer to a traverse run by a tacheometer fitted with an anallatic lens. The constant of the instrument was 100 and the staff was normal.

Line.	Bearing.	Vertical Angle.	Staff intercept.
AB	30° 24′	+ 5° 6′	1·875
BC	300° 48′	+ 3° 48′	1·446
CD	226° 12′	− 2° 36′	1·725

Find the length and bearing of DA

(i) The lengths of AB, BC, and CD may be calculated by $D = \frac{f}{i} S \cos \theta$, since the staff was held perpendicular to the *line of sight. The correction, viz. (axial reading $\times \sin \theta$).* being small, is neglected.

$\therefore$ AB $= 100 \times 1 \cdot 875 \cos 5^\circ 6' \quad = 186 \cdot 78$ m.
BC $= 100 \times 1 \cdot 446 \cos 3^\circ 48' \quad = 144 \cdot 24$ m.
CD $= 100 \times 1 \cdot 725 \cos 2^\circ 36' \quad = 172 \cdot 35$ m

(ii) The reduced bearings of the lines may be obtained from their W. C bearings.

R. B of AB = N. 30° 24′ E ;
R. B of BC = N. 59° 12′ W. ;
R B. of CD = S 46° 12′ W.

(iii) The latitudes and departures of the lines may now be computed by $L = l \cos \alpha$ and $D = l \sin \alpha$.

Line AB . Latitude $= 186 \cdot 78 \cos 30^\circ 24' = + 161 \cdot 10$
Departure $= 186 \cdot 78 \sin 30^\circ 24' = + 94 \cdot 53$
Line BC · Latitude $= 144 \cdot 24 \cos 59^\circ 12' = + 73 \cdot 86$
Departure $= 144 \cdot 24 \sin 59^\circ 12' = - 123 \cdot 90$
Line CD : Latitude $= 172 \cdot 35 \cos 46^\circ 12' = - 119 \cdot 31$
Departure $= 172 \cdot 35 \sin 46^\circ 12' = - 124 \cdot 41$

(iv) The latitude and departure of DA may be found from the known latitudes and departures of the lines.

Since the traverse is a closed one, the algebraic sum of the latitudes is equal to zero. Similarly, that of the departures equals zero.

$\therefore \quad +161{\cdot}10 + 73{\cdot}86 - 119{\cdot}31 + L = 0$ or $L = -115{\cdot}65$

and $\quad +\ 94{\cdot}53 - 123{\cdot}90 - 124{\cdot}41 + D = 0$ or $D = +153{\cdot}78$

where L = the latitude of DA, and D = the departure of DA.

(v) Now $\tan \alpha = \dfrac{153{\cdot}78}{115{\cdot}65}$, where α is the reduced bearing of DA.

or $\quad \alpha = 53^\circ\ 3'$.

From the signs of the latitudes and departures of DA, it is evident that it is in the second (S. E.) quadrant.

$\therefore$ R. B. of DA = S. 53° 3′ E.

Hence W. C. B. of DA = 180° − 53° 3′ = 126° 57′.

(vi) Length of DA = 153·78 cosec 53° 3′ = 192.45 m.

Check : ,, of DA $= \sqrt{(115{\cdot}65)^2 + (153{\cdot}78)^2}$

$= 192{\cdot}45$ m.

PROBLEMS

1. Explain the principles on which various methods of determining distance with the help of a telescope are based and state how each one differs from the other.

A tacheometer fitted with an anallatic lens was used to observe the following :

From	To	Bearing	Vertical angle.	Hair Readings.
C	A	320°	+ 12°	0·906, 1·728, 2·550
C	B	50°	+ 10°	0.744, 2.199, 3·654

The value of the constant was 100 and the staff was held vertically Determine the length and gradient of AB.

(Ans. 323.1 m.; 1 in 20·35.)

2. A tacheometer is set up at an intermediate point on a traverse course AB and the following observations are taken on a staff held vertically

Staff station	Bearing	Vertical Angle	Intercept	Axial hair Reading
A	40° 35	— 4° 24	2 172	1 962
B	220° 35	— 5° 12	1 986	1 866

The instrument is fitted with an anallatic lens and the multiplying constant is 100 The reduced level of A being given as 350 75, calculate the length of AB and the reduced level of B

(Ans 412 86 m 351 96)

3 The elevation of a point P is to be determined by observations from two adjacent stations of a tacheometric survey The staff was held vertically upon the point and the instrument is fitted with an anallatic lens the constant of the instrument being 100 Compute the elevation of the point P from the following data taking both the observations as equally trustworthy

Inst Station	Height of Axis	Staff Point	Vertical Angle	Staff Readings	Elevation of Station
A	1 44	P	+ 3°12	1 20, 1 94, 2 68	75 23
B	1 38	P	- 5°36	1 66, 2 27, 2 88	95 67

(Ans 82 90)

4 A line was levelled tacheometrically with a tacheometer fitted with an anallatic lens, the value of the constant being 100 The following observations were made, the staff having been held vertically

Instrument Station	Height of axis	Staff at	Vertical Angle	Staff Readings	Remarks R L.
A	1 44	B M	- 2° 24	1 20, 1 83, 2 46	37 725
A	1 44	B	+ 4° 36	1 35, 1 82, 2 29	
B	1 41	C	+ 6° 12	0 72, 1 38, 2 04	

Compute the elevations of A, B, and C

(Ans 43 42, 50 57, 64 77)

5 An old temple is on a small hill adjoining a provincial road With a view of determining the distance of the temple and the height of the tower of the temple above its plinth, observations were taken from the centre of the road upon vertically held staff (a) on the plinth of the entrance door of the temple and (b) on the top of the tower The tacheometer was fitted with an anallatic lens—the constant of the instrument being 100

Instrument Station.	Height of Instrument	Staff Station.	Vertical Angle	Staff Readings
Centre of the road	1 56 m	Plinth at entrance door	+ 14° 14	1 53, 2 10, 2 67
"	"	Top of the tower	+ 17° 28	1 26 1 90, 2 54

Calculate (1) the distance of the entrance door from the centre of the road (2) the height of the towe above the plinth, and (3) the R L of the plinth of the temple, if the R. L of road station be 850 74

{Ans (1) 107 1 m, (2) 9 63 m (3) 877 26}

6 Derive an expression for the distance D of a vertical staff from a tacheometer, if the line of sight of the telescope is horizontal How do you determine the constants of a tacheometer? Describe in detail the methods of finding distances using (i) a fixed intercept and (ii) a variable intercept (U.B)

7 What is a tacheometer? State the procedure of determining the constants of this instrument Levels were carried from a bench mark to the first station A of a tacheometric survey by tacheometric observations The instrument was fitted with an anallatic lens and the value of the constant was 100. The following observations were made, the staff having been held vertically

Inst Station	Height of Axis	Staff at	Vertical Angle	Staff Readings
O	4 8	B M	− 2° 40	4 20 6 42, 8 64
,,		Change Point	+ 5° 6	3 65 5 30, 6·95
A	5 2		− 5° 36	4 15 6 15 8 15

If the R L of the B M is 575 45 calculate the R L of the station A

(U P)

(Ans. 666 23)

8 Outline the tangential method of tacheometry and deduce expressions for both horizontal and vertical distances

The following readings were taken with an anallatic tacheometer The value of the constant was 100 and the staff was held vertically —

Inst. station	Height of axis	Staff station	Vertical Angle	Staff readings	Remarks
A	4 80	B M	− 5° 30′	3 02 5 76, 8 50	R L of B M
A	4 80	B	+ 3 24	3 12, 5 58, 8 04	= 685 40
B	4 60	C	+ 6° 12′	2 94, 6 46, 9 98	

Determine the horizontal distances between A, B, and C and also the elevations of these three stations. (U P)

(Ans AB = 490 3 ft , BC = 696 ft 738 65 767 01 840 75)

\+ + +

CURVES

Curves are usually employed in lines of communication in order that the change of direction at the intersection of the straight lines shall be gradual. The lines connected by the curve are tangential to it and are called *tangents* or *straights* The curves are generally circular arcs, but parabolic arcs are often used in some countries for this purpose

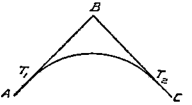


Fig 43

Circular curves are divided into three classes . (i) Simple, (ii) Compound, and (iii) Reverse.

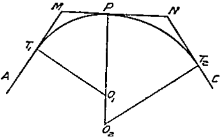


Fig 44

(i) Simple Curve —(Fig 43) A simple curve consists of a single arc connecting two straights

(ii) Compound Curve —(Fig 44) A compound curve consists of two arcs of different radii bending in the same direction and lying on the same side of their common tangent, their centres being on the same side of the curve

(iii) Reverse Curve —(Fig 45) A reverse curve is composed of two arcs of equal or different radii bending in opposite directions with a common tangent at their junction, their centres being on opposite sides of the curve.

Nomenclature of a curve :—A curve is designated either by the angle subtended by a chord of specified length or by the radius In America standard chord is 100 ft long and the curve is designated as a 2° curve or a 6° curve etc. In England the radius of the curve is expressed in terms of feet or chains (Gunter's) e g a 12 chain curve, a 24-chain curve etc The relation between the radius and the degree of the curve may be found as follows In India so far the standard chord was 100 ft With metric conversion. this may be changed to 30 m.

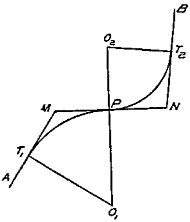


Fig 45

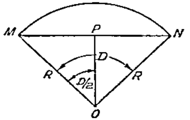


Fig 46.

Referring to Fig. 46, let R = the radius of a curve in m.
D = the degree of a curve.
MN = the chord 30 m long
P = its mid-point.

In the $\triangle$ OMP, OM = R ; MP = ½ MN = 15 m.; MOP $= \frac{D}{2}$.

Then $\sin \frac{D}{2} = \frac{MP}{OM} = \frac{15}{R}$ or $R = \frac{15}{\sin \frac{D}{2}}$ (exact) ... (1)

when D is small, $\sin \frac{D}{2}$ may be taken approximately equal to $\frac{D}{2}$ in radians

$$\therefore\ R = \frac{15}{\frac{D}{2} \times \frac{\pi}{180^\circ}} = \frac{15 \times 360^\circ}{\pi D} = \frac{1718\ 89}{D} \text{ (approximate)}$$

$$= \frac{1719}{D} \quad \text{,,} \quad (1a)$$

Notation for Circular Curve — (Fig 47) (1) The straight lines AB and BC, which are connected by the curve are called the tangents or straights to the curve

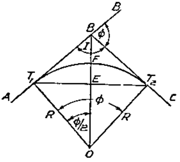


Fig 47

(2) The point (B) at which the two tangent lines AB and BC intersect is known as the point of intersection (P I) or the vertex (V)

(3) If the curve deflects to the right of the direction of the progress of survey (AB), it is called a right hand curve, if to the left, it is called a left hand curve.

(4) The tangent line AB is called the first tangent or rear tangent (also called the back tangent)

(5) The tangent line BC is known as the second tangent or forward tangent

(6) The points (T_1 and T_2) at which the curve touches the straights are called the tangent points (T P) The beginning of the curve (T_1) is called the point of curve (P C) or the tan gent curve (T C) The end of the curve (T_2) is known as the point of tangency (P T) or the curve tangent (C T)

(7) The angle ABC between the tangent lines AB and BC is called the angle of intersection (I) The angle B BO (i e the angle by which the forward tangent deflects from the rear tangent) is known as the deflection angle (ϕ) of the curve

(8) The distance from the point of intersection to the tangent point is called the tangent distance or tangent length (BT_1 and BT_2).

(9) The line T_1T_2 joining the two tangent points (T_1 and T_2) is known as the long chord (L)

(10) The arc T_1FT_2 is called the length of the curve (l)

(11) The mid point F of the arc T_1FT_2 is known as the apex or summit of the curve and lies on the bisector of the angle of intersection

(12) The distance BF from the point of intersection to the apex of the curve is called the apex distance or external distance

(13) The angle T_1OT_2 subtended at the centre of the curve by the arc T_1FT_2 is known as the central angle and is equal to the deflection angle (ϕ)

(14) The intercept EF on the line OB between the apex (F) of the curve and the mid point (E) of the long chord is called the versed sine of the curve

Elements of Simple Curve —(Fig 47)

$T_1BT_2 + B'BT_2 = 180°$ or $I + \phi = 180°$ (2)

The angle $T_1OT_2 = 180° - I = \phi$ (3)

Tangent length $= BT_1 = BT_2 = OT_1 \tan\dfrac{\phi}{2} = R\tan\dfrac{\phi}{2}$ (4)

Length of long chord $(L) = 2T_1E = 2OT_1 \sin\dfrac{\phi}{2} = 2R\sin\dfrac{\phi}{2}$ (5)

Length of the curve (l) = length of arc T_1FT_2

$= R \times \phi$ (in radians) $= \dfrac{\pi R\phi}{180°}$ (6)

If the curve be designated by the degree of the curve (D),

Length of the curve $= \dfrac{30\,\phi}{D}$ (6a)

Apex distance $= BF = BO - OF = OT_1 \sec\dfrac{\phi}{2} - OF$

$= R\left(\sec\dfrac{\phi}{2} - 1\right) = R \text{ exsec}\dfrac{\phi}{2}$ (7)

Versed sine of the curve $= EF = OF - OE = OF - OT_1\cos\dfrac{\phi}{2}$

$= R\left(1 - \cos\dfrac{\phi}{2}\right) = R \text{ versine}\dfrac{\phi}{2}$ (8)

Methods of Curve Ranging —The methods for setting out curves may be divided into two classes according to the instruments employed —

(1) Linear or chain and tape methods, and (2) Angular or instrumental methods

(1) Linear methods are those in which the curve is set out with a chain and tape only

(2) Instrumental methods are those in which a theodolite with or without a chain is employed to set out the curve

Peg Interval —It is the usual practice to fix pegs at equal intervals on the curve as along the straight The interval between the pegs is usually 20 to 30 m Strictly speaking this interval must be measured as the arc intercepted between them However, as it is necessarily measured along the chord the curve consists of a series of chords rather than of arcs In other words, the length of the chord is assumed to be equal to that of the arc In order that the difference in length between the arc and chord may be negligible, the length of the chord should not be more than $\frac{1}{20}$th of the radius of the curve

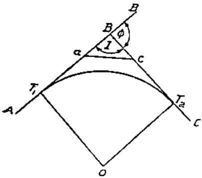


Fig 48

The length of unit chord (peg interval) is, therefore, 30 m for flat curves, 20 m for sharp curves and 10 m or less for very sharp curves When the curve is of a small radius, the peg intervals are considered to be along the arcs and the lengths of the corresponding chords are calculated to locate the pegs

Location of Tangent Points —(Fig 48) To locate the tangent points T_1 and T_2, proceed as follows —

(i) Having fixed the directions of the tangents produce them so as to meet at the point B

(ii) Set up a theodolite at the intersection point B and measure the angle of intersection T_1BT_2 Find the deflection angle ϕ from the relation $I + \phi = 180°$

(iii) Calculate the tangent lengths from formula (4)

(iv) Locate T_1 by measuring the tangent length backward along the rear tangent AB from the intersection point B

(v) Similarly, locate T_2 by measuring the same distance forward along the forward tangent BC from B

If an angle-measuring instrument is not available, the angle of intersection may be found by chain measurements thus

Set off from B equal distances Ba and Bc (say, 60 m) along BA and BC Measure ac accurately

$$\text{Then } \sin\frac{aBc}{2} = \sin\frac{I}{2} = \frac{ac}{2Ba} \text{ or } I = 2\sin^{-1}\left(\frac{ac}{2Ba}\right).$$

$$\text{From (2), } \phi = 180° - I = 180° - 2\sin^{-1}\left(\frac{ac}{2Ba}\right)$$

Having located the tangent points T_1 and T_2, their chainages may be determined The chainage of the first tangent point (T_1) may be obtained by subtracting the tangent length from the known chainage of the point of intersection (B) The chainage of the second tangent point (T_2) may be found by adding the *length of the curve* to the chainage of the first tangent point (T_1)

In railway and highway work, distances along the centre line are continuously measured from the point of beginning of the line, and it is customary to set stakes at an interval of 30 m or 20 m (or one chain) on it, the stations so fixed being called *full* stations, and record the distances in chain units as carried forward from the beginning of the line It rarely happens that the tangent point will be at a full station, i e its chainage is a whole number of chains (in 30 m or 20 m units) It is generally a *plus* station For example, let the chainage of T_1 be p chains and r links (written as $p + r$ in 30 m or 20 m units) The chainage of the first point on the curve will then be $p + 1$ chains, and the length of the first chord $150 - r$ or $100 - r$ links(chainage of the first point on the curve—chainage of the first tangent point) When the length of a chord is less than the length of the unit chord (i e less than the peg interval) the chord is called a sub chord Similarly, there will be a sub-chord at the end of the curve Thus the curve usually consists of two sub-chords and a number of unit chords.

Chain and Tape (or Linear) Methods of Setting out Curves.

(1) By Offsets or Ordinates from the Long Chord.—(Fig 49)

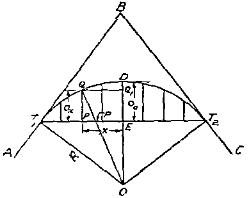


Fig 49

Let AB and BC = the tangents to the curve T_1DT_2.

T_1 and T_2 = the tangent points

T_1T_2 = the long chord of length L.

$ED = O_0$ = the offset at the midpoint of T_1T_2 (the versed sine)

$PQ = O_x$ = the offset at a distance x from E so that $EP = x$

$OT_1 = OT_2 = OD = R$ = the radius of the curve

The exact formula for the offset at any point on the long chord may be derived as follows:

Draw QQ_1 parallel to T_1T_2, meeting ED at Q_1. Join OQ cutting T_1T_2 in P_1

Now in the $\triangle\ OT_1E$, $OT_1 = R$, $T_1E = \dfrac{L}{2}$,

$$OE = OD - ED = R - O_0$$

$$OT_1^2 = T_1E^2 + OE^2 \text{ or } R^2 = \left(\frac{L}{2}\right)^2 + (R - O_0)^2.$$

$$\therefore\ O_0 = R - \sqrt{R^2 - \left(\frac{L}{2}\right)^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

Of the three quantities O_0, L, and R, two quantities L and R, or L and O_0 are usually known The remaining unknown may be calculated from the above formula

From the $\triangle$ OQQ_1, $OQ^2 = QQ_1^2 + OQ_1^2$.

$$\text{But } OQ_1 = OE + EQ_1 = OE + O_x = (R - O_0) + O_x$$

$$QQ_1 = x\,,\ OQ = R$$

$$\therefore \quad R^2 = x^2 + \left\{(R - O_0) + O_x\right\}^2$$

$$\text{or} \quad O_x + (R - O_0) = \sqrt{R^2 - x^2}$$

$$\text{Hence } O_x = \sqrt{R^2 - x^2} - (R - O_0) \ (\text{exact}) \qquad (10)$$

When the radius of the curve is large as compared with the length of the long chord, the offsets may be calculated from the approximate formula, which may be deduced as follows :

In this case PQ (the offset O_x at P) is very nearly equal to the radial ordinate QP_1

Then $QP_1 \times 2R = T_1P \times PT_2$

Now $T_1P = x$, $T_1T_2 = L$ $\quad PT_2 = L - x$, $QP_1 = O_x$

$$\therefore \quad O_x = \frac{x\,(L - x)}{2R} \quad (\text{approximate}) \quad .. \quad (11)$$

It must be remembered that in the first case, the distance of the point P is measured from the mid point of the long chord, while in the second case, it is measured from the first tangent point (T_1)

To set out the curve, (i) divide the long chord into an even number of equal parts (ii) Set out the offsets as calculated from formula (10) at each of the points of division, thus obtaining the required points on the curve Since the curve is symmetrical along ED, the offsets for the right half of the curve will be the same as those for the left half

If the offsets are calculated from formula (11), the long chord should be divided into a convenient number of equal parts and the calculated offsets set out at each of the points of division

This method is usually used for setting out short curves. e g curves for street kerbs.

When the offsets from the long chord T_1T_2 are too long to be set out with sufficient accuracy, the curve may be set out from the chords T_1D and DT_2 after the point D is located by the calculated offset ED The lengths of T_1D and DT_2 are now each equal to $2 R \sin \frac{\phi}{4}$ The rest of the procedure is the same as before

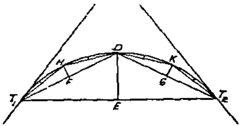


Fig 50

(2) By Successive Bisections of Arcs —(Fig 50) Let T_1 and T_2 be the tangent points Join T_1T_2 and bisect it at E Set out the offset ED (the versed sine) equal to $R\left(1 - \cos\frac{\phi_1}{2}\right)$, thus determining the point D on the curve Join T_1D and DT_2 and bisect them at F and G respectively At F and G set out the offsets FH and GK each equal to $R\left(1 - \cos\frac{\phi}{4}\right)$, thus obtaining two more points H and K on the curve By repeating the process, set out as many points as are required

(3) By Offsets from the Tangents —In this method the offsets are set out either radially or perpendicular to the tangents BA and BC according as the centre (O) of the curve is accessible or inaccessible

(*a*) *By Radial Offsets* —(Fig 51) Let T_1 = the first tangent point

$EE_1 = O_x$ = the radial offset at E at a distance of x from T_1 along the tangent AB

Now in the $\triangle\, OT_1E$ $OT_1 = R$, $T_1E = x$, $OE = OE_1 + E_1E = R + O_x$ Now $OE^2 = OT_1^2 + T_1E^2$.

$\therefore\ (R+O_x)^2 = R^2 + x^2$, i.e. $O_x = \sqrt{R^2 + x^2} - R$ (exact) ... (12)

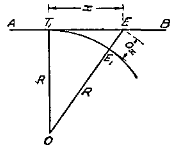


Fig 51

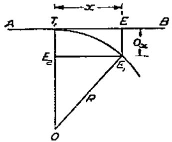


Fig 52

When the radius is large, the offsets may be calculated by the approximate formula, which may be deduced as follows:

$ET_1^2 = EE_1 \times (2R + EE_1)$, i e $x^2 = O_x \times (2R + O_x)$.

Since O_x^2 is very small as compared with 2R, it may be neglected.

$$\therefore\quad O_x = \frac{x^2}{2R} \qquad \text{(approximate)} \qquad \ldots \qquad (13)$$

This formula may also be obtained from the exact one thus:

Expanding the factor $\sqrt{R^2 + x^2}$, we have

$$O_x = R\left(1 + \frac{x^2}{2R^2} - \frac{x^4}{8R^4} + \qquad\right) - R$$

Neglecting the other terms except the first two, we get

$$O_x = R\left(1 + \frac{x^2}{2R^2}\right) - R = \frac{x}{2R} \text{ (approximate)}$$

(*b*) *By offsets perpendicular to the tangents* :— (Fig. 52). Let EE_1 be the perpendicular offset at a distance x measured along the tangent AB from the tangent point T_1 so that $T_1E = x$.

Through E_1 draw E_1E_2 parallel to BT_1, meeting OT_1 at E_2.

Then $E_1E_2 = T_1E = x$; $T_1E_2 = EE_1 = O_x$;

$OE_2 = OT_1 - T_1E_2 = (R - O_x)$.

Now from the $\triangle\ OE_1E_2$, $OE_1^2 = E_1E_2^2 + OE_2^2$

i.e. $R^2 = x^2 + (R - O_x)^2$ or $O_x = R - \sqrt{R - x^2}$ (exact) ... (14)

From which, the corresponding approximate formula may be obtained by expanding the factor $\sqrt{R^2 - x^2}$. Thus we have

$$O_x = R - R\left(1 - \frac{x^2}{2R^2} - \frac{x^4}{8R^4} - \quad\right)$$

Neglecting higher powers of R^2 we get

$$O_x = R - R\left(1 - \frac{x^2}{2R^2}\right) = \frac{x^2}{2R} \quad \text{(approximate)} \tag{14}$$

The method was formerly in common use for railway curves

To set out the curve, (i) locate the tangent points T_1 and T_2 by measuring a distance equal to the tangent length $R \tan\left(\frac{\phi}{2}\right)$ backward along the tangent BA from the point of intersection (B) and the same distance forward along the tangent BC

(ii) Measure equal distances say, 20 or 30 m along the tangent T_1B from T_1

(iii) Set out the offsets calculated from formula (14) or (18) perpendicular to T_1B at each distance thus obtaining the required points on the curve

(iv) Continue the process until the apex of the curve is reached

(v) Set out the remaining half of the curve from the second tangent

The objections to this method are (i) the points so located on the curve are not the same distance apart (ii) The points

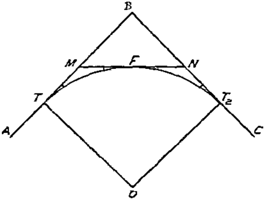


Fig 53

located by the offsets calculated from the approximate formula (18) do not lie on the circular arc, but lie on a parabola as is

evident from the expression for the offsets, which is the equation of a parabola. However, the curve approximates very closely to a circle, if the versed sine of the curve is less than one eighth of its chord. If the curve is set out by offsets calculated from the exact formula (14), the points so fixed will be on the circular arc. (iii) As the distances increase, the offsets become too long to be set out accurately, the errors of laying down the perpendicular direction and measuring long offsets being necessarily involved.

When the offsets become too long, the central portion of the curve may be set out from the third tangent through the apex of the curve as shown in Fig 53. When the curve is long, a new tangent is set out at the sixth point, the first six points being fixed by offsets from the first tangent. The next six points are then located by offsets from this new tangent. A third tangent is then set out at the twelfth point and the work repeated until the end of the curve is reached. The direction of the new tangent at any point may be determined as follows.

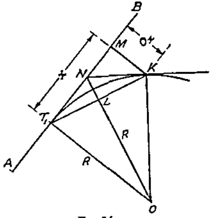


Fig 54

In Fig 54, let T_1B = the first tangent; K = the point on the curve at which a new tangent is to be set out.

$MK = O_x$ = the offset at a distance x from T_1

N = the point of intersection of the tangents T_1B and KN. Join ON, cutting the chord T_1K in L. The angles NMK and NLK being each a right angle, the points L, N, M, and K are concyclic. $\therefore$ $T_1N \times T_1M = T_1L \times T_1K$.

But $T_1L = \frac{1}{2} T_1K$ *and* $T_1K^2 = T_1M^2 + MK^2$.

$$\therefore \quad T_1N = \frac{T_1K^2}{2\,T_1M} = \frac{T_1M^2 + MK^2}{2\,T_1M}$$

Now $\quad T_1M = x$; $MK = O_x$

$$\text{Whence,} \quad T_1N = \frac{(x^2 + O_x^2)}{2x} \qquad \dots \qquad \dots \qquad \dots \quad (15)$$

To locate N, measure this distance along the tangent T_1B from T_1 and join NK, which gives the direction of the new tangent at K

(4) **By Offsets from Chords Produced** —(Fig 55)

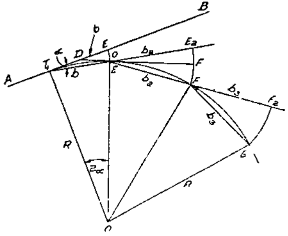


Fig 55

Let AB = the rear tangent, T_1 = the first tangent point

E, F, G, etc = the successive points on the curve

$T_1E_1 = T_1E$ = the first chord of length b_1

EF, FG, etc. = the successive chords of lengths b_2 b_3 etc each being equal to the length of unit chord

$BT_1E = \alpha$ in radian = the angle between the tangent BT_1 and the first chord T_1E

$E_1E = O_1$ = the offset from the tangent BT_1

$E_2F = O_2$ = the offset from the preceding chord T_1E produced

Draw the tangent DEF_1 at E, meeting the rear tangent at D Produce T_1E to E_2 so that $EE_2 = b_2$ Let F_1 be the point of intersection of DEF_1 and E_2F The formula for the offsets may be deduced as follows

The angle subtended at the centre (O) of the curve by the arc T_1E is obviously equal to 2α

But the chord T_1E = arc T_1E very nearly

$$= R \times 2 \alpha \quad \text{or} \quad \alpha = \frac{T_1E}{2R}.$$

Similarly, the chord E_1E = arc E_1E nearly.

$$\therefore \text{ The first offset } (O_1) = E_1E = T_1E \times \alpha = \frac{T_1E^2}{2R} = \frac{b_1^2}{2R} \qquad (16)$$

Now $\angle E_2EF_1 = \angle DET_1$ (vertically opposite), $\angle DET_1 = \angle DT_1E$, since $DT_1 = DE$, both being tangents to the circle

$$\therefore \quad \angle E_2EF_1 = \angle DT_1E = \angle E_1T_1E$$

The $\triangle s$ E_1T_1E and E_2EF_1 being nearly isosceles, may be considered approximately similar

$$\therefore \quad \frac{E_2F_1}{EE_2} = \frac{E_1E}{T_1E} \text{ i e } \frac{E_2F_1}{b_2} = \frac{O_1}{b_1}$$

$$\text{or} \quad E_2F_1 = \frac{b_2 \times O_1}{b_1} = \frac{b_2}{b_1} \times \frac{b_1^2}{2R} = \frac{b_2b_1}{2R}.$$

F_1F being the offset from the tangent at E, is equal to

$$\frac{EF^2}{2R} = \frac{b_2^2}{2R}$$

Now the second offset $(O_2) = E_2F = E_2F_1 + F_1F$

$$= \frac{b_2b_1}{2R} + \frac{b_2^2}{2R} = \frac{b_2(b_1 + b_2)}{2R} \quad . \quad \ldots \qquad (17)$$

$$\text{Similarly, the third offset } (O_3) = \frac{b_3(b_2 + b_3)}{2R} = \frac{b_2}{R} \quad .. \ (18)$$

since $b_2 = b_3 = b_4$ etc

Each of the successive offsets O_4, O_5, etc, except the last one (O_n) is equal to O_3 Since the length of the last chord is usually less than the length of unit chord (b_2),

$$\text{the last offset } (O_n) = \frac{b_n(b_{n-1} + b_n)}{2R} \qquad (19)$$

Mode of Procedure —(1) Having fixed the directions of the tangents AB and BC, locate the first tangent point T_1 by measuring backwards a distance equal to the tangent length $\left(R \tan \frac{\phi}{2}\right)$, along the tangent from the point of intersection (B)

Similarly, mark the other tangent point T_2 by measuring forwards the same distance along the tangent BC

(ii) Measure the distance equal to the length of the first chord (T_1E) along T_1B, thus marking the point E_1

(iii) With the zero end of the chain (or tape) pinned down at T_1, swing the portion of the chain ($= T_1E_1$) around the point T_1 through the calculated offset O_1, thus fixing the first point E on the curve

(iv) Pull the chain forward in the direction of T_1E produced, until EE_2 equals the length (1 chain or $\frac{1}{2}$ chain) of the second chord (b_2)

(v) Hold fast the zero end of the chain at E and swing the chain around E through the second calculated offset O_2, thus locating the second point F on the curve

(vi) Repeat the process until the end of the curve is reached The last point thus fixed should coincide with the previously located point T_2 If not, find the closing error If it is large, the whole curve must be set out again But if it is small all the points are moved sideways by an amount proportional to the square of their distances from the beginning of the curve (T_1), thus distributing the closing error among all the points

This method is largely used for road curves It gives better results than those obtained by the preceding method It can be used in confined situations, since all the work is done in the immediate proximity of the curve The most serious objection to this method is that if any point is inaccurately fixed, its error is carried forward through all the subsequent points

Instrumental Methods

(1) Rankine's Method of Tangential Angles —(Fig 56) In this method the curve is set out by the tangential angles (often called the deflection angles) with a theodolite and a chain or tape The derivation of the formula for calculating the deflection angles is as follows —

Let AB = the rear tangent to the curve

T_1 and T_2 = the tangent points.

D, E, F, etc. = the successive points on the curve.

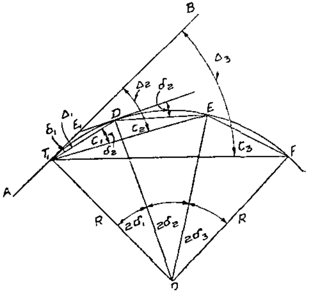


Fig 56

$\delta_1, \delta_2, \delta_3$, etc. = the tangential angles, which each of the successive chords T_1D, DE, EF, etc., makes with the respective tangents at T_1, D, E, etc.

$\Delta_1, \Delta_2, \Delta_3$, etc. = the total tangential or deflection angles (between the rear tangent AB and each of the lines T_1D, T_1E, T_1F, etc.) for the chords T_1D, DE, EF, etc.

c_1, c_2, c_3, etc. = the lengths of chords T_1D, DE, EF, etc.

R = the radius of the curve.

Chord T_1D = arc T_1D (very nearly) = c_1.

$BT_1D = \delta_1 = \frac{1}{2} T_1OD$ i. e. $T_1OD = 2\delta_1$.

Now $\frac{T_1OD}{c_1} = \frac{180°}{\pi R}$ i. e. $T_1OD = \frac{180° c_1}{\pi R}$ $\therefore$ $2\delta_1 = \frac{180° c_1}{\pi R}$

Hence $\delta_1 = \frac{90° c_1}{\pi R}$ degrees $= \frac{90 \times 60\, c_1}{\pi R}$ minutes.

$$= 1718 \cdot 9 \frac{c_1}{R} \text{ minutes} \quad \ldots \quad \ldots \quad \ldots \quad (20)$$

Similarly, $\delta_2 = 1718 \cdot 9 \frac{c_2}{R}$; $\delta_3 = 1718 \cdot 9 \frac{c_3}{R}$ and so on.

$$\delta_n = 1718 \cdot 9 \frac{c_n}{R} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (20a)$$

Since each of chord lengths $c_2, c_3 \ldots c_{n-1}$ is equal to the length of the unit chord (peg interval), $\delta_2 = \delta_3 = \delta_4 = \delta_{n-1}$

Now the total tangential (or deflection) angle (Δ_1) for the first chord (T_1D) $= BT_1D \qquad \Delta_1 = \delta_1$

The total tangential (or deflection) angle (Δ_2) for the second chord (DE) $= BT_1E$ But $BT_1E = BT_1D + DT_1E$

Now the angle DT_1E is the angle subtended by the chord DE in the opposite segment and, therefore, equals the tangential angle (δ_2) between the tangent at D and the chord DE.

Therefore, $\quad \Delta_2 = \delta_1 + \delta_2 = \Delta_1 + \delta_2$

Similarly, $\quad \Delta_3 = \delta_1 + \delta_2 + \delta_3 = \Delta_2 + \delta_3$

$$\Delta_n = \delta_1 + \delta_2 + \delta_3 + \ldots + \delta_n$$
$$= \Delta_{n-1} + \delta_n \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

Check —The total deflection angle $(BT_1T_2) = \Delta_n = \frac{\phi}{2}$, where ϕ is the deflection angle of the curve.

From the above, it will be seen that the deflection angle (Δ) for any chord is equal to the deflection angle for the preceding chord plus the tangential angle for that chord.

If the degree of the curve (D) be given, the deflection angle for 30 m chord is equal to $\frac{1}{2}$ D degrees, and that for the sub-chord Equals $\frac{c_1 \times D}{60}$ degrees, where c_1 is the length of the first sub-chord

Whence $\delta_1 = \frac{c_1 \times D}{60}$, $\delta_2 = \delta_3 = \delta_{n-1} = \frac{D}{2}$; $\delta_n = \frac{c_n \times D}{60} \ldots (22)$

In the case of a left-hand curve, each of the values Δ_1, Δ_2, Δ_3, etc should be subtracted from 360° to obtain the required value to which the vernier of the instrument is to be set

Mode of Procedure :—To set out the curve,

(i) set up a theodolite over the first tangent point (T_1) and level it.

(ii) With both plates clamped at zero, direct the telescope to the ranging rod at the point of intersection B and bisect it.

(iii) Release the vernier plate and set the vernier A to the first deflection angle Δ_1, the telescope being thus directed along T_1 D

(iv) Pin down the zero end of the chain or tape at T_1, and holding the arrow at a distance on the chain equal to the length of the first chord, swing the chain around T_1 until the arrow is bisected by the cross hairs, thus fixing the first point D on the curve

(v) Unclamp the upper plate and set the vernier to the second deflection angle Δ_2, the line of sight being now directed along T_1E

(vi) Hold the zero end of the chain at D and swing the other end around D until the arrow held at the other end is bisected by the line of sight, thus locating the second point on the curve.

(vii) Repeat the process until the end of the curve is reached

Check —The last point thus located must coincide with the previously located tangent point T_2 If not, find the distance between them which is the actual error If it is within the permissible limit, the last few pegs may be adjusted If it exceeds the limit, the entire work must be checked

The method gives more accurate results and is invariably used for railway and other important curves

(1) Two Theodolite Method —(Fig 57) The method is

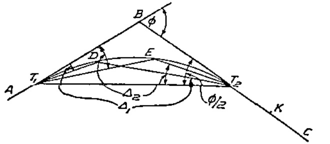


Fig 57

used when the ground is not favourable for accurate chaining

e g rough ground It is based on the fact that the angle between the tangent and the chord is equal to the angle which that chord subtends in the opposite segment

Let D, E etc be the points on the curve The angle (Δ_1) between the tangent T_1B and the chord $T_1D = BT_1D = T_1T_2D$ Similarly, the angle $BT_1E = \Delta_2 = T_1T_2E$, the total tangential or deflection angles Δ_1, Δ_2 Δ_3, being calculated as in the first method

To set out the curve,

(i) set up a theodolite over T_1 and another over T_2

(ii) Set the vernier of each of the instruments to zero

(iii) Direct the instrument at T_1 to the ranging rod at the point of intersection B and bisect it.

(iv) Direct the instrument at T_2 to the first tangent point T_1 and bisect it

(v) Set the vernier of each of the instruments to read the first deflection angle Δ_1 Thus the line of sight of the instrument at T_1 is directed along T_1D, and that of the other instrument at T_2 along T_2D Their point of intersection gives the required point on the curve

(vi) Move the ranging rod until it is bisected by the cross hairs of both instruments, thus locating the point D on the curve

(vii) To obtain the second point on the curve, set the vernier of each of the instruments to the second deflection angle Δ_2 and proceed as before

If the first tangent point T_1 cannot be sighted from the instrument at T_2 the ranging rod at the point of intersection B may be sighted The procedure will then be as follows —

(i) With both plates of the second instrument clamped at zero, bisect the signal at B

(ii) Release the vernier plate and swing the telescope clockwise) through $\left(360° - \frac{\phi}{2}\right)$, thus directing the line of sight along T_2T_1

(iii) To obtain the first point on the curve, set the vernier to the first deflection angle Δ_1 The vernier reading will then

be $\left(360^\circ - \frac{\phi}{2}\right) + \Delta_1$ instead of Δ_1 as in the first case The rest of the procedure is exactly the same as before Instead of sighting the intersection point B, any point K in the forward direction of the tangent line T_2C may be used In this case, however, the angle through which the telescop has to be turned, after having bisected the signal at K with both plates clamped at zero is equal to $\left(180^\circ - \frac{\phi}{2}\right)$ The line of collimation is thus directed along the line T_2T_1 To obtain the first point on the curve, the vernier reading must be $\left(180^\circ - \frac{\phi}{2}\right) + \Delta_1$

It will be seen that in this method no chain or tape is used to fix the points on the curve but they are located by the intersection of the lines of sight of the two instruments The method is simple and accurate but it is expensive since two surveyers and two instruments are requird to use this method Therefore, it is not so commonly used as the method of deflection angles

(3) **Tacheometric Method** —(Fig 58) The method is sometimes used when the curve is to be set out over rough ground In this method a tacheometer is used instead of a theodolite the use of a chain or tape being thus dispensed with

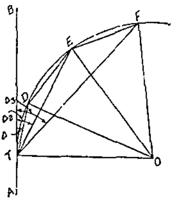


Fig 58

The deflection angles Δ_1, Δ_2, Δ_3 etc are calculated as before It is obvious that the lengths of the whole chords T_1D, T_1E T_1F, etc are respectively equal to $2R \sin \Delta_1$ $2R \sin \Delta_2$, $2R \sin \Delta_3$, etc The respective staff intercepts S_1 S_2 S_3 etc for these distances are then calculated from the formula

$$D = \frac{f}{i} S + (f + d) \text{ or } D = \frac{f}{i} S\cos^2\theta + (f + d)\cos\theta$$

according as the line of sight is horizontal or inclined

Procedure —(i) Set up a tacheometer at T_1 and level it accurately

(ii) With the vernier reading zero, bisect the signal at the intersection point (B)

(iii) Set the vernier to the first deflection angle Δ_1, thus directing the line of sight along T_1D, and sight the levelling staff held vertically in that direction

(iv) Move the staff backward or forward along T_1D until the staff intercept S_1 is obtained, thus locating the first point D on the curve

(v) Fix other points similarly

It is obvious that few points only can be located from T_1 When the distances along the whole chords become too large so that accurate staff reading is not possible the instrument requires to be moved to the last point located on the curve The method is laborious especially when the lines of sight are inclined

Obstacles in Setting out Simple Curves

The following obstacles occurring in common practice will now be considered

(1) When the Point of Intersection of the Tangent Lines is Inaccessible —e g when the intersection point falls in a lake, river, or wood Referring to Fig 59, let AB and BC be the two tangent lines intersecting at the point B, and T_1 and T_2 the tangent points It is required to determine the value of the deflection angle (ϕ) between the tangents and to locate the tangent points T_1 and T_2

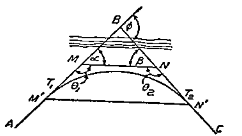


Fig 59

Procedure —(a) Fix points M and N suitably on the tangents AB and BC respectively so that M and N are intervisible, and the line MN runs on moderately level ground in order that accurate chaining may be possible If the ground beyond the curve is not suitable, the points may be fixed inside the curve as at M′ and N′. Measure MN accurately

(*b*) Set up the instrument at M and measure the angle AMN (θ_1) between AB and MN

Transfer the instrument to N and measure the angle CNM (θ_2) between BC and MN

Now in the $\triangle$ $\angle$BMN, $\angle BMN = \alpha = 180° - \angle AMN = 180° - \theta_1$

$\angle BNM = \beta = 180° - \angle CNM = 180° - \theta_2$

The deflection angle (ϕ) $= \angle BMN + \angle BNM = \alpha + \beta$

or $= 360°$—sum of the measured angles

$= 360° - (\theta_1 + \theta\ )$

(c) Solve the triangle BMN to obtain the distances BM and BN

$$BM = \frac{MN \sin \beta}{\sin\{180° - (\alpha + \beta)\}}, \quad BN = \frac{MN \sin \alpha}{\sin\ \{180° - (\alpha + \beta)\}}$$

(d) Calculate the tangent lengths BT_1 and BT_2 from formula $T = R \tan \dfrac{\phi}{2}$

(e) Obtain the distances MT_1 and NT_2

$MT_1 = BT_1 - BM$ and $NT_2 = BT_2 - BN$

(f) Measure the distance MT_1 from M along the tangent line BA, thus locating the first tangent point T_1

Similarly, locate the second tangent point T_2 by measuring the distance NT_2 from N along the tangent BC

If the points are fixed inside the curve, the procedure is the same as above, except for the distances to be measured from the points M and N to locate the tangent points T_1 and T_2 $M T_1$ and $N T_2$ being respectively equal to $(BM - BT_1)$ and $(BN' - BT_2)$

When it is found impossible to obtain a clear line MN, a traverse is run between M and N to find the length and bearing of the line MN From the known bearings of the tangent lines and the calculated bearing of the line MN, the angles α and β may easily be obtained The distances BM and BN are then calculated as before

(2) When the Whole Curve cannot be Set out from the Tangent Point, Vision being Obstructed —(Fig 60) As a rule, the whole curve is set out from the first tangent point T_1

But this is possible only when the curve is short and the ground

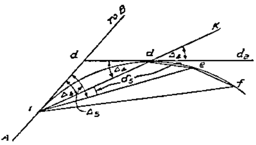


Fig 60

is moderately level and free from all obstructions However, it is often found that this cannot be done on account of great length of the curve, or obstructions intervening the line of sight such as buildings, cluster of trees plantations etc. In such a case, the instrument requires to be set up at one or more points along the curve

Procedure First Method —Suppose the first four points have been located by the deflection angles from the instrument at T_1, when it is found necessary to shift the instrument. Let d be the last point located from T_1 and its deflection angle Δ

(i) Shift the instrument and set it up at d

(ii) With both plates clamped at zero, take a backsight on T_1 and transit the telescope

(iii) To locate the next point e, set the vernier to read the deflection angle Δ_5, thus directing the line of sight along de.

(iv) Using the same tabulated deflection angles, continue the setting out of the curve from d as already explained

Proof —Draw the tangent d_1dd_2 at d, meeting the first tangent BT_1 at d_1. Produce T_1d to K.

$Kdd_2 = d_1dT_1$ (vertically opposite), $d_1dT_1 = d_1T_1d$ since $d_1T_1 = d_1d$. But $d_1T_1d = \Delta_4$ $\quad$ $Kdd_2 = d_1T_1d = \Delta_4$

The tangential angle for the chord $de = d_2de = \delta_5 = dT_1e$

$Kde = \Delta_4 + \delta_5 = \Delta_5$ The total tangential (or deflection) angle for the chord $de = d_1T_1e = d_1T_1d + dT_1e = \Delta_4 + \delta_5 = \Delta_5$ Thus it will be seen that when the telescope is transited after

a backsight was taken with the vernier reading zero, the line of sight is directed along dK and when the telescope is turned through the angle $\triangle_4$ it is in the direction of the tangent at d. It may be noted that the points on which backsights are to be taken or which are to be occupied by the instrument must be located very carefully. To use this method the instrument must be in perfect adjustment.

Second Method.—Suppose d is the last point located from the instrument at T_1. (i) While the instrument is at T_1, fix any point K in the line T_1d produced.

(ii) Move the instrument to d and with the vernier set to zero, bisect K.

(iii) Release the vernier plate and set the vernier to $\triangle_5$ to locate the next point e.

Third Method.—This method is used when the instrument is not in perfect adjustment. As before, d is the last point located from the instrument at T_1.

(i) Set up the instrument over d and backsight on T_1 with the vernier reading 180°.

(ii) Release the upper plate and swing, i.e. turn the telescope in azimuth through 180°. The telescope will now be pointing in the direction of dK (T_1d produced), and the vernier reading will be 360°. It is evident that if the telescope is further turned through the deflection angle $\triangle_4$, the line of sight will be directed along the tangent at d.

(iii) Set the vernier to $\triangle_5$ to locate the next point e.

It may be observed that if the vernier reading is zero or 360° when a backsight is taken on T_1, and if the telescope is turned through 180° in azimuth, the opposite vernier B or vernier 2 will have to be used for locating the new points on the curve. This is not, however, desirable especially when the verniers are eccentric.

It will be noticed from the above procedure that when the instrument is transferred to any point on the curve, no new calculations are required for continuing the curve, but the previously calculated deflection angles can be used. If more set ups are required the procedure to be adopted is as follows :

(1) Move the instrument to any point, say, h on the curve the deflection angle for that point being Δ_8

(2) Take a backsight on the last instrument station (d) with the vernier A set to the deflection angle (Δ_4) for that point (d) For a left hand curve the vernier reading will be $360° - \Delta_4$

(3) Plunge the telescope and set the vernier to read the deflection angle Δ_9 for the next point i, the line of sight being thus directed along hi

If the instrument is not in perfect adjustment, plunging the telescope should be avoided The procedure will then be as follows

(i) Backsight as before on the last instrument station d with the vernier A reading 180° plus the deflection angle for the point sighted (i e d) In this case the vernier reading will be $180° + \Delta_4$

(ii) Swing the telescope through 180° and set the vernier A to the deflection angle for the next point i, viz Δ_9

For a left hand curve, the reading to which the vernier is to be set when the last instrument station is backsighted is

$$180° - \Delta_4 \quad \{= 360° - (180° + \Delta_4)\}$$

(3) **When the Obstacle to Chaining Occurs** —(Fig 61) Suppose that the first four points have been located from T_1 in the usual way, d being the last point located and that the

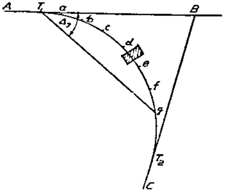


Fig 61

portion of the curve between d and f cannot be chained across In such a case, the following procedure may be adopted

Let g be the next point on the curve which can be seen from T_1

(i) Calculate the length of the whole (or long) chord T_1g from the formula, $T_1g = 2R \sin \Delta_7$

(ii) Set the vernier A to the deflection angle Δ_7, thus directing the line of sight along T_1g Measure the distance T_1g along this direction, and locate the point g on the curve Locate the remaining points in the usual way Alternatively, set out the curve T_2g in the reverse direction from the second tangent point T_2

(iii) The points e and f, which were left out, will be located after the obstruction is removed

Examples

Example 1 —Calculate the ordinates at 7 5 m intervals for a circular curve, given that the length of the long chord is 60 m and the radius 80 m

(See Fig 49)

Ordinate at the middle of the long chord

$$= \text{versed sine}$$

$$= 0_o = R - \sqrt{R^2 - \left(\frac{L}{2}\right)^2}$$

Now $\because$ $R = 60$ m, $L = 180$ m $\mid = 180 - \sqrt{180^2 - 30^2}$

$$= 2\ 52 \text{ m}$$

The various ordinates may be calculated from the formula

$$0_x = \sqrt{R^2 - x^2} - \sqrt{R^2 - \left(\frac{L}{2}\right)^2}, \text{ the distance } x$$

being measured from the mid-point of the long chord

$$0_{7\ 5} = \sqrt{180 - 7\ 5^2} - \sqrt{180^2 - 30^2} = 2{\cdot}34 \text{ m}$$

$$0_{15} = \sqrt{180 - 15^2} - \sqrt{180^2 - 30^2} = 1{\cdot}89 \text{ m}$$

$$0_{22\ 5} = \sqrt{180^2 - 22\ 5^2} - \sqrt{180^{\circ} - 30^2} = 1{\cdot}14 \text{ m}$$

$$0_{30} = \sqrt{180^2 - 30^2} - \sqrt{180^2 - 30^2} = 0 \text{ m.}$$

The ordinates for the other half of the curve are the same as above

By the approximate formula $0_x = \dfrac{x\,(L - x)}{2R}$, where x is the distance of the point measured from either end of the long chord, The ordinates are

$$0_{7.5} = \frac{7.5 \times 52.5}{360} = 1.095 = 1.10 \text{ m} = 0_{52.5}$$

$$0_{15} = \frac{15 \times 45}{360} = 1.875 = 1.88 \text{ m} = 0_{45}$$

$$0_{22.5} = \frac{22.5 \times 37.5}{360} = 2.343 = 2.34 \text{ m} = 0_{37.5}$$

$$= \frac{30 \times 30}{360} = 2.5 \text{ m} = 0_{30}$$

Example 2 —Calculate the offset at 15 m intervals along the tangents to locate a curve having a radius of 360 m

(See Fig 51)

(i) By the accurate formula $0x = \sqrt{R^2 + x^2} - R$, the radial offsets are

$$0_{15} = \sqrt{360^2 + 15^2} - 360 = 0.300 \text{ m}$$
$$0_{30} = \sqrt{360^2 + 30^2} - 360 = 1.245 \text{ m}$$
$$0_{45} = \sqrt{360^2 + 45^2} - 360 = 2.802 \text{ m}$$
$$0_{60} = \sqrt{360^2 + 60^2} - 360 = 4.965 \text{ m}$$
etc

(See Fig 52)

(ii) By the accurate formula $0_x = R - \sqrt{R^2 - x^2}$ the offsets perpendicular to the tangent are

$$0_{15} = 360 - \sqrt{360^2 - 15^2} = 0.315 \text{ m}$$
$$0_{30} = 360 - \sqrt{360^2 - 30^2} = 1.254 \text{ m}$$
$$0_{45} = 360 - \sqrt{360^2 - 45^2} = 2.790 \text{ m}$$
$$0_{60} = 360 - \sqrt{360^2 - 60^2} = 5.037 \text{ m}$$

(iii) By the approximate formula $0_x = \frac{x^2}{2R}$, the offsets are

$0_{15} = \frac{15^2}{2 \times 360} = 0.312$ m	$0_{30} = \frac{30^2}{2 \times 360} = 1.251$ m
$0_{45} = \frac{45^2}{2 \times 360} = 2.814$ m	$0_{60} = \frac{60^2}{2 \times 360} = 5.001$ m etc.

Example 3 —Two tangents intersect at chainage 1190 m the deflection angle being 36° Calculate all the data necessary for setting out a curve with a radius of 300 m by (i) deflection angles and (ii) offsets from chords the peg interval being 30 m

(*a*) Radius of the curve = 300 m

(*b*) Tangent length (T) $= R \tan \frac{\phi}{2} = 300 \tan 18° = 97.47$ m

(*c*) Length of the curve $(l) = \frac{\pi R \phi}{180} = \frac{\pi \times 300 \times 36°}{180°}$

$= 188.52$ m

Chainage of the intersection point	=	1190.00 m
Deduct tangent length (T)	= −	97.47 m
Chainage of the 1st tangent point	=	1092.53 m
Add length of the curve	= +	188.52 m
Chainage of the 2nd tangent point	=	1281.05 m

Thus the curve consists of six chords four unit chords and two sub chords

Length of the first sub chord = 1100 − 1092.53 = 7.47 m
Length of the second chord to the fifth chord = 30 m
Length of the last chord = 31.05 m

The last chord is chosen slightly greater than the unit chord as otherwise the last chord would have a length of of 1.05 m which is not convenient

(*d*) By deflection angles —By $\delta = 1718.9 \frac{c}{R}$ mins, the tangential angles are

δ_1 for the first chord $= 1718 \cdot 9 \, \dfrac{7 \cdot 47}{300} = 42'\ 48''$.

δ_2 for the second to the fifth chord

$$= 1718 \cdot 9 \times \frac{30}{300} = 2°51'53''.$$

$$\delta_6 \text{ for the last chord} = \frac{1718 \ 9 \times 31 \ 05}{300} = 2°\ 57'\ 50''.$$

The total tangential (or deflection) angles for the chords are

$$\begin{aligned}
\Delta_1 &= \delta_1 && = 42'48''. \\
\Delta_2 &= \delta_1+\delta_2 = 42'48''+2°51'53'' && = 3°32'24''. \\
\Delta_3 &= \delta_1+\delta_2+\delta_3 && = 3°32'24 + 2°51'53'' \\
& && = 6°20'34'' \\
\Delta_4 &= \delta_1+\delta_2+\delta_3+\delta_4 && = 6°26'34''+2°51'53'' \\
& && = 9°18'24'' \\
\Delta_5 &= \delta_1+\delta_2+\delta_3+\delta_4+\delta_5 && = 9°18'24'' +2°51'53'' \\
& && = 12°10'17'' \\
\Delta_6 &= \delta_1+\delta_2+\delta_3+\delta_4+\delta_5+\delta_6 && = 12°10'17''+2°57'50'' \\
& && = 18°0\ 04''
\end{aligned}$$

Check $-\Delta_6 = \frac{1}{2}\,\phi = \frac{1}{2} \times 36° = 18°$.

(*c*) *By offsets from chords.*—*By* $O_n = \dfrac{b_n\,(b_{n-1}+b_n)}{2R}$,

the offsets are

$$O_1 = \frac{b_1^2}{2R} = \frac{(7\ 47)^2}{2 \times 300} = 0\ 093 \text{ m.}$$

$$O_2 = \frac{b_2\,(b_1+b_2)}{2R} = \frac{30\,(7 \cdot 47 + 30)}{2 \times 300} = 1 \cdot 823 \text{ m}$$

$$O_3 \text{ to } O_5 = \frac{b_3\,(b_2+b_3)}{2R} = \frac{b_3^2}{R} = \frac{30^2}{300} = 3 \cdot 0 \text{ m.}$$

$$O_6 = \frac{b_6\,(b_5+b_6)}{2R} = \frac{31 \cdot 05\,(30+31 \cdot 05)}{2 \times 300} = 3 \cdot 159 \text{ m.}$$

Example 4 :—Two tangents PQ and QR to a railway curve meet at an angle of 140°. Find the radius of the curve which will pass through a point M 24 m from the intersection point Q, *the angle* PQM *being 100° (Fig. 62)*

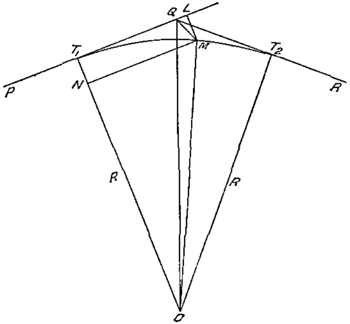


Fig 62

Let T_1 and T_2 be the tangent points. Join OM and OT_1. Draw ML perpendicular to PQ and MN perpendicular to OT_1.

Let R = the radius of the curve $= OT_1 = OT_2$.

Now in the $\triangle OMQ$, $OQ = OT_1 \operatorname{cosec} PQO = R \operatorname{cosec} 70°$.

$OM = R$; $\angle OQM = 100° - 70° = 30°$; $QM = 24$ m

$$\frac{OQ}{OM} = \frac{\sin OMQ}{\sin OQM} \text{ or } \frac{R \operatorname{cosec} 70°}{R} = \frac{\sin OMQ}{\sin 30°}$$

i e. $\sin OMQ = \sin 30° \operatorname{cosec} 70°$ $\qquad \angle OMQ = 147° 51'$.

$$\text{Now } \angle MOQ = 180° - \angle OMQ - \angle OQM$$
$$= 180° - 147° 51' - 30° = 2° 9'$$

and $\angle T_1OM = \angle T_1OQ + \angle MOQ = 20° + 2° 9' = 22° 9'$.

$ML = 24 \sin 100° = NT_1 = R - ON = R - R \cos 22° 9'$.
$= R \text{ versin } 22° 9'$.

$\therefore$ R versin $22^\circ 9' = 24 \sin 100^\circ$.

$$R = \frac{24 \sin 100^\circ}{(1 - \cos 22^\circ 9')} = 320.4 \text{ m}$$

Example 5 —The centre line of a light railway is to be tangential to each of the following lines

Line	W C B	Length
AB	0°	
BC	270°	165 m
CD	220°	

Find the radius of the curve and the tangent lengths (Fig 63)

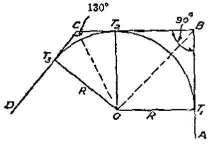


Fig 63

Let the bisectors of the angles ABC and BCD meet at O, which is then the centre of the curve From O draw perpendiculars OT_1, OT_2, and OT_3 to AB, BC, and CD respectively T_1, T_2 and T_3 are then the tangent points

Now in the $\triangle$ OBC $\angle OBC = 45^\circ$, $\angle OCB = 65^\circ$ and $\angle COB = 180^\circ - 45^\circ - 65^\circ = 70^\circ$, and BC = 165 m.

By the Sine rule we get

$$OC = \frac{165 \sin 45^\circ}{\sin 70^\circ} = 124.17 \text{ m and}$$

$$OB = \frac{165 \sin 65^\circ}{\sin 70^\circ} = 159.18 \text{ m}$$

Radius of the curve $= OT_2 = OC \sin 65^\circ$

$= 124.17 \sin 65^\circ = 112.56$ m

Check — , $= OB \sin 45^\circ$

$= 159.18 \times \sin 45^\circ = 112.56$ m.

Tangent length $BT_1 = BT_2 = 112.56 \tan 45^\circ = 112.56$ m

„ „ $CT_3 = CT_2 = 112.56 \tan 25^\circ = 52.5$ m.

Check — $BC = BT_2 + CT_2$

$= 112.56 + 52.5 = 165.06$m

Example 6.—Two straights BA and AC are intersected by a third line EF. The angles AEF and AFE are 27° 12′ and 32° 24′ and EF is 180 m Find the radius of the simple curve which will be tangential to the lines BA, EF, and AC and the chainages of the beginning and end of the curve, if the chainage of A = 1700·0 m (Fig. 64)

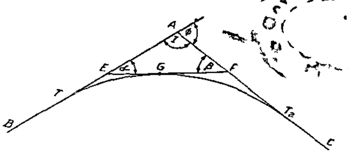


Fig 64

(i) In the $\triangle$ AEF, EF = 180 m, $\angle$AEF(α) = 27°12′; $\angle$AFE(β) = 32°24′; $\angle$EAF (I) = 180° − 27°12′ − 32°24′ = 120°24′; and ϕ = 59° 36′.

Therefore, by the Sine rule, we get

$$AE = \frac{EF \sin\beta}{\sin I} = \frac{180 \sin 32°24}{\sin 120°24} = 111\ 8\ \text{m}.$$

$$AF = \frac{EF \sin\alpha}{\sin I} = \frac{180 \sin 27°12}{\sin 120°24} = 95\cdot 39\ \text{m}$$

$$\angle BEF = 180° - \alpha = 180° - 27°\ 12' = 152°\ 48'.$$

$$\angle CFE = 180° - \beta = 180° - 32°\ 24' = 147°\ 36'.$$

(ii) Now the formula for the radius of the curve tangential to the lines BA, EF, and AC is derived as follows :—

In Fig. 64, $\angle T_1EF = \pi - \alpha$, $\angle EFT_2 = \pi - \beta$, $\angle EAF = I$.

Considering the main tangents AT_1 and AT_2, we get

$$R = AT_1 \tan \frac{I}{2} = AT_2 \tan \frac{I}{2}$$

While, considering the tangents ET_1 and EG, we have

$$R = ET_1 \tan \tfrac{1}{2}(\pi - \alpha);\ \text{but}\ ET_1 = AT_1 - AE.$$

$$\therefore\quad R = (AT_1 - AE)\tan \tfrac{1}{2}(\pi - \alpha) \qquad \text{(a)}$$

Substituting the value of R, viz

$$AT_1 = \tan \frac{I}{2} \text{ in (a), we have}$$

$$AT_1 \tan \frac{I}{2} = (AT_1 - AE) \tan (\pi - \alpha)$$

$$AT_1 = \frac{AE \tan \frac{1}{2}(\pi - \alpha)}{\left\{\tan \frac{1}{2}(\pi - \alpha) - \tan \frac{I}{2}\right\}}$$

Whence
$$R = \frac{AE \tan \frac{1}{2}(\pi - \alpha) \tan \frac{I}{2}}{\left(\tan \frac{\pi - \alpha}{2} - \tan \frac{I}{2}\right)}$$

Similarly, $R = FT_2 \tan \frac{1}{2}(\pi - \beta)$
$= (AT_2 - AF) \tan \frac{1}{2}(\pi - \beta)$

and $R = AT_2 \tan \frac{I}{2}$

Proceeding as above we get

$$AT_2 = \frac{AF \tan \frac{1}{2}(\pi - \beta)}{\left(\tan \frac{\pi - \beta}{2} - \tan \frac{I}{2}\right)}$$

and
$$R = \frac{AF \tan \frac{1}{2}(\pi - \beta) \tan \frac{I}{2}}{\left(\tan \frac{\pi - \beta}{2} - \tan \frac{I}{2}\right)}$$

$$\therefore \quad R = \frac{111.8 \tan 76^\circ 24' \tan 60^\circ 12'}{(\tan 76^\circ 24' - \tan 60^\circ 12')} = 338.1 \text{ m}$$

Check —It is also given by
$$R = \frac{AF \tan \frac{\pi - \beta}{2} \tan \frac{I}{2}}{\left(\tan \frac{\pi - \beta}{2} - \tan \frac{I}{2}\right)}$$

$$\therefore \quad R = \frac{95.39 \tan 73^\circ 48' \tan 60^\circ 12'}{(\tan 73^\circ 48' - \tan 60^\circ 12')} = 338.1 \text{ m}$$

(iii) Now tangent length $AT_1 = R \tan \frac{\phi}{2} = 338 \cdot 1 \tan 29°48'$

$= 193 \cdot 5$ m

(iv) Length of the circular curve $= \frac{\pi R \phi}{180°}$

$= \frac{\pi \times 338 \cdot 1 \times 59 \cdot 6}{180°}$

$= 351 \cdot 3$ m.

(v) Chainage of A	=	1700 0 m
Deduct tangent length AT_1	=	$-193 \cdot 5$ m
Chainage of T_1	=	1506 5 m
Add length of the curve	=	$+351 \cdot 3$ m
Chainage of (T_2)	=	1837 8 m

Example 7 —Two straights AB and BC meet in an inaccessible point B and were joined by a circular curve of 480 m radius. Two points M and N were selected on AB and BC respectively, and the following data were obtained ·

$\angle AMN = 159° \ 48'$, $\angle CNM = 155° \ 42'$; $MN = 127 \ 5$ m

Make the necessary calculations for setting out the curve by the method of tangential angles, given that the chainage of M was 1400 00 m (Fig 59)

(i) In the triangle BMN, $\angle BMN = 180° - \angle AMN$
$= 180° - 159° 48 = 20° 12'$.

$\angle BNM = 180° - \angle CNM = 180° - 155° \ 42' = 24° \ 18'$.

$\angle MBN = 180° - \angle BMN - \angle BNM = 180° - 20°12' - 24°18'$.
$= 135° \ 30'$, $MN = 127 \cdot 5$ m

Now $BM = \frac{127 \cdot 5 \sin 24° \ 18'}{\sin 135° \ 30'} = 74 \ 88$ m

$BN = \frac{127 \cdot 7 \sin 20° \ 12'}{\sin 135° \ 30'} = 62 \cdot 79$ m

(ii) Deflection angle $(\phi) = \angle BMN + \angle BNM$
$= 20° \ 12' + 24° \ 18' = 44° \ 30$

Tangent length $= BT_1 = BT_2 = R \tan \dfrac{\phi}{2} = 480 \tan 44°30$

$= 196\ 38$ m

$MT_1 = BT_1 - BM = 196\ 38 - 74\ 88 = 121\ 50$ m

$NT_2 = BT_2 - BN = 196\ 38 - 62\ 79 = 133\ 59$ m.

(iii) Length of the curve $= \dfrac{\pi R \phi}{180} = \dfrac{\pi \times 480 \times 44°\ 5}{180°}$

$= 372\ 9$ m

(iv) Chainages —	Chainage of M	= 1400 0 m
	Deduct MT_1	= 121 5 m
	Chainage of T_1	1278 5 m
	Add length of the curve	372 9 m
	Chainage of T_2	1651 4 m

If the normal length of the chord is taken as 25 m there will be 13 normal chord and 2 sub chords of lengths 21 5 m and 26 4 m respectively

(v) Tangential angles —By $\delta = 1718\ 9 \dfrac{c}{R}$ minutes

Chainage of T_1 $= 1278\ 5$ m

Length of the first sub chord $= 21\ 5$ m

Hence $\delta_1 = 1718\ 9\ \dfrac{21\ 5}{480} = 77\ 0$ min $= 1°17\ 0''$

δ_2 to δ_{14} $= 1718\ 9 \times \dfrac{25}{480} = 85\ 54$ min $= 1°29\ 32''$

$\delta_{15} = 1718\ 9\ \dfrac{(26\ 4)}{480} = 1°35\ 33''$

(vi) The total tangential angles for the various chords may be obtained from $\Delta_n = \delta_1 + \delta_2 + \delta_3 + \quad + \delta_n$.

$\Delta_1 = \delta_1 = 1°17\ 0''$ $\Delta_2 = \delta_1 + \delta_2 = 1°17\ 0'' + 1°29\ 32''$

$= 2°46\ 32''$ and so on

$\Delta_{15} = \delta_1 + \delta_2 + \delta_3 \quad + \delta_{15} = 1°17 + 13\ (1°29\ 32'') + 1°35\ 33''$

$= 22°16\ 29''$

Check —$\Delta_{15} = \frac{1}{2}\phi = \frac{1}{2}\ (44°\ 30) = 22°\ 15$

Example 8 —Two straights BA and AC the bearings of which are 30° and 82° respectively are to be connected by a curve with a radius of 3o0 m The intersection point is inaccessible and the following traverse is run from a point P on the first straight to a point S on the second straight

Line	Length in m	Bearing
PQ	132 0 m	55°
QR	118 8 m	120°
RS	22o 6 m	3o°

The chainage of P is ?2b0 m Make all the necessary calculations for setting out the curve by the method of offsets from chords

(i) Since the traverse PQRS is a closed one the length and bearing of PS may be obtained from the known lengths and bearings of PQ QR and RS

The latitudes and departures of the sides by $L = l \cos \alpha$ and $D = l \sin \alpha$ are

Line	Lat	Dep
PQ	+ 75 63	+ 108 12
QR	− 59 40	+ 102 90
RS	+ 184 80	+ 129 39

Total lat of S with respect to P $= \Sigma L = + 201\ 03$ m
dep of S $= \Sigma D = + 340\ 41$ m

$$\text{Tan } \alpha = \frac{\text{dep}}{\text{lat}} = \frac{340\ 41}{201\ 03} \quad \text{where } \alpha \text{ is the R B of PS}$$

or $\alpha = 59° 27$ R B of PS = N 59° 27 E W C B = 59°27′
Length of PS − 340 41 cosec 59° 27 − 39o 4 m

(ii) Now in the $\triangle$ APS,

$\angle$ APS = bearing of PS − bearing of BA = 59°27 − 30° = 29°27′.

$\angle$ASP = bearing of CA − bearing of SP
= 262° − 239° 27 = 22° 33

$\angle$PAS = 180° − 29° 27 − 22° 33 = 128° PS = 395 4 m

By the sine rule $AP = \dfrac{395\ 4 \sin 22° 33}{\sin 128°} = 192\ 45$ m

(iii) Let T_1 and T_2 be the tangent points

$$R = 350 \text{ m}$$

$$\text{Tangent length } AT_1 = AT_2 = R \tan\frac{\phi}{2} = 350 \tan 26^\circ$$

$$(\phi = 180^\circ - 128^\circ = 52^\circ) \qquad = 170.7 \text{ m}$$

$$\text{Whence } PT_1 = AP - AT_1 = 192.45 - 170.70$$
$$= 21.75 \text{ m.}$$

(iv) Length of the curve $= \dfrac{\pi R \phi}{180} = \dfrac{\pi \times 350 \times 52}{180}$

$$= 317.7 \text{ m}$$

(v) Chainage of P	= 2260.00 m
Add PT_1	= +21.75 m
Chainage of the first tangent point (T_1)	= 2281.75 m
Add curve length	= +317.70 m
Chainage of the second tangent point (T_2)	= 2599.45 m

(vi) The curve consists of 11 chords of 25 m each and two sub chords at T_1 and T_2. The offsets from chords may be calculated from $O_n = \dfrac{b_n(b_{n-1} + b_n)}{2R}$

Length of the first sub chord $= b_1 = 2300.00 - 2281.75$
$$= 18.25 \text{ m}$$

$$\text{First offset } O_1 = \frac{b_1^2}{2R} = \frac{18.25^2}{2 \times 350} = 0.476 \text{ m}$$

$$\text{Second offset } O_2 = \frac{b_2(b_1 + b_2)}{2R} = \frac{25(18.2 + 25)}{2 \times 350} = 1.543 \text{ m}$$

Third to 12th offset O_3 to O_{12} $= \dfrac{{}^{2}}{350} = 1.785$ m

$$\text{Last offset } O_{13} = \frac{24.45(25 + 24.45)}{2 \times 350} = 1.727 \text{m}$$

The results are tabulated as under

Point	Chainage	Chord length in m	Offset in m
T_1	2281 75		
1	2300 00	18 25	0 476
2	2325 00	25	1 543
3 to 12	2350 00 to 2575 00	25 ,,	1 785 ,,
T_2	2599 45	24 75	1 727

Example 9 —The bearings of two straights AB and BC intersecting at B are 120° 40 and 100° 25 respectively They are to be connected by a curve of 180 m radius The chainage of A is 1076 00 m Submit in a tabular form, the calculations necessary for setting out the curve by means of a theodolite, given the following co ordinates of A and C

Point	Co ordinates	
	North	East
A	153 12	13 68
C	64 74	330 12

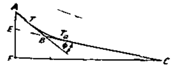


Fig 65

Through A draw a line parallel to the north and south line (Fig 65) and produce CB so as to meet it in E Through C draw a line parallel to the east and west line, meeting the line AE in F

(1) Now the deflection angle (ϕ) at B = bearing of AB − bearing of BC = 120° 40 − 100° 25 = 20° 15 (left)

In the $\triangle$ AEB, $\angle$EAB = 180° − 120° 40 = 59° 20′;

$\angle$EBA = bearing of BA − bearing of BE
= 300° 40 − 280° 25 = 20° 15

. $\angle$AEB = 180 − 59° 20 − 20° 15 = 100° 25′

Now $\angle$ECF = bearing of CB − 270° = 280° 25′ − 270°
= 10° 25

$\therefore$ EF = FC tan 10° 25'.

Now FC = total departure of C − total departure of A
$= 330.12 - 13.68 = 316.44$ m.

EF = 316.44 tan 10° 25' = 58.17 m.

AE = AF − EF, but EF = north co-ordinate of A − north co-ordinate of C = 153.12 − 64.74 = 88.38 m.

AE = 88.38 − 58.17 = 30.21 m.

Whence, by the Sine rule $AB = \dfrac{30.21 \sin 100°25'}{\sin 20°15'} = 85.84$ m.

(ii) Let T_1 and T_2 be the first and second tangent length on AB and BC respectively.

Tangent length $BT_1 = BT_2 = R \tan \dfrac{\phi}{2} = 180 \tan 20° 15'$
$= 32.14$ m.

Length of the curve $= \dfrac{\pi R \phi}{180°} = \dfrac{\pi \times 180 \times 20.25}{180} = 63.62$ m.

(iii) Chainage of A	= 1076.00 m.
Add length of AB	= − 85.84 m.
Chainage of the intersection point (B)	= 991.84 m.
Ded. ct tangent length (BT_1)	− 32.14 m.
Chainage of the first tangent point (T_1)	= 959.70 m.
Add length of the curve	+ 63.62 m.
Chainage of (T_2)	= 1023.32 m.

(iv) Tangential angles —The curve will be set out with pegs at 25 m intervals of through chainage. The curve is made up of 3 chords one normal chord, and two sub-chords.

Length of the first sub-chord = 975.00 − 959.70 = 15.30 m.

Tangential angle $\delta_1 = \dfrac{1718.9 \times 15.30}{180} = 146.1$ mins
$= 2°26.1'$

Length of the second chord = 25 m.

Tangential angle $\delta_2 = \dfrac{1718 \cdot 9 \times 25}{180} = 238 \cdot 7$ mins

$= 3°58' \cdot 7$

Length of the last sub chord $= 1023 \cdot 32 - 1000 \cdot 00$

$= 23 \; 32$ m.

Tangential angle $\delta_3 = \dfrac{17189 \times 23 \; 32}{180} = 222 \cdot 7$ mins.

$= 3°42' \cdot 7$

The results may be tabulated as under :

Point	Chainage in metres	Chord length in m	Tangential angle (δ) °	Total tangential angle (Δ) ° ′ ″	Actual Theodolite reading ° ′ ″
T_1	959·70				
1	975 00	15 30	2 26 1	2 26 6	2 26 10
2	1000 00	25	3 58 7	6 24 48	6 24 50
T_2	1023 32	23 32	3 42 7	10 7 30	10 7 30

N. B. Accuracy of theodolite = 10″

Check :—$\Delta_3 = \frac{1}{2} \phi = \frac{1}{2} (20° \; 15') = 10° \; 7' \; 30''$.

Compound Curves :—In Fig 66 is shown a compound curve which is tangential to the three straights AB, BC, and KM at T_1, T_2 and N respectively. The two circular arcs, T_1N and NT_2 having centres at O_1 and O_2 meet at the point N, which is called the point of *compound curvature*, the points N, O_1, and O_2 being in a straight line. The arc having a smaller radius may be first or second. Let the tangents AB and BC intersect at the point B, AB and KM at K, and KM and BC at M

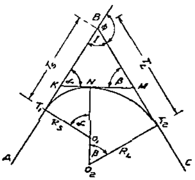


Fig 66

Notation —Let R_S = the smaller radius (O_1T_1).
R_L = the greater radius (O_2T_2).
T_S = the smaller tangent length (BT_1)
T_L = the greater „ „ (BT_2).

ϕ = the deflection angle between the end tangents AB and BC.

α = the deflection angle between the rear and common tangents AB and KM = $\angle$ BKM

β = the deflection angle between the common and forward tangents KM and BC = $\angle$ BMK

t_S = the length of the tangent to the arc (T_1N) having a smaller radius

t_L = the length of the tangent to the arc (NT_2) having a greater radius

Elements of the Compound Curve :—

$$\phi = \alpha + \beta \quad \ldots \quad \ldots \quad \ldots \quad (23)$$

$$KN = KT_1 = t_S = R_S \tan\frac{\alpha}{2};\ MN = MT_2 = t_L = R_L \tan\frac{\beta}{2}.$$

$$\therefore\ KM = KN + MN = t_S + t_L = R_S \tan\frac{\alpha}{2} + R_L \tan\frac{\beta}{2} \quad (24)$$

From the $\triangle$ BKM, $BK = \dfrac{KM \sin\beta}{\sin(\alpha+\beta)}$, $BM = \dfrac{KM \sin\alpha}{\sin(\alpha+\beta)}$

$$= \frac{(t_S + t_L)\sin\beta}{\sin\phi};\quad = \frac{(t_S + t_L)\sin\alpha}{\sin\phi}.$$

Now $T_S = BT_1 = KT_1 + BK = t_S + \dfrac{(t_S + t_L)\sin\beta}{\sin\phi}$. (25)

$$T_L = BT_2 = MT_2 + BM = t_L + \frac{(t_S + t_L)\sin\alpha}{\sin\phi} \ \ldots \ (26)$$

Substituting the values of t_S and t_L in the equations (25) and (26), we get

$$T_S = R_S \tan\frac{\alpha}{2} + \left(R_S \tan\frac{\alpha}{2} + R_L \tan\frac{\beta}{2}\right)\frac{\sin\beta}{\sin\phi} \ \ldots \ (25a)$$

$$T_L = R_L \tan\frac{\beta}{2} + \left(R_S \tan\frac{\alpha}{2} + R_L \tan\frac{\beta}{2}\right)\frac{\sin\alpha}{\sin\phi} \qquad (26a)$$

Of the seven quantities R_S, R_L, T_S, T_L, ϕ, α, and β, four *must be known The remaining three may then be calculated* from equations (23), (25), and (26)

R_S, R_L, *and* ϕ *are usually known and the fourth known* quantity may be either α or β, or T_S or T_L

The following equations give the relationships between the seven elements involved in a compact form

$$T_S \sin\phi = (R_L - R_S)\ \text{versin}\,\beta + R_S\ \text{versin}\,\phi \qquad (25\,b)$$

$$T_L \sin\phi = (R_L - R_S)\ \text{versin}\,\alpha + R_L\ \text{versin}\,\phi \qquad (25\,c)$$

$$\phi = \alpha + \beta \qquad (23)$$

Setting out the Compound Curve —The curve may be set out by the method of deflection angles from the two points T_1 and N, the first branch from T_1 and the second one from N

Procedure —(i) The four parts of the curve being known, calculate the other three

(ii) Locate B, T_1 and T_2 as already explained Obtain the chainage of T_1 from the known chainage of B

(iii) Calculate the length of the first arc and add it to the chainage of T_1 to obtain the chainage of N Similarly, compute the length of the second arc which, when added to the chainage of N, gives the chainage of T_2

(iv) Calculate the deflection angles for both the arcs

(v) With a theodolite set up over T_1, set out the first branch as already explained

(vi) Shift the instrument and set it up over N With the vernier set to $\frac{\alpha}{2}$ behind zero, i e $\left(360° - \frac{\alpha}{2}\right)$, take a backsight on T_1 and plunge the telescope which is thus directed along T_1N produced (If the telescope is now swung through the angle $\frac{\alpha}{2}$,

the line of sight will be directed along the common tangent NM and the vernier will read 360°)

(vii) Set the vernier to the first deflection angle Δ_1 as calculated for the second branch thus directing the line of sight to the first point on the second arc

(viii) Continue the process until the end of the second arc is reached

Check —Measure the angle T_1NT_2 which must equal $180^\circ - \left(\frac{\alpha + \beta}{2}\right)$ or $\left(180^\circ - \frac{\phi}{2}\right)$

Example 1 —Two straights BA and AC are intersected by a line EF The angles BEF and EFC are 140° and 145° respectively The radius of the first arc is 600 m and that of the second arc 400 m Find the chainages of the tangent points and the point of compound curvature given that the chainage of the intersection point A is 3115 m (Fig 67).

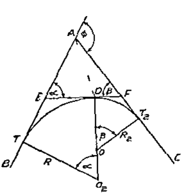


Fig 67

Let T_1 and T_2 be the tangent points and D the point of compound curvature

ET_1 and ED the tangents to the first branch of the compound curve

FD and FT_2 the tangents to the second branch of the compound curve

α and β the central angles of the first and second branches respectively

R_1 and R_2 the radii of the first and second branches respectively

Then $\alpha = 180^\circ - 140^\circ = 40^\circ \qquad \beta = 180^\circ - 145^\circ = 35^\circ$.

$$ET_1 = ED = R_1 \tan\frac{\alpha}{2} = 600 \tan 20^\circ = 218.4 \text{ m}$$

$$FT_2 = FD = R_2 \tan\frac{\beta}{2} = 400 \tan 17^\circ 30' = 126.1 \text{ m}$$

$\therefore$ EF = ED + FD = 21.84 + 126.1 = 344.5 m

Now in the $\triangle$ AEF, $\angle$AEF = 40°, $\angle$AFE = 35°, $\angle$EAF = 105°, and EF = 344.5 m

$$\therefore \text{AE} = \frac{344.5 \sin 35^\circ}{\sin 105^\circ} = 204.7 \text{ m}$$

$$\text{AF} = \frac{344.5 \sin 40^\circ}{\sin 105^\circ} = 229.4 \text{ m}$$

Hence tangent length $AT_1 = AE + ET_1 = 204.7 + 218.4$
$= 423.1$ m

" " $AT_2 = AF + FT_2 = 229.4 + 126.1$
$= 355.5$ m

$$\textit{Length of the first branch} = \frac{\pi R_1 \times \alpha}{180^\circ} = \frac{\pi \times 600 \times 40^\circ}{180^\circ} = 418.9 \text{ m}$$

$$\text{" " second "} = \frac{\pi R_2 \times \beta}{180^\circ} = \frac{\pi \times 400 \times 35^\circ}{180^\circ} = 244.4 \text{ m},$$

Chainage of the intersection point (A)	=	3415.0 m
Deduct tangent length (AT_1)	=	− 423.1 m
Chainage of the tangent point (T_1)	=	2991.9 m
Add length of the first branch	=	*+ 418.9 m*
Chainage of the point of compound curvature (D)	=	3410.8 m
Add length of the second branch	=	244.4 m
Chainage of the tangent point (T_2)	=	3655.2 m

Reverse (or Serpentine) Curves —A reverse curve is composed of two circular arcs curving in opposite directions with a common tangent at their junction The point at which the two arcs join is called the point of *reverse curvature or contrary flexure*

Reverse curves are used when the straights are parallel or intersect at a very small angle They are frequently used in railway sidings, and sometimes on roads, and railway tracks designed for low speeds They should be avoided as far as possible on highways and main railway lines where speeds are necessarily high for the following reasons :

(1) They involve a sudden change of cant from one side to the other

(2) The curves cannot be properly superelevated at the point of reverse curvature

(3) The sudden change in direction is objectionable

(4) Steering is very dangerous in the case of highways It is always preferable, whenever practicable, to insert a short straight length or a reversed spiral between the two branches of the reverse curve

It is not possible to determine the elements of the reverse curve directly unless some condition is given, e g , equal radii or equal central angles

The equations for the general case may be deduced as follows (Fig 68).

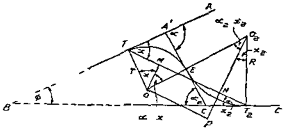


Fig 68

Notation —Let AB and CB = the straights to the curve

B = their point of intersection

ϕ = the angle of intersection ABC.

R = the greater radius

r = the smaller radius

T_1 and T_2 = the tangent points

E = the point of reverse curvature

α_1 = the angle (T_1O_1E) subtended at the centre by the arc having a smaller radius

α_2 = the angle (T_2O_2E) subtended at the centre by the arc having a greater radius

x_1 = the angle (AT_1T_2) between the tangent AB and the line joining the tangent points T_1 and T_2.

x_2 = the angle ($C'T_2T_1$) between the tangent BC and the line joining the tangent points T_1 and T_2

Join T_1T_2 Draw O_1M and O_2N perpendicular to T_1T_2. Though O_1 draw O_1P parallel to T_1T_2, cutting O_2N produced in P. Through the point of reverse curvature (E) draw the common tangent, meeting the tangent AB at A′ and the tangent BC at C′.

The points O_1, E, and O_2 are in a line.

$A'T_1$ and A′E being tangents to the first arc,

$$\angle AA'E = \angle T_1O_1E = \alpha_1.$$

C′E and $C'T_2$ being tangents to the second arc,

$$\angle BC'E = \angle T_2O_2 \alpha_2$$

From the triangle A′BC′, $\angle AA'C' = \angle A'BC' + \angle A'C'B$

i. e $\quad \alpha_1 = \phi + \alpha_2$ or $\phi = \alpha_1 - \alpha_2 \quad \ldots \quad \ldots \quad (27)$

Similarly, considering the triangle T_1BT_2,we have

$\angle A'T_1T_2 = \angle T_1BT_2 + \angle T_1T_2B \quad$ i e $x_1 = \phi + x_2$

or $\phi = x_1 - x_2 \ldots \quad \ldots \quad . \quad (28)$

Now $\angle T_1O_1M = \angle A'T_1T_2 = x_1$; $\angle T_2O_2N = \angle C'T_2T_1 = x_2$.

$\therefore \angle MO_1E = \angle T_1O_1E - \angle T_1O_1M = \alpha_1 - a_1$;

$\angle NO_2E = \angle T_2O_2E - \angle T_2O_2N = \alpha_2 - x_2$.

But $\angle MO_1E = \angle NO_2E \;\therefore\; \alpha_1 - x_1 = \alpha_2 - x_2$ or

$\alpha_1 - \alpha_2 = x_1 - x_2$

$T_1M = r \sin x_1$; $MN = O_1P = (R + r) \sin (\alpha_2 - x_2)$;

$NT_2 = R \sin x_2$.

Now $T_1T_2 = T_1M + MN + NT_2$

$= \{r \sin x_1 + (R + r) \sin (\alpha_2 - x_2) + R \sin x_2 \} \; .. \; (29)$

Again, $O_1M = r \cos x_1$; $O_2N = R \cos x_2$;

$O_2P = (R + r) \cos (\alpha_2 - x_2) = (R + r) \cos (\alpha_1 - x_1)$.

But $O_2P = O_2N - NP = O_2N + O_1M = R \cos x_2 + r \cos x_1$.

$$(R + r)\cos(\alpha_2 - x_2) = (R + r)\cos(\alpha_1 - x_1)$$
$$= (r\cos x_1 + R\cos x_2)$$

or $$\cos(\alpha_1 - x_1) = \cos(\alpha_2 - x_2) = \frac{(r\cos x_1 + R\cos x_2)}{(R + r)} \quad (30)$$

It may be noted that when the central angle α_1 is greater than α_2 the point of intersection occurs before the reverse curve When α_1 is less than α_2, it occurs after the reverse curve in which case $\phi = \alpha_2 - \alpha_1 = x_2 - x_1$

A few cases of reverse curves of common occurrence will now be considered

Case I —When the two straights are parallel (Fig 69),

Notation —R = the greater radius (O_2D)

r = the smaller radius (O_1C)

α_1 = the angle subtended at the centre by the arc having a smaller radius (r)

α_2 = the angle subtended at the centre by the arc having a greater radius (R)

v = the perpendicular distance between the straights CC and DD

l = the length of the line joining the tangent points C and D

h = the distance between the perpendiculars at C and D

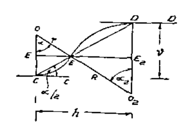


Fig 69

Let CC and DD be the parallel tangents

Through the point of reverse curvature E draw a line parallel to the straights cutting the perpendiculars CO_1 and DO_2 in E_1 and E_2 respectively

The perpendicular distance (v) $= CE_1 + DE_2$

But $CE_1 = O_1C - O_1E_1 = r - r\cos\alpha_1 = r(1 - \cos\alpha_1)$

$= r \text{ versin } \alpha_1$

Similarly, $DE_2 = O_2D - O_2E_2 = R - R\cos\alpha_2 = R(1-\cos\alpha_2)$
$= R \text{ versin } \alpha_2.$

It is evident from the figure that $\alpha_1 = \alpha_2$

$\therefore \quad v = R \text{ versin } \alpha_1 + r \text{ versin } \alpha_1 = (R+r) \text{ versin } \alpha_1$.(31)

The points C, E, and D obviously lie in a straight line

Now $CD = CE + ED$. But $CE = 2r\sin\frac{\alpha_1}{2}$ and $ED = 2R\sin\frac{\alpha_1}{2}$

$\therefore \quad l = CD = 2(R-r)\sin\frac{\alpha_1}{2}$ .. (32)

Since $\sin\frac{\alpha_1}{2} = \frac{v}{l}$ we get

$l = 2(R+r)\frac{v}{l}$ i e $l^2 = 2v(R+r)$

$\therefore \quad l = \sqrt{2v(R+r)}$. (32a)

$E_1E_2 = E_1E + EE_2 = r\sin\alpha_1 + R\sin\alpha_2.$

$\therefore \quad h = (R+r)\sin\alpha_1$ .. ... . (33)

When thetwo radii are equal ($R = r$), we have

$v = 2R \text{ versin } \alpha_1$ ... ... (34)

$l = 4R\sin\frac{\alpha_1}{2}$ ... . (35)

$l = \sqrt{4vR}$ ... ... (35a)

$h = 2R\sin\alpha_1$ ... ... (36)

If the curve is short, it may be set out by offsets from the long chords CE and ED.

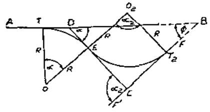


Fig 70

Case II — When the straights are non-parallel :

Data—The central angles α_1 and α_2, and the length of the common tangent are given (Fig. 70).

It is required to determine the common radius.

Let AB and BF = the straights intersecting at B

DC = the common tangent of length (d).

E = the point of reverse curvature

T_1 and T_2 = the tangent points

D = the point of intersection of the tangents AB and DC.
C = " " " " " " " DC and BF.
R = the common radius

DT_1 and DE are equal, since they are the tangents to the first arc

Similarly, CE and CT_2 are equal, since they are the tangents to the second arc

Now $DT_1 = DE = R \tan \frac{\alpha_1}{2}$, the angle BDE being equal to α_1

$CT_2 = CE = R \tan \frac{\alpha_2}{2}$, " F'CD " " to α_2

$$d = DC = DE + EC = R \tan \frac{\alpha_1}{2} + R \tan \frac{\alpha_2}{2}$$

$$= R \left(\tan \frac{\alpha_1}{2} + \tan \frac{\alpha_2}{2} \right)$$

$$R = \frac{d}{\left(\tan \frac{\alpha_1}{2} + \tan \frac{\alpha_2}{2} \right)} \tag{37}$$

From which R may be calculated Knowing R, and the central angles α_1 and α_2, the lengths of the two arcs of the curve may be determined

If the chainage of D be given, the chainage of the points of tangency T_1 and T_2, and the point of reverse curvature may be obtained thus

Chainage of T_1 = chainage of D $- DT_1$
" of E = " of T_1 + length of the first arc
" of T_2 = " of E + length of the second arc

The deflection angles for the two arcs may be calculated in the usual way. The first arc may be set out from T_1 and the second one from E.

Case *III*:—When the straights are non-parallel:—Given the tangent points, their distance apart, and the angles x_1 and x_2 which the line joining the tangent points makes with the two tangents. It is required to find the common radius of the two branches of the curve (Fig. 71)

Let AA′ and CC′ = the tangents to the curve.

T_1 and T_2 = the tangent points.

E = The point of contrary flexture.

R = the common radius.

d = the length of the line joining the tangent points T_1 and T_2.

x_1 = the angle ($A'T_1T_2$) between T_1T_2 and AA′.

x_2 = the angle (T_1T_2C') between T_1T_2 and CC′.

The points O_1, E, and O_2 lie in a straight line. Join T_1 and T_2.

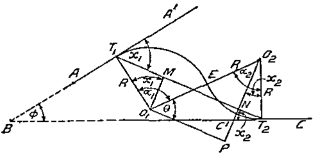


Fig. 71

Draw O_1M and O_2N at right angles to T_1T_2. Through O_1 draw a line parallel to T_1T_2, meeting O_2N produced at P. Let the angle O_2O_1P be θ.

From the triangle O_2O_1P, $\sin O_2O_1P = \sin\theta = \dfrac{O_2P}{O_1O_2}$

and $O_1P = O_1O_2 \cos\theta = 2R\cos\theta$

But $O_2P = O_2N + NP = O_2N + O_1M$, since $NP = O_1M$

Now $O_1M = O_1T_1 \cos T_1O_1M = R\cos x_1$

and $O_2N = O_2T_2 \cos T_2O_2N = R\cos x_2$

$$\therefore \quad \sin\theta = \frac{R\cos x_1 + R\cos x_2}{2R}, \text{ since } O_1O_2 = 2R$$

or $$\theta = \sin^{-1}\left(\frac{\cos x_1 + \cos x_2}{2}\right) \tag{38}$$

Again, $T_1T_2 = T_1M + MN + NT_2$

$T_1M = R\sin x_1$, $MN = O_1P = 2R\cos\theta$, $NT_2 = R\sin x_2$

$$T_1T_2 = R\sin x_1 + 2R\cos\theta + R\sin x_2 = d$$

Whence, $$R = \frac{d}{(\sin x_1 + 2\cos\theta + \sin x_2)} \tag{39}$$

The angle subtended at the centre by the first arc $= \alpha$

$$= \angle T_1O_1E = \angle T_1O_1M + \angle MO_1E$$
$$= x_1 + (90° - \theta) \tag{40}$$

The angle subtended at the centre by the second arc $= \alpha$

$$= \angle T_1 OE = \angle T_1 O_1 M + \angle M O_1 E$$
$$= x_1 + (90 - \theta)$$
$$= \angle T_2 O_2 E = T_2 O_2 N + N O_2 E$$
$$= x_2 + (90 - \theta)$$
$$= \angle T_2O_2E = \angle T_2O_2N + \angle NO_2E$$
$$= x + (90° - \theta) \tag{40a}$$

Knowing θ, R, α_1, and α_2, the lengths of the two branches of the curve may be computed The calculations necessary for setting out the curve may be made as already explained

Case IV —When the straights are non parallel and the radii unequal —The data are the same as in case III, and in addition one of the radii is given (Fig 72)

The notation will be the same as in Case III

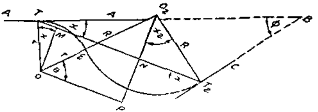


Fig 72

Let R be the greater radius (O_2E) and r the smaller one (O_1E)

$$O_1O_2 = O_2E + O_1E = R + r \quad O_1P = MN$$

$$O_2P = O_2N + NP = O_2N + O_1M = R \cos x_2 + r \cos x_1$$

From the $\triangle O_1O_2P$ $\quad O_1P = \sqrt{O_1O_2^2 - O_2P^2}$

$$= \sqrt{\{(R + r)^2 - (R \cos x_2 + r \cos x_1)^2\}} = MN$$

$$T_1M = r \sin x_1 \quad NT_2 = R \sin x_2$$

$$T_1T_2 = d = T_1M + MN + NT_2$$

$$= r \sin x_1 + \sqrt{\{(R + r)^2 - (R \cos x_2 + r \cos x_1)^2\}} + R \sin x_2 \text{ or}$$

$$\{d - (R \sin x_2 + r \sin x_1)\}^2 = \{(R + r)^2 - (R \cos x_2 + r \cos x_1)^2\}$$

On reduction, we get

$$d^2 - 2d\,(r \sin x_1 + R \sin x_2) = 4\ Rr \sin^2\left(\frac{x_1 - x_2}{2}\right) \qquad (41)$$

If one of the radii say R is given the other radius (r) may be found from the above quadratic equation

Knowing R and r the angle O_2O_1P (θ) may be obtained With θ known the necessary calculations may be made as explained in the preceding case

Case V —When the straights are non parallel and the radii unequal —Given the angle of intersection (ϕ) of the two tangents, the two radii, and one tangent length (Fig 73)

Let AB and BC = the first and second tangents intersecting at B

T_1 and T_2 = the tangent points

T = the length of the first tangent BT_1.

T'' = „ of the second tangent BT_2

α_1 = the angle subtended at the centre O_1 by the first arc

α_2 = the angle subtended at the centre O by the second arc

ϕ = the angle of intersection of the tangent AB and BC

R = the greater radius O_2E

r = the smaller radius O_1E

Draw O_2N perpendicular to AB Through O_2 draw a line Parallel to AB meeting O_1T_1 produced in P

Let $O_2P = a$, $O_2N = b$, $NM = c$, $MT_2 = d$, $MB = e$

(1) $T_2MB = 90^\circ - \phi = O_2MN$

$EO_2T_2 = \alpha_2$, $NO_2M = \phi$, $T_1O_1E = \alpha_1$

$\therefore$ $EO_2N = EO_2T_2 - NO_2M = \alpha_2 - \phi = T_1O_1E = \alpha_1$

i e $\alpha_2 - \phi = \alpha_1$ or $\phi = (\alpha_2 - \alpha_1)$ (42)

(2) From the triangle MBT_2,

$d = MT_2 = BT_2 \tan\phi = T'' \tan\phi$

$e = MB = BT_2 \sec\phi = T'' \sec\phi$

T'' being given, d and e can be calculated

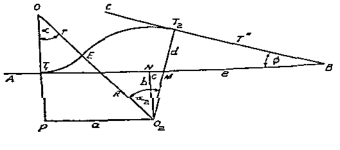


Fig 73

(3) From the triangle O_2NM, $b = O_2N = O_2M \cos \phi$

$c = NM = O_2M \sin \phi$.

But $O_2M = O_2T_2 - MT_2 = R - d$.

$\therefore \quad b = (R - d) \cos \phi$; and $c = (R - d) \sin \phi$.

R, d, and ϕ being known, b and c can be found.

(4) From the triangle O_1PO_2, $a = O_2P = O_1P \tan \propto_1$.

But $O_1P = O_1T_1 + T_1P = r + b$

$\therefore \quad a = (r + b) \tan \propto_1$.

With R, r, and b known, we can calculate $\propto_1$ from

$$\cos \propto_1 = \frac{O_1P}{O_1O_2} = \frac{(r + b)}{(R + r)}.$$

(5) Having calculated a, c, and e, the tangent distance T_1B may be found from

$$T' = T_1B = T_1N + NM + MB = a + c + e \qquad (43)$$

To Locate the Tangent Points on a Given Deviation consisting of Three Curves of Equal Radius: Given the common radius and amount of the deviation.—Sometimes it is found necessary to deviate from a given straight line in order to avoid intervening obstructions such as a building, a bend of a river, etc. To locate the tangent points on a given deviation, the procedure is as follows —

Referring to Fig. 74, BC is the original line, M the point of deviation, MD (d) the amount of deviation, the angle T_1MD being a right angle; T_1 and T_2 the tangent points; R the common radius. It is required to determine the distances MT_1 and MT_2. To do this, drop a perpendicular O_1N on the chord T_1E. Then

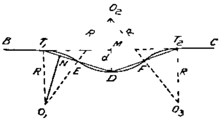


Fig 74

$T_1N = \frac{1}{2} T_1E = \frac{1}{4} T_1D$ Since the triangles T_1O_1N and T_1MD are similar, $\dfrac{T_1N}{T_1O_1} = \dfrac{MD}{T_1D}$.

But $T_1N = \frac{1}{4} T_1D$ and $T_1O_1 = R$.

$$\therefore \quad T_1D^2 = 4R \times MD \text{ or } T_1D = 2\sqrt{R \times MD} \quad \ldots \quad (44)$$

$$\text{and} \quad T_1E = \tfrac{1}{2} T_1D = \sqrt{R \times MD} \quad \ldots \quad (44a)$$

$$\text{Now } MT_1^2 = T_1D^2 - MD^2 = 4R \times MD - MD^2$$

$$MT_1 = \sqrt{MD\,(4R - MD)} \qquad (45)$$

Similarly, $MT_2 = \sqrt{MD\,(4R - MD)}$

Example 1.—A reverse curve is to be set out between two parallel tangents 18 m apart. The distance between the tangent points C and D is 180 m and the two arcs of the curve have the same radius. Calculate the radius, and the offsets at 7.5 m intervals, the curve being set out by means of offsets from CD (Fig. 69).

(i) Radius of the curve.—Let α be the central angle.

Then $\sin \dfrac{\alpha}{2} = \dfrac{v}{l} = \dfrac{18}{180} = \dfrac{1}{10}$ $\quad \alpha = 11^\circ\, 28'$.

$$\text{Radius} = \frac{v}{2 \text{ versin } \alpha} = \frac{18}{2\,(1 - \cos 11^\circ\, 18')} = 450 \text{ m}$$

(ii) Offsets from the long chord CE of the first arc of the curve —

(a) By approximate formula, $O_x = \dfrac{x\,(L - x)}{2R}$, the distances of the points on the chord CE being measured from C.
Length of CE $= L = 90$ m

Offset at C $= O_0 = 0.00$ = offset at 90 m

" at 7.5 m $= O_1 = \dfrac{7.5 \times 82.5}{2 \times 450} = 0.687$ m = " " 82.5 m

" at 15 m $= O_2 = \dfrac{15 \times 75}{2 \times 450} = 1.251$ " = " " 75 m

Offset at 22·5 m $= O_3 = \dfrac{22\ 5 \times 67\ 5}{2 \times 450} = 1\cdot689$,, = ,, ,, 67 5 m

,, at 30 m $= O_4 = \dfrac{30 \times 60}{2 \times 450} = 2\ 001$,, = ,, ,, 60 m.

,, at 37 5 m $= O_5 = \dfrac{37\ 5 \times 52\ 5}{2 \times 450} = 2\ 187$,, = ,, ,, 52 5 m

,, at 45 m $= O_6 = \dfrac{45 \times 45}{2 \times 450} = 2\ 25$,, = ,, ,, 45 m.

(b) By exact formula, $O_x = \sqrt{R^2 - x^2} - \sqrt{R^2 - \left(\frac{L}{2}\right)^2}$

the distances of the points being measured from the mid point F of the long chord CE

Offset at F $= O_0 = \sqrt{450^2 - 0} - \sqrt{450^2 - 45^2} = 2\ 25$ m.

,, at 7 5 m $= O_1 = \sqrt{450^2 - 7\ 5^2}$ − ,, $= 2\ 103$ m.

,, at 15 m $= O_2 = \sqrt{450^2 - 15^2}$ − ,, $= 1\ 986$ m

,, at 25 5 m $= O_3 = \sqrt{450^2 - 22\ 5^2}$ − ,, $= 1\cdot638$ m

,, at 30 m $= O_4 = \sqrt{450^2 - 30^2}$ − ,, $= 1\ 230$ m.

,, at 37 5 m $= O_5 = \sqrt{450^2 - 37\ 5^2}$ − ,, $= 0\ 750$ m.

,, at 45 m $= O_6 = \sqrt{450^2 - 45^2}$ − ,, $= 0\cdot000$ m.

The offsets for the second arc are the same as above.

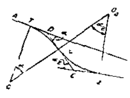


Fig 75

Example 2 —Two straights AD and CF intersect at B The common tangent DC intersects AB at D and BF at C respectively It is proposed to introduce a reverse curve of radius R between them The angles ADC and DCF are 142° 36′ and 132° 48′ respectively. The length of the common tangent is 465 m Find the common radius, and the chainages of the tangent points and the point of reverse curvature, if the chainage of D is 785 m. (Fig. 75).

(i) The deflection angle (α_1) between AD and DC
$= 180° - 142° 36' = 37° 24'$

The deflection angle (α_2) between DC and CF
$= 180° - 132° 48 = 47° 12'$

Now $DC = DE + EC = R\left(\tan\frac{\alpha_1}{2} + \tan\frac{\alpha_2}{2}\right)$

$= 465$ m

$R = 600$ m

(ii) Length of the tangent $DT_1 = R \tan\frac{\alpha_1}{2}$

$= 600 \times 18° 42'$

$= 203\ 07$ m

Length of the tangent $CT_2 = R \tan\frac{\alpha_2}{2}$

$= 600 \times \tan 23° 36'$

$= 262\ 11$ m

(iii) Length of the first arc of the curve $= \frac{\pi R \alpha_1}{180}$

$= \frac{\pi\ 600 \times 37° 4}{180°} = 391\ 68$ m

Length of the second arc of the curve $= \frac{\pi R \alpha_2}{180}$

$= \frac{\pi\ 600 \times 47° 2}{180°} = 494\ 28$ m

(iv) Chainage of D	= 735 00 m.
Deduct tangent length DT_1	= 203 07 m
Chainage of the first tangent point (T_1)	= 531 93 m
Add length of the first arc	= 391 68 m
Chainage of the point of reverse curvature (E)	= 923 61 m
Add length of the second arc	= 494 28 m
Chainage of the second tangent point (T_2)	= 1417 89 m

Example 3:—A reverse curve is to be run from a point T_1 on AA′ to the point T_2 on CC′. Determine the common radius, and the lengths of the two parts of the curve, given that T_1T_2 is 720 m and the angles $A\,T_1T_2$ and T_1T_2C are 47° 30′ and 25° 12′ respectively (Fig. 71).

Here $\alpha_1 = 47° 30'$, $\alpha_2 = 25° 12'$, $T_1T_2 = 720$ m,

$\angle O_2O_1P = \theta$.

Let R denote the common radius.

$$\text{Then } \sin\theta = \frac{R\cos\alpha_1 + R\cos\alpha_2}{2R} = \frac{\cos\alpha_1 + \cos\alpha_2}{2}$$

$$= \frac{\cos 47° 30' + \cos 25° 12'}{2}$$

$\therefore \qquad \theta = 52° 12'$.

The common radius (R) may be obtained from

$$R = \frac{T_1T_2}{\sin\alpha_1 + 2\cos\theta + \sin\alpha_2}$$

$$\therefore \qquad = \frac{720}{\sin 47° 30' + 2\cos 52° 12' + \sin 25° 12'}$$

$$= 301{\cdot}396 \text{ m}$$

Now the central angle $\propto_1$ of the first arc

$$= \alpha_1 + 90° - \theta = 47° 30' + 90° - 52° 12' = 85° 18'.$$

The central angle $\propto_2$ of the second arc

$$= \alpha_2 + 90° - \theta = 25° 12' + 90° - 52° 12' = 63°.$$

$$\therefore \text{ Length of the first arc} = \frac{\pi R \propto_1}{180} = \frac{\pi \times 301{\cdot}496 \times 85°{\cdot}3}{180°}$$

$$= 448{\cdot}4 \text{ m}$$

$$\text{,, of the second arc} = \frac{\pi R \propto_2}{180} = \frac{\pi \times 301{\cdot}496 \times 63°}{180°}$$

$$= 331{\cdot}40 \text{ m}$$

Example 4:—Two straights AA′ and CC′ are to be connected by a reverse curve which begins from T_1 on AA′ and

ends at T_2 on CC′. The angles $A'T_1T_2$ and T_1T_2C' are 32° 14′ and 16° 48′ respectively. The radius of the first branch commencing from T_1 is 320 m Find the radius of the second branch of the curve and the lengths of the two branches, if the length of T_1T_2 is 496 8 m (Fig. 72).

Let r be the radius of the second branch of the curve.

Here $x_1 = 32°14'$; $x_2 = 16°48'$; $T_1T_2 = 496$ m; $R = 320$ m

$$\sin\theta = \frac{R\cos x_1 + r\cos x_2}{R+r} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

$$T_1T_2 = T_1M + MN + NT_2 = 496 \text{ m.}$$

Now $T_1M = R\sin x_1$; $MN = (R+r)\cos\theta$; $NT_2 = r\sin x_2$.

$$T_1T_2 = R\sin x_1 + (R+r)\cos\theta + r\sin x_2 = 496{\cdot}8$$

$$\text{Hence } \cos\theta = \frac{T_1T_2 - R\sin x_1 - r\sin x_2}{R+r}. \quad \ldots \quad \ldots (2)$$

Eliminating θ, from the relation $\sin^2\theta + \cos^2\theta = 1$, we have

$$\left\{\left(\frac{R\cos x_1 + r\cos x_2}{R+r}\right)^2 + \left(\frac{T_1T_2 - R\sin x_1 - r\sin x_2}{R+r}\right)^2\right\} = 1 \ldots (3)$$

Substituting the given values in equation (3), we get

$$\left\{\left(\frac{320\cos 32°\,14' + r\cos 16°48'}{320+r}\right)^2 + \left(\frac{496\ 8 - 320\sin 32°\,14' - r\sin 16°\,48'}{320+r}\right)^2\right\} = 1$$

or $\{640 - 640\cos 32°\,14'\cos 16°\,48' - 640\sin 32°\,14'\sin 16°\,48.$ $+ 993\ 6\sin 16°\,48'\}\,r$

$$= \{(496\ 8)^2 - 640 \times 496{\cdot}8\sin 32°\,14'\}$$

$$r = \frac{77440}{300{\cdot}16} = 219{\cdot}64 \text{ m}$$

Whence from (1), $\sin\theta = \dfrac{320\cos 32°14' + (249{\cdot}64)\cos 16°48'}{320 + 249{\cdot}64}$

$$= \frac{509{\cdot}68}{569{\cdot}64} \qquad \therefore\ \theta = 63°\,28'$$

Now the central angle of the first arc

$= \measuredangle_1 = x_1 + 90^\circ - \theta = 32^\circ\, 14' + 90^\circ - 63^\circ\, 28' = 58^\circ\, 46'.$

The central angle of the second arc

$= \measuredangle_2 = x_2 + 90^\circ - \theta = 16^\circ\, 48 + 90^\circ - 63^\circ\, 28' = 43^\circ\, 20'.$

Length of the first branch of the curve

$$= \frac{\pi \times 320 \times 58^\circ\, 77}{180^\circ} = 328\ 24 \text{ m}$$

Length of the second branch of the curve

$$= \frac{\pi \times 249\ 64 \times 43^\circ\, 33}{180^\circ} = 188\ 84 \text{ m}$$

Transition Curves (from page 161)

On railways and highways it is the common practice to introduce a curve of varying radius called a *transition* curve between the tangent and a circular curve The transition curve is also called the *spiral* or *easement* curve It is also inserted between the two branches of a compound or reverse curve

The objects of introducing a transition curve at each end of the circular curve are as follows

(1) To accomplish gradually the transition from the tangent to the circular curve and from the circular curve to the tangent

(2) To obtain a gradual increase of curvature from zero at the tangent point to the specified quantity at the junction of the transition curve with the circular curve

(3) To provide a satisfactory means of obtaining a gradual increase of superelevation from zero on the tangent to the specified amount on the main circular curve so that the full superelevation is attained simultaneously with the curvature of the circular curve at the junction of the transition curve with the circular curve

A transition curve should fulfil the following conditions when it is inserted between the tangent and the circular curve

(1) It should meet the original straight tangentially

(2) It should meet the circular curve tangentially

(3) Its radius at the junction with the circular curve should be the same as that of the circular curve

(4) The rate of increase of curvature along the transition curve should be the same as that of increase of superelevation

(5) Its length should be such that the full superelevation is attained at the junction with the circular curve

The types of the transition curve which are in common use are (1) a *cubic parabola*, (2) a *clothoid* or *spiral*, and (3) a *lemniscate* the first two being used on railways, and the third on highways

In order to admit a transition curve, the main circular curve requires to be shifted inwards When the transition curves are inserted at each end of the main circular curve, the resulting curve is called the *combined* or *composite* curve

Superelevation —When a vehicle passes from a straight path to a curved one, the forces acting on it are (i) the weight of the vehicle, and (ii) the centrifugal force, both acting through the centre of gravity of a vehicle Since the centrifugal force always acts in a direction perpendicular to the axis of rotation which is vertical, its direction is always horizontal The effect of the centrifugal force is to push the vehicle off the rails or track In order to counteract this action, the plane of the rails or the road surface is made perpendicular to the resultant of the centrifugal force and the weight of the vehicle In other words, the outer rail is superelevated or raised above the inner one Similarly, the road should be ' banked ', i e the outer edge of the road should be raised above the inner one, the raising of the outer rail or outer edge above the inner one, being called the *superelevation* or *cant* The amount of superelevation depends upon the speed of the vehicle and the radius of the curve

In Fig 76, let

W = the weight of the veh cle

P = the centrifugal force

v = the speed of the vehicle in m per second

g = the acceleration due to gravity, 9 81 m per sec²

R = the radius of the curve in m

h = the superelevation in m

b = the width of the road in m

G = the distance between the centres of the rails in m

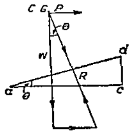


Fig 76

Then for equilibrium the resultant of the weight and the centrifugal force must be equal and opposite to the reaction perpendicular to the road or rail surface

Now $P = \frac{Wv^2}{gR}$ $\quad \frac{P}{W} = \frac{v^2}{gR}$

If θ be the inclination of the road or rail surface, the inclination of the resultant to the vertical is also θ Therefore, we have

$$\text{Tan } \theta = \qquad = \frac{v^2}{gR}$$

Hence the amount of superelevation h

$$= b \tan \theta = \frac{bv^2}{gR} \text{ on roads} \qquad (44)$$

$$= \frac{Gv^2}{gR} \text{ on railways} \qquad (45)$$

The amount of superelevation is limited to 15 cm on a standard gauge 1 435 m (4' 8½") the distance between the centres of the rails being 1 486 m (4' 11½") {16 cm for B G 1 677m (5' 6") and 10 cm for M G 1 m (3' 3⅜")} for safety of the vehicles It should be applied gradually along the transition curve so that it is proportional to the distance from the beginning of the curve full superelevation being attained at the junction of the transition curve with the circular curve In applying it, it is the common practice in America to raise the outer edge of the road or the outer rail and to depress the

inner edge or inner rail by half the amount of superelevation However, in India and England the practice is to raise the outer edge of the road or the outer rail by the full amount of superelevation

Length of Transition Curve —The length of a transition curve may be determined in the following ways

(1) *By an arbitrary gradient* —The length may be such that the superelevation is applied at a uniform rate of 1 in n the value of n varying from 300 to 1200

Therefore $\quad L = nh \qquad (46)$

where $\quad L$ = the length of the transition curve in m

h = the superelevation in m

1 in n = the rate of canting

(2) *By the time rate* —The transition curve may be of such a length that the cant is applied at an arbitrary time rate of a cm per second a varying from 2 5 cm to 5 cm

Let L = the length of the transition curve in m

v = the speed in m per second

h = the amount of superelevation in cm

a = the time rate (cm/sec)

Time taken by a vehicle in passing over the transition curve

$$= \frac{L}{v} \text{ seconds}$$

Superelevation attained in this time $= \dfrac{La}{v} = h'$.

$$L = \frac{h\,v}{a} \qquad (47)$$

(3) *By the rate of change of radial acceleration* —This rate should be such that the passengers should not experience any sensation of discomfort when the train is travelling over the curve It is taken as 30 cm per sec ^{3}, which is the maximum that will pass unnoticed

Now the radial acceleration on the circular curve $= \dfrac{v^2}{R}$ (m/sec²)

Time taken by a vehicle to pass over the transition curve

$$= \frac{L}{v} \text{ seconds}$$

Radial acceleration attained in $\dfrac{L}{v}$ seconds at the rate of

$$0.3 \text{ m per sec}^3 = \frac{L}{v} \times .3 \text{ m/sec}^2$$

$$\frac{v^2}{R} = \frac{L}{v} \times 0.3 \text{ or } L = \frac{v^3}{0.3\,R} \tag{48}$$

$$L = \frac{V^3}{14\,R} \quad \text{if } V = \text{speed in km/hr} \tag{48a}$$

Of these methods the third method is commonly used in determining the length of a transition curve

The ratio of the centrifugal force and the weight is called the centrifugal ratio

$$\text{Centrifugal ratio} = \frac{P}{W} = \frac{Wv^2}{gRW} = \frac{v^2}{gR} \tag{49}$$

The maximum value of the centrifugal ratio on roads is taken as $\frac{1}{4}$ and for railways as $\frac{1}{8}$

On roads — $\dfrac{v^2}{gR} = \dfrac{1}{4}$

$$v^2 = \frac{gR}{4} = 2.452\,R \text{ or } v = \sqrt{2.452\,R}$$

Now from formula (48) $L = \dfrac{v^3}{0.3\,R}$

$$L = \frac{2.452^{3/2}\,R^{3/2}}{0.3\,R} = 12.80\sqrt{R} \tag{50}$$

On railways — $\dfrac{v^2}{gR} = \dfrac{1}{8}$ $\quad v^2 = 1.226\,R$ or $v = \sqrt{1.226R.}$

$$\text{Now } L = \frac{v^3}{0.3R} = \frac{1.226^{3/2}R^{3/2}}{0.3R} = 4.526\sqrt{R} \qquad (50a)$$

The expression (50) or (50a) is to be used only when the full centrifugal ratio is developed and when the rate of gain of radial acceleration is 0.3 m per sec.^{3}

Ideal Transition Curve.—The intrinsic equation of the ideal

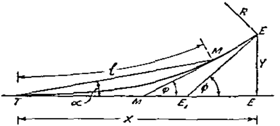


Fig. 77

transition curve (clothoid spiral) may be deduced as follows:—

In Fig. 77, let TB = the initial tangent

T = the beginning of the transition curve

E = the point of junction of the transition curve with the circular curve

M = any point on the transition curve l m. along it from T

ρ = the radius of the transition curve at M

R = the radius of the circular curve

ϕ = the inclination of the tangent to the transition curve at M to the initial tangent TB

ϕ_1 = the angle between the tangent TB and the tangent to the transition curve at the junction point E

(This angle is known as the *spiral* angle.)

L = the length of the transition curve

The fundamental requirement of the spiral curve is that its radius of curvature at any point shall vary inversely as the distance (l) from the beginning of the curve.

Therefore, $\rho \propto \frac{1}{l}$ or $\frac{1}{\rho} = ml$

Now for all curves, $\frac{d\phi}{dl}$ = curvature $= \frac{1}{\rho}$.

$\therefore \quad d\phi = \frac{1}{\rho} dl = ml \times dl.$

Integrating, we get $\phi = \frac{ml^2}{2}$ .. .. (51)

The constant of integration being zero, since $\phi = 0$, when $l = 0$.

At the junction point E, $l = L$, $\rho = R$, and $\phi = \phi_1$

$\therefore \quad \frac{1}{R} = mL$ or $m = \frac{1}{RL}$ and $\phi_1 = \frac{1}{RL} \cdot \frac{L^2}{2} = \frac{L}{2R}$ .. (52)

Substituting the value of m in equation (51), we get

$\phi = \frac{l^2}{2RL}$ or $l = K\sqrt{\phi}$ . ... .. 53)

in which $K = \sqrt{2RL}$.

If the curve is to be set out by offsets from the tangent at the commencement of the curve (T), it is necessary to calculate the rectangular (Cartesian) co ordinates, the 'axes of co-ordinates' being the tangent at T as the x-axis and a line perpendicular to it as the y-axis.

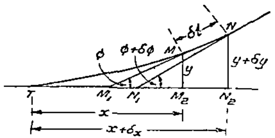


Fig 78

(i) Referring to Fig 78, let M and N be the two points at a distance δl apart on the curve. Let the co ordinates of M and N be (x, y) and $(x + \delta x, y + \delta y)$, and the respective inclinations

of the tangents at M and N to the initial tangent (TB) at T, ϕ and $\phi + \delta\phi$

Then we have $\quad dx = dl\cos\phi \quad$ and $\quad dy = dl\sin\phi$

$$\therefore \quad dx = dl\left(1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} - \text{etc}\right)$$

$$\text{Now } l = K\sqrt{\phi} \qquad \therefore \quad dl = \frac{K}{2\sqrt{\phi}}\,d\phi$$

$$\therefore \quad dx = \frac{K}{2}\left(\phi^{-\frac{1}{2}} - \frac{\phi^{\frac{3}{2}}}{2!} + \frac{\phi^{\frac{7}{2}}}{4!} - \text{etc}\right)d\phi$$

By integration, $x = K\left(\phi^{\frac{1}{2}} - \dfrac{\phi^{\frac{5}{2}}}{10} + \dfrac{\phi^{\frac{9}{2}}}{216} - \text{etc}\right)$

$$= K\sqrt{\phi}\left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \text{etc}\right)$$

Substituting l for $K\sqrt{\phi}$ we get

$$x = l\left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \text{etc}\right) \tag{51}$$

Writing $\dfrac{l^2}{K^2}$ for ϕ $\quad x = l\left(1 - \dfrac{l^4}{10K^4} + \dfrac{l^8}{216\,K^8} - \text{etc}\right) \qquad (52)$

(ii) $dy = dl\sin\phi = dl\left(\phi - \dfrac{\phi^3}{3!} + \dfrac{\phi^5}{5!} - \text{etc}\right)$

But $dl = \dfrac{K}{2\sqrt{\phi}}\,d\phi \qquad dy = \dfrac{K}{2}\left(\phi^{\frac{1}{2}} - \dfrac{\phi^{\frac{5}{2}}}{6} + \dfrac{\phi^{\frac{9}{2}}}{120} - \text{etc}\right)d\phi$

By integration $\quad y = K\left(\dfrac{\phi^{\frac{3}{2}}}{3} - \dfrac{\phi^{\frac{7}{2}}}{42} + \dfrac{\phi^{\frac{11}{2}}}{1320} - \text{etc}\right)$

$$= K\sqrt{\phi}\,\frac{\phi}{3}\left(1 - \frac{\phi^2}{14} + \frac{\phi^4}{440} - \text{etc}\right)$$

Writing l for $K\sqrt{\phi}$ and $\frac{l^2}{K^2}$ for ϕ, we have

$$y = \frac{l^3}{3K^2}\left(1 - \frac{\phi^2}{14} + \frac{\phi^4}{440} - \text{etc}\right)$$

$$= \frac{l^3}{3K^2}\left(1 - \frac{l^4}{14\,K^4} + \frac{l^8}{440\,K^8} - \text{etc}\right)$$

Substituting the value of K $(=\sqrt{2RL})$, we get

$$y = \frac{l^3}{6RL}\left(1 - \frac{\phi^2}{14} + \frac{\phi^4}{440} - \text{etc}\right) \tag{55}$$

$$= \frac{l^3}{6RL}\left(1 - \frac{l^4}{14\,(2RL)^2} + \frac{l^8}{440\,(2RL)^4} - \text{etc}\right) \tag{55a}$$

(iii) Rejecting all terms of the expansions (54) and (55) except the first, we have

$$x = l \quad\text{and}\quad y = \frac{l^3}{6RL} = \frac{x^3}{6RL} \tag{56}$$

which is the equation of a cubic parabola, the length of the curve being measured along the x axis (*along the tangent* TB). The cubic parabola is known as Froude's transition or easement curve. The offset to any point on the curve for a given distance along TB may be obtained from equation (56).

(iv) Taking the first two terms of the above expansions (54) and (55), we get

$$x = l\left(1 - \frac{\phi}{10}\right) = l\left(1 \text{ [illegible]}\right) \tag{57}$$

$$y = \frac{l^3}{6RL}\left(1 - \frac{\phi}{14}\right) = \text{[illegible]}\left(\text{[illegible]} - \frac{l^4}{14\,(2RL)^2}\right) \tag{58}$$

From which the co-ordinates of any point on the true or clothoid spiral may be computed, the length l being measured *along the curve*.

(v) If we take the first term only of the expansion (55), we get an equation for the cubic spiral, $y = \frac{l^3}{6RL}$ (59)

Thus it will be seen that in the case of a cubic parabola, we make two approximations, viz $\cos\phi = 1$ and $\sin\phi = \phi$, while in the case of a cubic spiral only one approximation is made, viz $\sin\phi = \phi$. Since the cosine series is less rapidly converging than the sine series, greater error is involved in assuming $\cos\phi = 1$ than that made in assuming $\sin\phi = \phi$. The cubic parabola is, therefore, inferior to the cubic spiral.

For all practical purposes, however, there is very little difference between these two forms of the transition curve.

Now $\tan\alpha = \dfrac{y}{x}$, where α = the deflection angle, i.e. the angle (MTB) between the tangent at T and the line from T to any point (M) on the curve (Fig. 76).

Dividing the expansion (55) by the expansion (54), we get $\tan\alpha = \dfrac{\phi}{3} + \dfrac{\phi^3}{105} + \text{etc.}$, which very closely resembles the expansion of $\tan\dfrac{\phi}{3}$, viz. $\left(\tan\dfrac{\phi}{3} = \dfrac{\phi}{3} + \dfrac{\phi^3}{81} + \text{etc}\right)$

$$\therefore \tan\alpha = \tan\frac{\phi}{3}$$

Since ϕ is usually small (a small fraction of a radian)

$$\alpha = \frac{\phi}{3} \text{ very nearly} \quad \ldots \quad (60)$$

But $\phi = \dfrac{l^2}{K^2}$ and $K = \sqrt{2RL}$.

$$\text{Hence } \alpha = \frac{1}{3}\frac{l^2}{K^2} = \frac{l^2}{6RL} \text{ radians}$$

$$= \frac{1800\, l}{\pi RL} \text{ minutes} \quad \ldots \quad \ldots \quad \ldots \quad (61)$$

Characteristics of a Transition Curve :—

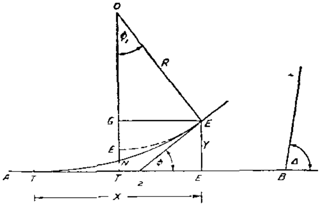


Fig 79

In Fig 79, let TB = the original tangent.

T = the commencement of the transition curve.

E = the end of the transition curve

EE_2 = the tangent to both the transition and the circular curve at E

$Y = EE_1$ = the offset to the junction point (E) of both curves.

$X = TE_1$ = the x co-ordinate of E

EE′ = the redundant circular curve.

T_1 = the point of intersection of the line (OE′) perpendicular to the tangent to the circular curve at E′ and the original tangent TB.

$S = E'T_1$ = the shift of the circular curve

N = the point in which OE′ cuts the transition curve.

ϕ_1 = the spiral angle (EE_2B) between the common tangent EE_2 and the original tangent TB

R = the radius of the circular curve (OE)

L = the length of the transition curve.

(a) Now $EE_1 = R\phi_1$ but $\phi_1 = \dfrac{L}{2R}$ $\quad EE_1 = R\,\dfrac{L}{2R} = \dfrac{L}{2}$

But EN is very nearly equal to EE_1 $\quad EN = \dfrac{L}{2}$ (62)

i e the shift ($E\,T_1$) bisects the transition curve at N

Hence $TN = \dfrac{L}{2}$ (62a)

(b) Draw EG perpendicular to OE′

$$\text{Now } S = E\,T_1 = GT_1 - GE = EE_1 - GE$$
$$= Y - R(1 - \cos\phi_1) \tag{63}$$

$$\text{or} \quad S = Y - 2R\sin^2\frac{\phi}{2} \tag{63a}$$

$$\text{But } Y = \frac{L^3}{6RL} = \frac{L^2}{6R} \quad \text{and} \quad \phi_1 = \frac{L}{2R}$$

$$S = Y - 2R\sin^2\frac{\phi_1}{2} = Y - 2R\frac{\phi_1^{\,2}}{4} = \frac{L^2}{6R} - \frac{L^2}{8R} = \frac{L^2}{24R} \tag{64}$$

$$\text{Also } NT_1 = \frac{TN^3}{6RL} = \frac{\left(\frac{L}{2}\right)^3}{6RL} = \frac{L^2}{48R} = \frac{1}{2}S = \frac{1}{2}E\,T_1 \tag{65}$$

i e the transition curve bisects the shift

(c) Total tangent length (BT) —(a) True Spiral (Clothoid)

$$BT = BT_1 + T_1T$$

$$\text{Now } T_1T = TE_1 - E_1T_1 = TE_1 - EG = X - R\sin\phi_1$$

$$OT_1 = OE + E\,T_1 = R + S$$

$$BT_1 = (R + S)\tan\frac{\Delta}{2}$$

where Δ is the deflection angle between the straights

$$BT = (R + S)\tan\frac{\Delta}{2} + (X - R\sin\phi_1) \tag{66}$$

$$\text{Now } X = L\left(1 - \frac{\phi_1^{\,2}}{10}\right), \text{ and } \phi_1 = \frac{L}{2R}$$

$$BT = (R + S)\tan\frac{\Delta}{2} + L\left(1 - \frac{\phi_1^2}{10}\right) - R\left(\phi_1 - \frac{\phi_1^3}{6}\right)$$

$$= (R + S)\tan\frac{\Delta}{2} + L\left(1 - \frac{L^2}{40R^2}\right) - R\left(\frac{L}{2R} - \frac{L^3}{48R^3}\right)$$

$$= (R + S)\tan\frac{\Delta}{2} + \frac{L}{2}\left(1 - \frac{L^2}{120R^2}\right)$$

$$= (R + S)\tan\frac{\Delta}{2} + \frac{L}{2}\left(1 - \frac{S}{5R}\right) \tag{66a}$$

(b) *Cubic Parabola* —In the case of a cubic parabola, the length of the curve is measured along the x axis (TB) Therefore, $TE = L = TE_1 = X$

Also, $\sin\phi_1 = \phi_1 = \frac{L}{2R}$ radians

Hence the equation (66) may be written as

$$BT = (R + S)\tan\frac{\Delta}{2} + L - R\phi_1 = (R+S)\tan\frac{\Delta}{2} + L - R\frac{L}{2R}$$

$$= (R + S)\tan\frac{\Delta}{2} + \frac{L}{2} \tag{67}$$

Hence it follows that $T_1T = \frac{L}{2} = TN$

The amount $S\tan\frac{\Delta}{2}$ is called the shift increment, and $(X - R\sin\phi_1)$ the spiral extension

Thus it will be noticed that when a transition curve is inserted between the tangent and the circular curve, the length of the tangent of the combined curve is greater than that of the simple curve $\left(= R\tan\frac{\Delta}{2}\right)$ by an amount depending upon the form of the transition curve used In the case of a cubic parabola, this increase is equal to $\left(S\tan\frac{\Delta}{2} + \frac{L}{2}\right)$, while in the case of a true spiral or clothoid, it equals

$$\left\{S \tan\frac{\Delta}{2} + (X - R \sin\phi_1)\right\} \text{ or } \left\{S \tan\frac{\Delta}{2} + \frac{L}{2}\left(1 - \frac{S}{5R}\right)\right\}.$$

(1) Elements of a Cubic Parabola —(Fig 80)

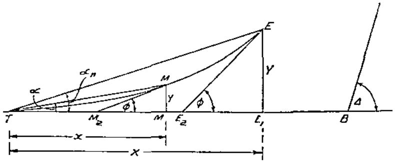


Fig 80

Let x	=	TM_1	= the distance of any point M on the curve measured along the tangent TB from the commencement T of the curve
y	=	M_1M	= the perpendicular offset to the point of
l			= the length of the transition curve TM
X	=	TE_1	= the distance of the junction point E of the transition curve with the circular curve from T measured along TB
Y	=	E_1E	= the perpendicular offset to the junction point E
L			= the length of the transition curve
ϕ	=	$\angle MM_2B$	= the angle between the tangent line TB and the tangent to the transition curve at any point M
ϕ_1	=	$\angle EE_2B$	= the angle between TB and the tangent a E (Spiral angle)
R			= the radius of the circular curve
$\propto$	=	$\angle MTB$	= the deflection angle to any point M
$\propto_n$	=	$\angle ETB$	= the deflection angle to the junction E

$$y = \frac{x^3}{6\,RL} \qquad (68)$$

in which $x = l$

Now $\tan \alpha = \frac{y}{x} = \frac{x^2}{6RL} = \frac{l^2}{6RL}$

From the intrinsic equation $\phi = \frac{l^2}{2RL}$ radians

$$\alpha \text{ (in radians)} = \frac{l^2}{6RL} = \frac{1}{3}\phi \text{ i e } \phi = 3\alpha \qquad (69)$$

$$\alpha = \frac{l^2}{6RL} \text{ radians} = \frac{l^2 \times 180^\circ \times 60}{6RL \times \pi} = \frac{1800\,l^2}{\pi RL} \text{ minutes} \qquad (70)$$

$$= \frac{573\,l^2}{RL} \text{ minutes} \qquad (70a)$$

Since $l = L$, $\alpha_n = \frac{1800\,L}{\pi R} = \frac{573\,L}{R}$ minutes (71)

$$\phi_1 = \frac{L}{2R} \text{ radians} = \frac{3 \times 573\,L}{R} \text{ minutes} \qquad (72)$$

If the degree (D) of the curve be given instead of the radius, the corresponding values of α α_n and ϕ_1 may be found by substituting the value of R in terms of D, viz $\left(R = \frac{1719}{D}\right)$ in equations (70a) to (72)

$$\text{Hence } \alpha = \frac{573\,l^2 \times D}{L \times 1719} = \frac{Dl^2}{3L} \text{ minutes} \qquad (73)$$

$$\alpha_n = \frac{DL}{3} \text{ minutes} \qquad (74)$$

$$\phi_1 = DL \text{ minutes} = \frac{DL}{60} \text{ degrees} \qquad (75)$$

From equation (68) the co ordinates of E are

$X = L$ and $Y = \frac{L^2}{6R}$ = four times the shift

$$\text{Total tangent length} = BT = (R + S)\tan\frac{\Delta}{2} + \frac{L}{2},$$

in which S = the shift and Δ = the deflection angle between the two tangents.

(2) **Elements of True Spiral :—**

Using the same notation, the elements are :

The co-ordinates of any point M :

$$x = l\left(1 - \frac{\phi^2}{10}\right) = l\left(1 - \frac{l^4}{40\,R^2L^2}\right) \quad \ldots \quad \ldots \quad \ldots \quad (76)$$

$$y = \frac{l^3}{6RL}\left(1 - \frac{\phi^2}{14}\right) = \frac{l^3}{6RL}\left(1 - \frac{l^4}{56R^2L^2}\right) \quad \ldots \quad \ldots \quad (77)$$

The co-ordinates of the end (E) of the curve :—

$$X = L\left(1 - \frac{\phi_1^2}{10}\right) = L\left(1 - \frac{L^2}{40\,R^2}\right) = L\left(1 - \frac{3S}{5R}\right) \quad (76a)$$

$$Y = \frac{L^2}{6R}\left(1 - \frac{\phi_1^2}{14}\right) = \frac{L^2}{6R}\left(1 - \frac{L^2}{56\,R^2}\right) \quad \ldots \quad \ldots \quad (77a)$$

The expressions for the deflection angles are the same as above.

$$\alpha = \frac{573\,l^2}{RL}\text{ minutes} = \frac{Dl^2}{3L}\text{ minutes.}$$

$$\alpha_n = \frac{573\,L}{R}\text{ minutes} = \frac{DL}{3}\text{ minutes.}$$

$$\phi_1 = \frac{3 \times 573\,L}{R}\text{ minutes} = DL\text{ minutes} = \frac{DL}{60}\text{ degrees}$$

$$\text{Total Tangent length (BT)} = BT_1 + T_1T$$

$$= (R + S)\tan\frac{\Delta}{2} + X - R\sin\phi_1.$$

$$= (R + S)\tan\frac{\Delta}{2} + \frac{L}{2}\left(1 - \frac{S}{5R}\right).$$

(3) **Elements of Cubic Spiral :—**

$$y = \frac{l^3}{6RL}, \quad l \text{ being measured along the curve} \quad \ldots \quad (78)$$

$$\alpha = \frac{1800 l^2}{\pi RL} \text{ minutes} = \frac{573 l^2}{RL} = \frac{Dl^2}{3L} \text{ minutes} \quad \ldots \quad (79)$$

$$\alpha_n = \frac{1800L}{\pi R} = \frac{573L}{R} = \frac{DL}{3} \text{ minutes.}$$

$$\phi_1 = 3\alpha_n = \frac{3 \times 573L}{R} = \frac{3DL}{DL} \text{ minutes} = \frac{DL}{60} \text{ degrees.}$$

$$\text{Total tangent length} = (R + S) \tan \frac{\Delta}{2} + \frac{L}{2},$$

if the deflection angle of the transition curve is small as is usually the case. But if large, formula (66) or (66a) should be used.

Length of the Combined Curve :—(Fig. 81).

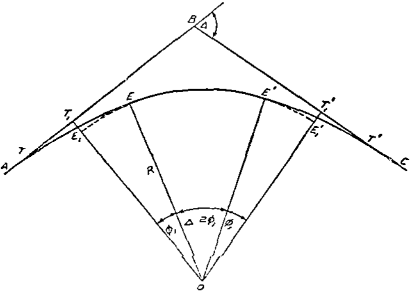


Fig. 81

The angle subtended at the centre by the circular arc (EE') = ($\Delta - 2\phi_1$) degrees.

$\therefore$ Length of the circular arc $EE' = \dfrac{\pi R(\Delta - 2\phi_1)}{180^\circ}$ ... (80)

Hence, the length of the combined curve

$$= \frac{\pi R(\Delta - 2\phi_1)}{180^\circ} + 2L \quad \ldots \quad \ldots \quad \ldots \quad (81)$$

The length of the curve may also be found in another way. The central angle subtended by the circular arc E_1E_1' $= \Delta$ degrees

$\therefore$ Length of the circular arc $E_1E'_1 = \dfrac{\pi R\Delta}{180^\circ}$.

Hence, the length of the combined curve $= \dfrac{\pi R\Delta}{180^\circ} + L$. (81a)

The calculations required for setting out the combined curve may be made in the following steps :

Data (a) the deflection angle (Δ) between the straights, (b) the radius (R) of the circular curve, (c) the length of the transition curve (L), and (d) the chainage of the point of intersection of the two straights

If the bearings of the two straights be given instead of the deflection angle between them, the latter may be found by subtracting one bearing from the other.

(1) Find the shift (S) of the circular curve from $S = \dfrac{L^2}{24R}$.

(2) Compute the total tangent length from formula (67) or (66), according as a cubic parabola or a spiral is used.

(3) Calculate the spiral angle ϕ_1 from $\phi_1 = \dfrac{L}{2R}$ radians.

(4) Calculate the length of the circular curve from formula (80); and the length of the combined curve from formula (81), and also from formula (81a) for checking the results

(5) Find the chainage of the beginning (T) of the combined curve by subtracting the total tangent length from the given chainage of the point of intersection (B).

(6) Obtain the chainage of the junction point (E) of the transition curve with the circular curve by adding the length of the transition curve to the chainage of T

(7) Determine the chainage of the other junction point (E′) of the circular arc with the other transition curve by adding the length of the circular arc to the chainage of E

(8) Obtain the chainage of the end point (T′) of the combined curve by adding the length of the transition curve to the chainage of E′

Check —The chainage of T′ thus obtained should agree with its chainage found by adding the length of the combined curve to the chainage of T

(9) Calculate the deflection angles for the transition curve from $\alpha = \frac{573 l^2}{RL}$ minutes or $\alpha = \frac{D l^2}{3L}$ minutes and also for the circular curve from $\delta = 1718.9 \frac{c}{R}$ mins bearing in mind that the points are staked on the combined curve with through chainage so that there will be sub chords at each end of the transition curves and of the circular curve In the case of a transition curve the peg interval may be 10 m or 15 m

(10) Find the total tangential angles for the circular curve from $\Delta_n = \Sigma\delta$ and check the results by observing if Δ_n equals $\frac{1}{2}(\Delta - 2\phi_1)$

Tabulate the results as under

Station No	Chainage	Length of chord	Deflection angle α or δ in minutes	Total tangential angle Δ in minutes	Δ in		Actual instrument reading			Remarks
					°	′	°	′	″	

(11) Calculate the offsets for the transition curves from $y = \frac{x^3}{6RL}$ in the case of a cubic parabola and from

$y = \dfrac{l^3}{6RL}\left(1 - \dfrac{\phi^2}{14}\right)$ or $y = \dfrac{l^3}{6RL}\left(1 - \dfrac{l^4}{56R^2L^2}\right)$ in the case of a true spiral

(12) Finally compute the offsets from chords (produced) from $O_n = \dfrac{b_n(b_{n-1} + b_n)}{2R}$ for the circular curve

Setting out the Combined Curve By Deflection Angles – The first transition curve may be set out from T (i) by the deflection angles or (ii) by the tangent offsets and the circular curve from the junction point E The second transition curve may then be set out from T checking on the junction point E previously located

(1) Having fixed the tangents AB and BC, locate the tangent point T by measuring backward the total tangent length from the intersection point B along the initial tangent AB and the other tangent point T by measuring forward the same distance from B along the forward tangent BC

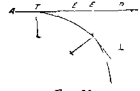


Fig 82

(2) From T measure along TB the distances equal to $\frac{1}{3}$L $\frac{2}{3}$L and L and peg these points which are lettered T_1 E_2 and E_3 respectively (Fig 82)

(3) Set up a theodolite over T and with both plates clamped at zero bisect B

(4) Release the vernier plate and set the vernier to the first deflection angle (α_1) as obtained from the table thus directing the line of sight to the first point on the transition curve

(5) With the zero end of the tape pinned down at T, hold the arrow at a distance on the tape corresponding to the length of the first sub chord and swing the tape until the arrow is bisected by the cross hairs thus fixing the first point on the transition curve It may be observed that the distance to each of the successive points on the transition curve is measured from T

(6) Repeat the procedure until the end of the curve (E) is reached Check the location of E by measuring the distance EE_3 which should equal 4S $\left(i\ e\ \frac{L^2}{6R}\right)$

(7) To set out the circular curve shift the instrument and set it up at E

(8) With the vernier set to $\frac{2}{3}\phi_1$ behind zero (i e $360^\circ - \frac{2}{3}\phi_1$) for a right hand curve take a backsight on T (When the telescope is turned in azimuth through the angle $\frac{2}{3}\phi_1$ it will be pointing in the direction of the comm n tangent EE_o and the vernier will read zero (360° O i dep ssir tl telescope the point E_o previously fixed at ⅔I from 1 will be bisected When the telescope is plunged it will point in the forward direction of the common tangent the vernier reading being 360°)

(9) Transit the telescope and set the vernier A to the first tabulated deflection angle for the circular curve and locate the first po nt on the circular curve as aire i ly explaine l

(10) Continue the setting out of tl circul ir curve upto E in the usual way

(11) Set out the other transition irve from T as before

(12) Check the points thus pegged on the transit on curve and the circular curve by tangent of cts an l by off ets from chords produced respe tively

It may be noted that through ch ur ge is sometimes not carried forward on the transition curve so that there will be no sub chord at the beginning of the transition curve However, the e will necessarily be the sub caor ls at tl e end of the transit on curve, and at the beginning and at the end of circular curve

Setting out the Transition Curve by Tangent Offsets —

(a) Cubic parabola (i) From T measure the x co ordinates of the points along TB (ii) Locate the points on the curve by setting out the respective offsets perpendicular to TB at each distance

(b) Cubic spiral Each point is located by swinging the chord length from the preceding point through the calculated offset.

Setting out the Transition Curve by Offsets from the Tangent (TT_1) and from the Circular Arc E_1E.—(Fig. 81) This method is based upon the fact that the offset from the circular arc (E_1E) to the transition curve at a distance x from E is equal to the offset from the tangent (TT_1) to the transition curve at a distance x from T, the tangent offsets being calculated from $y = \frac{x^3}{6RL}$. In this method, therefore, half the transition curve is set out by means of offsets from the tangent (TT_1) and the remaining half by means of offsets from the circular arc E_1E.

Spiralling Compound Curves.—(Fig. 83.) When it is required to insert a transition curve between the two arcs of a compound curve the following procedure may be adopted:—

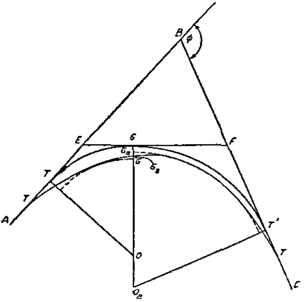


Fig. 83

(1) With the given radii of the two circular arcs, the maximum speed, and the distance between the centres of the rail heads,

calculate the amount of cant or superelevation (h_1 and h_2) for each arc by the relation $h = \frac{v^2}{gR} G$

(2) From the given rate of application (n) of the cant, find the lengths (L_1 and L_2) of the transition curves from the equation $L = nh$.

(3) Knowing the lengths of the transition curves for both branches, calculate the amount of shift (S_1 and S_2) for each branch by the formula $S = \frac{L^2}{24R}$ The distance between the tangents of the shifted curves $= G_1G_2 = S_1 - S_2$.

The transition curve at the common point of tangency is bisected at G_3 The point G_3 is, therefore, midway between G_1 and G_2

(4) Determine the length of the transition curve required at the common point (G) of tangency by the equation $L = n (h_1 - h_2)$ Alternatively, the length L is fixed empirically.

(5) Obtain the chainages of the points of tangency and the junctions of the transition curves with the circular arc from the calculated lengths of the tangent and circular arcs.

(6) Compute the deflection angles for the transition curve by the relation $\alpha = \frac{1800\ l^2}{\pi R L}$ mins

(7) Calculate the offsets by the equation $y = \frac{4S}{L^3} x^3$

(8) Compute the deflection angles for the circular arcs by the formula $\delta = 1718.9 \frac{c}{R}$ mins

(9) Find the offsets for the transition curve at the common point (G) of tangency by the equation $y = \frac{4 (S_1 - S_2)}{L^3} x^3$

(10) Locate the points at which the transition curve meets the circular curves by setting out $\frac{L}{2}$ from G_3 in each direction Set out the transition curve by means of offsets from the circular curves as calculated in (9), commencing from the junction points

The method is illustrated in example 5

Spiralling Reverse Curves —(Fig 84) If it is required to insert a transition curve between the two branches of a reverse curve, the procedure is similar to that adopted in the case of a compound curve

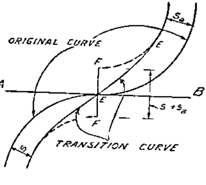


Fig 84

From the known data the amount of superelevation (h_1 and h_2) for each branch may be calculated

Then the greatest change of cant $= h_1 + h_2$

Length of the transition curve required at the point of reverse curvature $= L = n\ (h_1 + h_2)$ in which $n =$ the rate of canting

The distance between the shifted circular arcs $= S_1 + S_2$ where S_1 and S_2 are the amounts of shift for the two branches

The offsets for the transition curve at the point (E) of reverse curvature may then be calculated from the formula

$$y = \frac{4\,(S_1 + S_2)}{L^3}\,x^3$$

To set out the transition curve, the points at which the transition curve joins the shifted arcs are located by measuring half the length $\left(\frac{L}{2}\right)$ of the transition curve on either side of the original point of reverse curvature The transition curve is then set out by means of offsets from the shifted arcs

Examples on Composite (or Combined) Curve

Example 1 :—The following data refer to a composite curve. Deflection angle (Δ) $= 60° \; 30'$, maximum speed 90 km per hour; centrifugal ratio $= \frac{1}{4}$; max rate of change of acceleration $= 0{\cdot}3$ m per sec^3; chainage of intersection point $= 2570$ m.

Find (a) the radius of the circular curve, (b) the length of the transition curve, and (c) the chainages of the beginning and end of the transition curves, and of the junctions of the transition curves with the circular arc

(i) Radius (R) :

$$\text{Speed 90 km/hr} = \frac{90 \times 1000}{60 \times 60} = 25 \text{ m/sec.}$$

$$\text{Now the centrifugal ratio} = \frac{v^2}{gR} = \frac{1}{4} = \frac{25^2}{9{\cdot}81 \times R}$$

$\therefore$ $R = 254{\cdot}8$ m.

(ii) Length of transition curve (L) :—By $L = \dfrac{v^3}{0{\cdot}3R}$

$$\text{Length of the transition curve} = \frac{25^3}{0{\cdot}3 \times 254{\cdot}8}$$

$$= 204{\cdot}4 \text{ m say, } 205 \text{ m.}$$

(iii) Spiral angle ϕ_1 :—By $\phi_1 = \dfrac{L}{2R}$ radians.

$$\phi_1 = \frac{205 \times 180}{2 \times 254{\cdot}8 \times \pi} = 23° \; 6'{\cdot}6$$

(iv) Central angle $= \Delta - 2\phi_1 = 60° \; 30' - 23° \; 6'{\cdot}6$
$= 14° \; 16'{\cdot}8.$

(v) Length of the circular arc $= \dfrac{\pi R \,(\Delta - 2\phi_1)}{180°}$

$$= \pi \, \frac{254{\cdot}8 \times 14°{\cdot}28}{180°} = 63.5\text{m}$$

(vi) Shift of the curve (S) $= \dfrac{L^2}{24R} = \dfrac{205^2}{24 \times 254.8}$

$= 6{\cdot}9$ m

(vii) Tangent length (T) $= (R + S) \tan \dfrac{\Delta}{2}$

$$= (254.8 + 6.9) \tan 30° 15' = 263.9 \text{ m}$$

(viii) Chainages —

Chainage of the intersection point	= 2570.0 m
Deduct tangent length (T)	− 263.9 m
	2306.1 m
Deduct half the length of the transition curve	− 102.5 m
Chainage of the beginning of the first transition curve	= 2203.6 m
Add length of the transition curve	+ 205.0 m
Chainage of the junction of the transition curve with the circular arc	= 2408.6 m
Add length of the circular curve	+ 63.5 m
Chainage of the junction of the transition curve with the circular arc	= 2472.1 m
Add length of the transition curve	+ 205.0 m.
Chainage of the end of the second transition curve	= 2677.1 m

Check —Total length of the composite curve

$= 63.5 + 2 \times 205$	= 473.5 m
Chainage of the beginning of the first transition curve	= 2203.6 m
Add length of the combined curve	+ 473.5 m
Chainage of the end of the other transition curve	= 2677.1 m

Example 2 —Two straights intersect at chainage 3000 m with a deflection of 36° 48. It is proposed to insert a circular curve 750 m radius with cubic spirals 150 m long at each end. Find the chainage (i) at the beginning and at the end of the combined curve and (ii) at the junctions of the transition curves with the circular arc

(i) $L = 150$ m ; $R = 750$ m

$$\text{Shift (S)} = \frac{L^2}{24\,R} = \frac{150^2}{24 \times 750} = 1{\cdot}25 \text{ m.}$$

(ii) $\Delta = 36° \, 48'$, $S = 1{\cdot}25$ m ; $R = 750$ m.

$$\text{Total tangent length} = (R+S) \tan\frac{\Delta}{2} + \frac{L}{2}\left(1 - \frac{S}{5R}\right)$$

$$= (751{\cdot}26) \tan 18° \, 24' + \frac{150}{2}\left(1 - \frac{1\ 25}{5 \times 750}\right)$$

$$= 324\ 90 - 0\ 025 = 324{\cdot}87 \text{ m}$$

(iii) $$\text{Spiral angle } (\phi_1) = \frac{L}{2R} \text{ radians} = \frac{150}{2 \times 750}$$

$$= 5° \, 43' \, 48''.$$

$$\text{Central angle of circular arc} = \Delta - 2\phi_1 = 36° \, 48' - 2(5° \, 43' 48'')$$

$$= 25° \, 20' \, 24''.$$

(iv) $$\text{Length of circular arc} = \frac{\pi R(\Delta - 2\phi_1)}{180}$$

$$= \frac{\pi \times 750 \times 25\ 34}{180} = 331{\cdot}80 \text{ m.}$$

(v) Chainage :—

Chainage of the intersection point	= 3000·0 m.
Deduct the total tangent length	= 324·9 m.
Chainage of the beginning of the combined curve	= 2675 1 m.
Add length of the cubic spiral	+ 150 0 m.
Chainage of the junction of the cubic spiral with the circular arc	= 2825·1 m.
Add length of the circular arc	= 331·8 m.
Chainage of the junction of the circular arc with the cubic spiral	= 3156·9 m.
Add length of the cubic spiral	= + 150·0 m.
Chainage of the end of the combined curve	= 3206·9 m.

Example 3.—Two clothoid spirals for a road transition between two straights meet at a common tangent point. If the deflection angle between the straights is 20° 12′, the chainage of their point of intersection 371·0 m, and the maximum speed 108 km per hour, calculate the chainages of the tangent points and the point of compound curvature.

The curve may be designed on the basis of the comfort condition or centrifugal ratio.

(i) The maximum rate of change of acceleration

$$= 0\cdot3 \text{ m/sec}^3, \quad 2\phi = 20° \, 12'$$

$$\therefore \quad L = \frac{v^3}{0\cdot3R} = 2R\,\phi \quad \therefore \; R = \sqrt{\frac{v^3}{2\phi \times \cdot3}}$$

$$\text{Now } V = 108 \text{ km/hr} = \frac{108000}{60 \times 60} = 30 \text{ m/sec}$$

$$\phi = \frac{20\cdot2 \times \pi}{2 \times 180°} \text{ radians} = 0\cdot1763 \text{ radians.}$$

$$\text{Whence, } R = \sqrt{\frac{(30)^3}{0\cdot3 \times 2 \times 0\cdot1763}} = \sqrt{\frac{27000}{0\cdot1058}}$$

$$= 505\cdot3 \text{ m.}$$

$$L = 2R\phi = 2 \times 505\cdot3 \times 0\cdot1763 = 178\cdot1 \text{ m}$$

(ii) Centrifugal ratio $= \frac{1}{4}$.

$$\therefore \quad \frac{v^2}{gR} = \frac{1}{4} \quad \text{or } R = \frac{4v^2}{g} = \frac{4 \times 30^2}{9\cdot81} = 366\cdot9 \text{ m}$$

$$L = 2R\phi = 2 \times 366\cdot9 \times 0\cdot1763 = 129\cdot3 \text{ m}$$

The curve may, therefore, be designed on the basis of the comfort condition.

Now the tangent length $= X + Y \tan \phi/2$, where X and Y are the co-ordinates of the end of the first clothoid.

$$\text{From (73a), } X = L\left(1 - \frac{L^2}{40R^2}\right)$$

$$\text{from (74a), } Y = \frac{L^2}{6R}\left(1 - \frac{L^2}{56R^2}\right), \text{ where } \phi = 10° \, 6'.$$

$$\text{Tangent length} = L\left(1-\frac{L^2}{40R^2}\right)+\frac{L^2}{6R}\left(1-\frac{L^2}{56R^2}\right)\tan\frac{\phi}{2}$$

$$=177.4+1.8=179.2 \text{ m}$$

Whence

Chainage of the intersection point	= 371.0 m
Deduct tangent length	− 179.2 m
Chainage of the beginning of the curve	= 191.8 m
Add length of the curve	+ 178.1 m
Chainage of the point of compound curvature	= 369.9 m
Add length of the curve	+ 178.1 m
Chainage of the end of the curve	= 548.0 m

Example 4 —Two straights AB and BC intersect at chainage 1642.5 m the deflection angle being 48° 24'. It is proposed to insert a circular curve of 300 m radius with a transition curve 60 m long at each end. The circular curve is to be set out with pegs at 25 m intervals and the transition curve with pegs at 15 m intervals of through chainage. Make all the necessary calculations for setting out the combined curve.

Data — $\Delta = 48^\circ 24'$ $R = 300$ m $L = 60$ m

(i) Shift (S) $= \dfrac{60^2}{24\times 300} = 0.5$ m

(ii) Tangent length $(T) = (R+S)\tan\dfrac{\Delta}{2}$

$$=(300+0.5)\tan 24^\circ 12' = 135.1 \text{ m}$$

$$\text{Total tangent length (BT)} = T+\frac{L}{2} = 135.1+30 = 165.1 \text{ m}$$

(iii) Spiral angle $(\phi_1) = \dfrac{L\times 180^\circ}{2R\pi} = \dfrac{60\times 180}{2\times 300\times\pi} = 5^\circ.729$

$$\text{Central angle} = \Delta - 2\phi_1 = 48^\circ 24' - 5^\circ.73\times 2 = 36^\circ.94 = 36^\circ 56'.4$$

(iv) Length of the circular curve $(l) = \dfrac{\pi R\,(\Delta-2\phi_1)}{180^\circ}$

$$=\frac{\pi\times 300\times 36^\circ.94}{180^\circ} = 193.4 \text{ m}$$

Check.—Length of the circular curve $(l) = \dfrac{\pi R \Delta}{180^\circ} - L$

$$= \frac{\pi \times 300 \times 48^\circ 4'}{180^\circ} - 60 = 253.4 - 60.0 = 193.4 \text{ m}$$

(v) Chainage —

Chainage of the intersection point (B)	= 1642.5 m.
Deduct tangent length (T)	− 135.1 m.
Chainage of T_1	= 1507.4 m.
Deduct half the length of the transition curve	− 30.0 m.
Chainage of the beginning (T) of the transition curve	= 1477.4 m.
Add length of the transition curve (L)	+ 60.0 m.
Chainage of the junction (E) of the transition curve with the circular curve.	= 1537.4 m.
Add length of the circular curve (l)	+ 193.4 m.
Chainage of the junction (E') of the circular curve with the transition curve	= 1730.8 m
Add length of the transition curve (L)	+ 60.0
Chainage of the end (T') of the transition curve	= 1790.8 m.

Check.—Length of the combined curve $= 193.4 + 2 \times 60$
$= 313.4$ m.

Chainage of T' = chainage of T + length of the combined curve = 1477.4 + 313.4 = 1790.8 m.

(vi) Deflection angles for the first transition curve —

$$\delta = \frac{1800\, l^2}{\pi R L} = \frac{1800\, l^2}{\pi \times 300 \times 60} = \frac{l^2}{10\pi} \text{ minutes}$$

Chainage of T = 1477.4 m.

,, of the 1st point = 1480.0 $l = 2.6$ m $\delta_1 = \dfrac{2.6^2}{10\pi} = 12.9$ sec.

,, of the 2nd ,, = 1495.0 m $l = 17.6$ m $\delta_2 = \dfrac{17.6^2}{10\pi} = 9.86$ mins

„ of the 3rd „ $=1510 \cdot 0$m; $l=32 \cdot 6$m, $\alpha_3=\dfrac{32 \ 6^2}{10\pi}=33 \cdot 83$mins

„ of the 4th „ $=1525 \cdot 0$m, $l=47 \cdot 6$m, $\alpha_4=\dfrac{47 \cdot 6^2}{10\pi}=72 \cdot 11$ „

„ of the 5th „ $=1537 \cdot 4$ m ; $l=60$ m , $\alpha_5=\dfrac{60^2}{10\pi}=114 \cdot 7$ „

Check·— $\alpha_5=\dfrac{1}{3}\phi_1=\dfrac{1}{3}\times 5^\circ \ 729=114 \ 7$ mins.

(vii) Tangential angles, and total tangential (or deflection) angles for the circular arc :—

By $\delta=1718 \cdot 9 \dfrac{c}{R}$ mins , and $\Delta_n=\delta_1+\delta_2+ \ . \ +\delta_n$,

where Δ = the total tangential angle

Chainage of E = 1537 4 m

„ of the 1st point on the circular arc = 1550 0 m

∴ Length of the first subchord = 12 6 m

and $\delta_1=\dfrac{1718 \cdot 9 \times 12 \cdot 6}{300}=72 \ 21$ min $=1^\circ \ 12 \ 21'$.

Chainage of the second point = 1575 0 m

∴ length of the unit chord = 25 m

$\because \ \delta_2 = \dfrac{1718 \cdot 9 \times 25}{300}=143 \cdot 2' \ =2^\circ \ 23' \cdot 2$

$=\delta_3=\delta_4= \qquad =\delta 8$

Chainage of the last point E′ = 1730·8 m

∴ The length of the last sub chord = 5 8 m.

$\therefore \ \delta_9=\dfrac{1718 \cdot 9 \times 5 \ 8}{300}=33' \cdot 70$

Now

$\Delta_1=\delta_1=1^\circ \ 12 \cdot 21'$

$\Delta_2=\delta_1+\delta_2=1^\circ \ 12 \cdot 21'+2^\circ 23 \cdot 2'=3^\circ \ 35 \cdot 41'$

$\Delta_3 - \delta_1 + \delta_2 + \delta_3 = 1°\ 12\ \ 21\ + 2\ (2°\ 23\ \ 2) = 5°\ 58\ \ 61$

$\Delta_4 = \delta_1 + \delta_2 + \delta_3 + \delta_4 = 1°12\ \ 21\ + 3\ (2°\ 23\ \ 2) - 8°\ 21\ \ 81$

$\Delta_5 - \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 = 1°1?\ \ ?1 + 4(2°23\ \ 2) = 10°45\ \ 01$

$\Delta_6 = \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 - 1°1?\ \ 21 + 5\ (2\ 23\ \ 2) - 13°\ 8\ \ 21$

$\Delta_7 = \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 - 1°12\ \ 21 + 6\ (2°23\ \ 2)$
$= 15°\ 31\ \ 41$

$\Delta_8 - \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta\ + \delta_8 - 1°\ 1?\ \ 21\ +\ 7\ (2°\ 23\ \ 2)$
$= 17°\ 54\ \ 4$

$\Delta_9 = \delta_1 + \delta_2 + \delta_3 - \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 - 1°\ 12\ \ 21 +$
$7(2°\ ?3\ \ 2) + 33\ \ ?0 = 18°28\ \ 31$

Check $-\Delta_9$ should equal $\frac{1}{2}(\Delta - 2\phi_1) - \frac{1}{2}\ (36°56\ \ 4) = 18°\ 28\ \ 2$

(viii) Deflection angles for the second transition curve —

$$\text{BY}\quad \propto\ = \frac{l^2}{100\pi}\ \text{min}$$

In this case l should be calculated from T and not from E

Point	Chainage	l	$\propto$	
E	1730 8	60	$\frac{60^2}{10\pi} = 114\ \ 7$	mins
1	1745 0	45 8	$\frac{45\ \ 8^2}{10\pi} = 66\ \ 77$	
2	1760 0	30 8	$\frac{30\ \ 8^2}{10\pi} = 30\ \ 20$	,
3	1775 0	15 8	$\frac{15\ \ 8^2}{10\pi} = 7\ \ 947$	
T	1790 8	0	$= 0$	,

(ix) Offsets from the tangent TB —

$$\text{By } y = \frac{x^3}{6\text{RL}} - \frac{x^3}{6 \times 300 \times 60}\ \text{m}$$

$$y_1 = \frac{?\ \ 6^3}{108000} = 0\ \ 00016\ \text{m} = 0\ \ 16\ \ \text{mm}$$

$$y_2 = \frac{17\ \ 6^3}{108000} = 0\ \ 05048\ \text{m} = ?\ \ 05\ \text{cm}$$

$$y_3 = \frac{32 \cdot 6^3}{108000} = 0 \cdot 3207 \text{ m} = 32 \cdot 1 \text{ cm.}$$

$$y_4 = \frac{47 \cdot 6^3}{108000} = 0 \cdot 9986 \text{ m} = 99\ 9 \text{ cm}$$

$$y_5 = \frac{60^3}{108000} = 2 \text{ m} = 2 \text{ m.}$$

Check :—$y_5 = 4S = 4 \times 0\ 5 = 2$ m.

(x) Offsets from chord :—By $O_n = \dfrac{b_n\,(b_{n-1} + b_n)}{2R}$.

Here $b_1 = 12 \cdot 6$ m ; $b_2 = b_3 =$ . . $b_8 = 25$ m
$b_9 = 5 \cdot 8$ m

$$O_1 = \frac{b_1^2}{2R} = \frac{1\ 62^2}{2 \times 300} = 0\ 2646 = 26 \cdot 5 \text{ cm.}$$

$$O_2 = \frac{25\,(12\ 6 + 25)}{2 \times 300} = 1 \cdot 563 \text{ m} = 1\ 56 \text{ m}$$

$$\left.\begin{matrix} O_3 \\ \text{to} \\ O_8 \end{matrix}\right\} = \frac{25^2}{300} = 2 \cdot 074 \text{ m} = 2\ 07 \text{ m}$$

$$O_9 = \frac{5\ 8\ (25 + 5 \cdot 8)}{2 \times 300} = 0 \cdot 2977 \text{ m} = 30 \text{ cm.}$$

(xi) Offsets from the tangent T′B from the second transition curve at T′.—By $y = \dfrac{x^3}{6RL}$, x being measured from T′.

$$y_1 = \frac{60^3}{6 \times 300 \times 60} = 2 \text{ m} = 2 \text{ m.}$$

$$y_2 = \frac{45\ 8^3}{108000} = 0 \cdot 8898 \text{ m} = 89\ 0 \text{ cm}$$

$$y_3 = \frac{3 \cdot 8^3}{108000} = 0 \cdot 2707 \text{ m} = 27\ 1 \text{cm}$$

$$y_4 = \frac{15 \cdot 8^3}{108000} = 0 \cdot 03654 \text{m} = 3.7 \text{ cm}$$

Example 5 —Two straights AB and BC are intersected by a third line EF the angles AEF and EFC being 135° and 150° respectively It is proposed to introduce a compound curve tangential to AB EF and BC The radii of the two branches of the curve are 320 m and 400 m respectively The maximum speed is 72 km per hour and the chainage of B is 2210 0 m Compute the necessary data for the location of the curve if the transition curves are to be inserted between the two branches and at the junctions with the straights AB and BC Take the distance between the centres of the rails = 1 62 m. (Fig 81)

(i) $72 \text{ km per hr} = \dfrac{72 \times 1000}{60 \times 60} = 20$ m per second

Now cant on the first branch $(h_1) = \dfrac{Gv^2}{gR} = \dfrac{1\ 62 \times 20^2}{9\ 81 \times 320}$

$= 0\ 2064$ m

cant on the second branch $(h_2) = \dfrac{Gv^2}{gR} = \dfrac{1\ 62 \times 20^2}{9\ 81 \times 400}$

$= 0\ 1651$m

(ii) Length of the transition curve —Assuming that the cant is applied at a uniform rate of 1 in 300 we have

Length of the first transition at the junction with AB

$(L_1) = nh_1 = 300 \times 2064 = 61\ 92$ say 60 m

Length of the second transition curve at the junction with BC

$(L_1) - nh = 300 \times 0\ 1651 = 49\ 53$ m say $= 50$ m.

Length of the third transition curve at the junction (G) of the two branches $(L_3) = n\,(h_1 - h_2) - 12\ 39$ m say $= 10$ m

(iii) Shift — By $S = \dfrac{L^2}{24R}$

Shift of the first branch $= S_1 = \dfrac{60^2}{24 \times 320} = 0\ 4312$ m

" second $- S_2 = \dfrac{50^2}{24 \times 400} = 0\ 2604$ m

(iv) Tangent length :—

Deflection angle for the first branch $= \alpha = 180° - 135° = 45°$

" " second " $= \beta = 180° - 150° = 30°$

Now $EG = ET_1 = (R_1 + S_1) \tan \frac{\alpha}{2} = (320 + 0 \cdot 4312) \tan 22° 30'$

$= 132 \cdot 6$ m

$FG = FT'_1 = (R_2 + S_2) \tan \frac{\beta}{2} = (400 + 0 \cdot 2604) \tan 15°$

$= 107 \cdot 3$ m

$\therefore \; EF = EG + FG = 132 \cdot 6 + 107 \cdot 3 = 239 \cdot 9$ m

Now consider the $\triangle$ BEF. By the sine rule, we get,

$BE = \frac{239 \; 9 \; \sin 30°}{\sin 105°}$ and $BF = \frac{239 \; 9 \; \sin 45°}{\sin 105°}$

$= 124 \cdot 3$ m. $\quad = 174 \cdot 7$ m

Hence $BT_1 = BE + ET_1 = 124 \cdot 3 + 132 \cdot 6 = 256 \cdot 9$

$BT'_1 = BF + FT'_1 = 174 \cdot 7 + 107 \cdot 3$ m $= 282 \cdot 0$ m.

Now chainage of B $= 2210 \cdot 0$ m

" of $T_1 = 2210 \cdot 0 - 256 \cdot 9 = 1953 \cdot 1$ m

" of T $= 1953 \cdot 1 - 30 \cdot 0 = 1923 \cdot 1$ m.

" of E_1 (end of the first transition curve) $= 1923 \; 1 + 60 = 1983 \cdot 1$ m

(v) The offsets for setting out the first transition curve (L_1) at 10 m interval from equation $y = \frac{4 S_1 x^3}{L_1^3}$, are

$$y_1 = \frac{4 \times (0 \cdot 4312) \times 10^3}{60^3} = 0 \cdot 0080 \text{m} = 8 \text{ mm}$$

$$y_2 = \frac{4 \times (0 \cdot 4312) \times 20^3}{60^3} = 0 \; 06389 = 6 \; 39 \text{ cm.}$$

$$y_3 = \frac{4 \times (0 \cdot 4312) \times 30^3}{60^3} = 0 \cdot 2156 \text{ m.}$$

$$y_4 = \frac{4 \times (0 \; 4312) \times 4 \; ^3}{60^3} = 0 \; 5111 \text{ m}$$

$$y_5 = \frac{4 \times (0\ 4312) \times 50^3}{60^3} = 1 \cdot 002 \text{ m.}$$

$$y_6 = \frac{4 \times (0 \cdot 4312) \times 60^3}{60^3} = 1\ 7248 \text{ m}$$

Similarly, for locating the second transition curve (L_2), The offsets from equation $y = \frac{4S_2 x^3}{L_2^3}$ at 10 m intervals are:

$$y_1 = \frac{4 \times 0\ 2604 \times 10^3}{50^3} = 0\ 0083 \text{ m} = 0 \cdot 83 \text{ cm.}$$

$$y_2 = \frac{4 \times 0\ 2604 \times 20^3}{50^3} = 0\ 0667 \text{ m} = 6\ 67 \text{ cm.}$$

$$y_3 = \frac{4 \times 0 \cdot 2604 \times 30^3}{50^3} = 0\ 225 \text{ m.}$$

$$y_4 = \frac{4 \times 0\ 2604 \times 40^3}{50^3} = 0\ 5333 \text{ m.}$$

$$y_5 = \frac{4 \times 0\ 2604 \times 50^3}{50^3} = 1 \cdot 0416 \text{ m.}$$

Example 6 —It is proposed to connect the straights AB and CD by a composite reverse curve with the point of reverse curvature on BC The points B and C are the intersections points of the tangents of the first and second circular curves, which have a common radius R metres. The transition curves are to be introduced at each end of the circular curves. Given the following total co-ordinates of A, B, C and D, and that the length of the transition curve is $4 \cdot 472 \sqrt{R}$ metres, find the common radius of the circular curves (Fig. 85).

Point	Total Lat. in metres	Total Dep. in metres
A	+ 711·6	+ 3309·6
B	+ 769·2	+ 3792 6
C	+ 1435·6	+ 4249·6
D	+ 1448·4	+ 4691·2

(i) Let α_1, α_2, and α_3 be the reduced bearings of the lines AB, BC, and CD respectively.

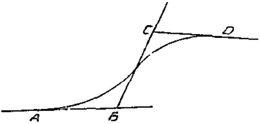


Fig 85

Then by $\tan \alpha = \dfrac{\text{dep}}{\text{lat}}$, the bearings α_1, α_2, and α_3 are

$$\tan \alpha_1 = \frac{(3792\ 6 - 3309\ 6)}{(769\ 2 - 711{\cdot}6)} = \frac{483{\cdot}0}{57{\cdot}6} \text{ or } \alpha_1 = \text{N. } 83°\ 12'\ \text{E.}$$

$$\tan \alpha_2 = \frac{(4249{\cdot}6 - 3792\ 6)}{(1435\ 6 - 769{\cdot}2)} = \frac{457{\cdot}0}{666{\cdot}4} \text{ or } \alpha_2 = \text{N. } 34°\ 27'\ \text{E.}$$

$$\tan \alpha_3 = \frac{(4691{\cdot}2 - 4249\ 6)}{(1448\ 4 - 1435\ 6)} = \frac{441{\cdot}6}{12{\cdot}8} \text{ or } \alpha_3 = \text{N. } 88°\ 20'\ \text{E.}$$

$\therefore$ Deflection angle (Δ_1) between AB and BC
$= 83°\ 12' - 34°\ 27' = 48°\ 45'$.

Deflection angle (Δ_2) between BC and CD
$= 88°\ 20' - 34°\ 27 = 53°\ 53'$

(ii) Length of BC $=(1435{\cdot}6 - 769{\cdot}2)\sec 34°27' = 808{\cdot}2$ metres.

(iii) Shift :— Length of the transition curve $= 4.472\sqrt{R}$.

$$\therefore\quad S = \frac{L^2}{24R} = \quad = 0{\cdot}833 \text{ metres}$$

Total tangent length of the first circular curve

$$= (R + S)\tan\frac{\Delta_1}{2} + \frac{L}{2}$$

$$= (R + 0{\cdot}833)\tan 24°\ 22'{\cdot}5 + 4\ 472\ \frac{\sqrt{R}}{2}\,.$$

Total tangent length of the second circular curve

$$= (R + 0\ 833)\ \tan 26^\circ\ 56'\ 5\ + 4\ 472\ \frac{\sqrt{R}}{2}$$

Now BC = the sum of these two tangent lengths

$$\therefore\quad BC = (R + 0\ 833)\,(0\ 45309 + 0\ 50825)\ + 4\ 472\sqrt{R}$$
$$= 808\ 2\text{m}$$

$$0\ 96134\ R + 0\ 833 \times 0\ 96134 + 4\ 472\ \sqrt{R}$$
$$= 808{\cdot}2\text{m}$$

$$R + 4\ 652\ \sqrt{R} = 840\ 1$$

Solving this quadratic equation, we get

$\sqrt{R} = 26\ 75$ m $\qquad R = 715\ 4$ m

Vertical Curves

When two different or contrary gradients meet, they are connected by a curve in a vertical plane called a vertical curve It is advisable to introduce a vertical curve in road and railway work in order to round off the angle and to obtain a gradual change in grade so that abrupt change in grade is avoided at the apex The vertical curve may be a circular arc or an arc of a parabola, but for simplicity of calculation work, the latter is preferred and is invariably used

The grade or gradient of a road or railway is expressed in two ways

(i) As a percentage e g 2% or 1 5%, and (ii) as 1 in *n* where *n* is the horizontal distance in metres corresponding to 1 m rise or fall e g 1 in 80 or 1 in 100, an ascending or up grade rising to the right being denoted by a plus (+) sign and a descending or down grade falling to the right by a minus (−) sign

For first class railways the rate of change in gradient recommended *is 0 1% per 30 m station at summits and 0 05% per* 30 m station at sags Twice these values are adopted for second class railways

For small gradient angles there is hardly any difference between a parabola and a circular arc

Types of Vertical Curves —

(1) An up grade followed by a down grade (Fig 86)

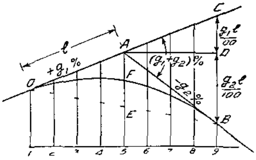


Fig 86 Summit or Convex

(2) A down grade followed by an up grade (Fig. 87)

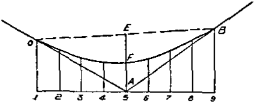


Fig 87 Sag or Concave

(3) An up grade followed by another up grade (Fig 88)

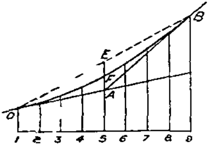


Fig. 88 Sag or Concave

(4) A down grade followed by another down grade (Fig. 89)

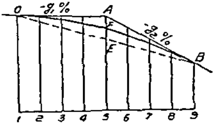


Fig 89 Summit or convex

Properties of the parabola —

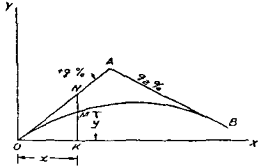


Fig 90

In Fig 90, let

OX and OY = the axes of the rectangular co-ordinates passing through the point of beginning (O) of the vertical curve

O = the origin of the co-ordinates

OA and AB = the tangents to the vertical curve

$+g_1\%$ = the grade of the tangent OA

$-g_2\%$ = „ „ „ AB

M = any point on the curve whose co ordinates are x and y

Now it may be shown that the equation of a parabola with respect to OX and OY is . $y = cx^2 + g_1x$ ... ... (82)

Now $OK = x$; $KM = y$; $KN = g_1 x$, and $NM = KN - KM$

$\therefore \quad NM = g_1 x - y = -cx^2$

from which it follows that the vertical distance from the tangent to any point on the curve varies as the square of its horizontal distance from the point of commencement of the curve (the point of tangency) This vertical distance is called the *tangent correction*. As the vertical curve set is out with respect to the tangents, the equation $y = cx^2$, in which y represents the tangent correction, may be employed to calculate the tangent corrections.

Note —In order to obtain a true curve, the ordinates (offsets) should be parallel to the main axis But in practice, they are made vertical to simplify calculations When the grades are equal, the theoretical condition is fulfilled. But when the grades are unequal, the main axis is slightly tilted Consequently, there will be a slight distortion by making the ordinates vertical instead of making them parallel to the tilted axis However, for all practical purposes, it is negligible and calculations are made on the basis of vertical offsets

$$y = cx^2$$

$$\therefore \quad \frac{dy}{dx} = 2\ cx$$

$$\frac{d^2y}{dx^2} = 2\ c = \text{a constant}$$

This shows that the parabola gives an even rate of change of gradient

When $x = 2l = L$

$$y = BC = \frac{(g_1 - g_2)\ l}{100}$$

$$\therefore \quad \frac{(g_1 - g_2)\ l}{100} = c\ 4\ l^2$$

$$\therefore \quad c = \frac{g_1 - g_2}{400 l} \quad \dots \quad \dots \quad \dots \quad (\cdot 3)$$

$$y = \frac{(g_1 - g_2)}{400l} \quad x^2$$

When $x = l$

$$y = AF = \frac{g_1 - g_2}{400l} \quad l^2$$

$$= \frac{(g_1 - g_2)l}{400} = FE.$$

In the above expressions $(g_1 - g_2)$ denotes the algebraic difference of the two grades

Knowing c, the tangent corrections (or tangent offsets) may be computed in the following ways

First Method By Tangent Corrections — Let the chainage and elevation of the apex A be given

(1) The length of the curve on either side of A being l m' determine the chainages of the points of tangency (O and B) (Fig 86)

Chainage of O = chainage of A $- l$
of B = , of A $+ l$

(2) Knowing the grades of the tangents OA and AB, and the elevation of A compute the elevations of the tangent points O and B

$$\text{Elevation of O} = \text{elevation of A} - \frac{lg_1}{100}$$

$$\text{Elevation of B} = \text{elevation of A} - \frac{lg_2}{100}$$

(3) Compute the tangent corrections from $y = cx^2$ for the stations on the curve

$$y_x' = c_x{}^2$$

(4) Determine the elevations of the corresponding stations on the tangent OAC

Elevation of tangent at any station = elevation of the point of tangency (O) $+ x\,g_1$

where x = the distance of that station from O

(5) Find the elevations of the stations on the curve by adding algebraically the tangent corrections to the elevations of the corresponding stations on the tangent OA (tangent elevations)

Elevation of the station at a distance x on the curve = elevation of the station on the tangent $\pm$ tangent correction y_x

The results may be tabulated as under

Station	Chainage	Tangent or grade Elevation	Tangent Correction	Elevation of the curve	Remarks.

Alternative Method —The alternative method of calculating the tangent corrections is as follows (Fig 86)

In this case the tangent corrections are calculated from both the tangents OA and AB

(1) Calculate as before the elevations of the points of tangency (O and B)

(2) Find the elevation of the mid point E of the chord OB (the line joining the points of tangency)

Elevation of E = $\frac{1}{2}$ (elevn of O + elevn of B)

(3) Determine the elevation of the mid point F of the vertical curve by finding the mean of the elevations of the mid point E of the chord and the point of intersection A of the two tangents

It may be noted that from the properties of a parabola the mid point E of the chord OB is situated on the vertical through the point of intersection A of the two tangents and that the mid point F of the vertical curve is mid way between these two points

Elevation of F = $\frac{1}{2}$ (elevn of E + elevn of A)

Hence the tangent offset from A to the curve = AF = elevn of A − elevn of F

(4) Compute the tangent offsets from the tangent OA to the various points on the curve using the well known property of a parabola that the tangent offset is proportional to the square of the horizontal distance from the point of tangency

The corresponding offsets from the other tangent AB have the same values

(5) Having obtained the tangent corrections find the elevations of the points on the curve as explained above

Second Method By Chord Gradients —In this method the successive differences in elevation between the points on the curve are calculated the differences being called the *chord gradients* The elevation of each point is then determined by adding the chord gradient to the elevation of the preceding point with due regard being paid to the signs of the chord gradients

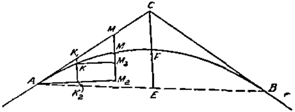


Fig 91

In Fig 91 let AC and CB = the tangents to the vertical curve meeting at C

$+g_1\%$	= the grade of AC
$-g\ \%$	= of CB
A and B	= the points of tangency
K, M etc	= the successive points on the curve at an interval of I metres

Through K draw a vertical line meeting AC in K_1 and the horizontal line AD through A in K_2 Similarly, draw the vertical line through M meeting AC and AD at M_1 and M_2 respectively Through K draw a horizontal line, cutting M_1M_2 in M_3

Now the difference in elevation between K and A $= KK_2$ $= K_1K_2 - K_1K$.

But $K_1K_2 = \dfrac{g_1 I}{100}$ and $K_1K = \dfrac{g_1 - g_2}{400\, l} \cdot I^2 = Cg$ from the equation $y' = cx^2$

$\therefore$ The first chord gradient $= \dfrac{g_1 I}{100} - Cg$

Similarly, $M_1M_2 = \dfrac{2g_1 I}{100}$; $M_1M = 4Cg$; $M_3M_2 = KK_2 = \dfrac{g_1 I}{100} - Cg$.

$\therefore$ The difference in elevation between M and K

$$= MM_3 = M_1M_2 - M_1M - M_3M_2$$

$$= \frac{2g_1 I}{100} - 4Cg - \left(\frac{g_1 I}{100} - Cg\right) = \frac{g_1 I}{100} - 3Cg$$

or the second chord gradient $= \dfrac{g_1 I}{100} - 3Cg$

the Nth " " $= \dfrac{g_1 I}{100} - (2N - 1)\, Cg.$ (84)

Hence, elevn. of K = elevn of A $+ \left(\dfrac{g_1 I}{100} - Cg\right)$

" of M = elevn. of K $+ \left(\dfrac{g_1 I}{100} - 3Cg\right)$ and so on.

To determine the length of the vertical parabola connecting two grades when it is made to pass through a point at a given distance above or below the point of intersection (apex) of the two grades, we proceed as follows :

Let g_1 and g_2 = the two grades.
p = the distance above or below the apex
l = *length of curve on either side of the apex.*

From the equation of the parabola $y = cx^2$, we have

$y_l = cl^2$, where $c = \dfrac{g_1 - g_2}{400l}$, and y_l = the tangent offset at a distance l metres from O.

$$\therefore \quad y = p \text{ i.e. } p = \frac{(g_1 - g_2)}{400l} l^2 = \frac{g_1 - g_2}{400} l$$

or $l = \dfrac{400p}{(g_1 - g_2)}$. Hence the length of the parabola $= 2l$

Location of Highest or Lowest Point —The position and

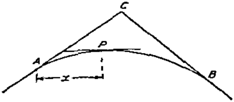


Fig 92

elevation of the highest point (Summit) (Fig 92) or the lowest

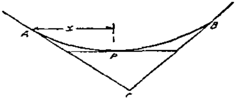


Fig 93

point at sag (Fig 93) may be calculated as follows :

In Figs 92 & 93 let P be the required point at a distance x m from the beginning A of the curve The tangent to the curve at this point P being a horizontal line, its slope is zero

The general equation of the parabola is $y = cx^2 + g_1 x$

The slope of the tangent at any point on the parabola

$= \dfrac{dy}{dx}$ Now $\dfrac{dy}{dx} = 2cx + g_1$

Since the slope of tangent at P $= 0$, we have

$$2cx + g_1 = 0 \quad \text{or} \quad x = -\frac{g_1}{2c} \qquad (93)$$

in which $c = \dfrac{g_1 - g_2}{400l}$

Knowing the distance of P from the beginning of the curve the tangent correction at P may be computed from $y = cx^2$, which, when added algebraically to the elevation of the corresponding point on the tangent, gives the elevation of P

Elevation of P = elevn of the tangent at P $\pm$ tangent correction at P

Length of the vertical curve—The length of a vertical curve is influenced by (i) centrifugal effect and (ii) visibility When sags and summits are formed by flat gradients centrifugal effect is the chief consideration while at summits where the algebraic change of gradient is large, visibility is the main consideration

Centrifugal effect A minimum radius of 1000 m should be used at brows and sags This gives a centrifugal acceleration of 0 75 m/sec² at 100 km p h

Parabolas on vertical curves can be approximated to circular curves If R is the radius of the curve,

$$\frac{d^2y}{dx^2} = 2c = \frac{1}{R}$$

$$\frac{1}{R} = 2\frac{g_1 - g_2}{400l} = \frac{g_1 - g_2}{200l}$$

$$R = \frac{200l}{g_1 - g_2}$$

$$\text{Lmin with } R = 1000 = \frac{1000\,(g_1 - g_2) \times 2}{200}$$

$$= 10\,(g_1 - g_2) \qquad (86)$$

Thus if two in 25 gradients meet in a sag, the minimum length of the curve should be = 10 (4 + 4) = 80 m

At summits where speeds of 100 km p h are contemplaed the requirement of visibility i e the sight line will lead to longer curves than one obtained by the above formula

Sight distances Let two points on the curve at a height 'h' metres from the ground be intervisible and let the distance between be S A value of 1 1 m is usually taken as the eye level height above the road surface for an observer sitting in a car Sight distances are laid down in

the interest of road safety and the choice for any distance depends on the nature of the road and the speed of the traffic using it.

There are three cases to consider (a) Sight distance equal to the length of the curve

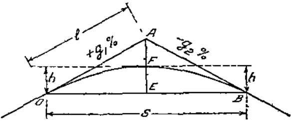


Fig. 94

$$S = 2l \text{ and } EF = h = \frac{g_1 - g_2}{400}\,l.$$

Given h, g_1, g_2 l may be determined and the offsets computed from $y = \left(\dfrac{g_1 - g_2}{400l}\right) x^2$

(b) Sight distance longer than the curve.

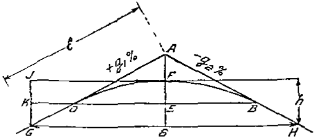


Fig 95

$$h = GK + KJ = GK + EF = GK + AF$$

Height of A above O $= \dfrac{g_1 l}{100}$.

Height of B above O $= \dfrac{g_1 l}{100} + \dfrac{g_2 l}{100}$.

$$= \frac{l}{100}\,(g_1 + g_2).$$

The angle between O B and the horizontal

$$= \frac{l}{100}\,\frac{(g_1-g_2)}{2l} = \frac{g_1+g_2}{200} \text{ radians}$$

$$\angle \text{ KOG} = \frac{g_1}{100} - \frac{g_1+g_2}{200} = \frac{g_1-g_2}{200} \text{ radians}\cdot$$

$$\text{But OK} = \frac{S}{2} - l$$

$$\therefore \text{ KG} = \left(\frac{S}{2} - l\right)\left(\frac{g_1-g_2}{200}\right)$$

$$h = \text{KG} + \text{AF}$$

$$= \left(\frac{S}{2} - l\right)\left(\frac{g_1-g_2}{200}\right) + \frac{l}{100}\,(g_1-g_2)\cdot$$

$$= \left(\frac{S-l}{400}\right)(g_1-g_2)\cdot$$

(c) Sight distance less than length of the curve

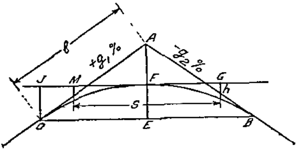


Fig 96.

The offsets from the tangent MFL are given by the equation $y = cx^2$

$$\therefore h = c\left(\frac{s}{2}\right)^2$$

$$\therefore c = \frac{4h}{S^2}.$$

At point J $x = l$.

$$\text{JO} = \frac{4h}{S^2}.\,l^2$$

But $JO = EF = AI = \dfrac{l}{400}(g_1 - g_2)$.

$$\therefore \quad \frac{4h}{S^2} l^2 = \frac{l}{400}(g_1 - g_2)$$

$$l = \frac{(g_1 - g_2)}{1600} \frac{S^2}{h} \tag{87}$$

Examples on vertical curves

Example 1 Find the length of the vertical curve connecting two uniform grades from the following data

(a) $+ 8\%$ and $- 6\%$
rate of change of grade $= 1\%$ per 30 m

(b) $- 5\%$ and $+1\%$
rate of change of grade $=$ 05% per 30 m

(a) Total change of grade $=$ the algebraic difference of two grades $= 8 - (- 6) = 1\ 4$

Length of the curve $= \dfrac{1\ 4}{1} \times 30$
$= 420$ m

(b) Length of the curve $= \dfrac{- 5 - (+1)}{05} \times 30$

$= \dfrac{-1\ 5}{05} \times 30 = 900$ m

Example 2 —Calculate the reduced levels of the various station pegs on a vertical curve connecting two uniform grades of $+ 5\%$ and $- 7\%$ The chainage and the reduced level of the point of intersection are 500 m and 350 750 m respectively Take the rate of change of grade as 1% per 30 m

(i) Length of the vertical curve —
Total change of grade $= 5 - (- 7) = 1\ 2$,
Rate of change of grade per 30- m chain $= 1\%$

∴ Length of the vertical curve $= \dfrac{1\ 2}{\cdot 1} \times 30 = 360$ m

Length of the curve on either side of the apex $= 180$ m

(ii) Chainages of the tangent points of the curves :—

Chainage of the point of intersection = 500 m.
Chainage of the beginning of the curve (first T. P.)

$$= 500 - 180 = 320 \text{ m}$$

" of the end of the curve (second T. P.)

$$= 500 + 180 = 680 \text{ m.}$$

(iii) Reduced levels of the various points —

R. L of the point of intersection = 330 75

R. L. of the beginning (O) of the curve

$$= 330\ 75 - \frac{\cdot 5 \times 180}{100} = 329.85$$

R L. of the end-point (B) of the curve

$$= 330\ 75 - \frac{7 \times 180}{100} = 329\ 49$$

" of the mid point (E) of the chord OB

$$= \tfrac{1}{2}\,[329\ 85 + 329 \cdot 49] = 329\ 27$$

" of the vertex (F) of the curve

$$= \tfrac{1}{2}\,[\text{R. L of E} + \text{R. L. of A}]$$

$$= \tfrac{1}{2}\,[329\ 67 + 330 \cdot 75] = 330\ 21$$

∴ The difference AF between A and F

$$= 330\ 75 - 330\ 21 = 0 \cdot 54 \text{ m}$$

check with the formula $AF = \frac{g_1 - g_2}{400}\, l = \frac{1\ 2}{400} \times 180 = 0\ 54$

(iv) The reduced levels of the points on the curve may be calculated by applying the tangent corrections to the reduced levels of the corresponding points of the tangents as shown below.

First point on the curve chainage 350, R. L. of the first point on the tangent = R. L. of O − 0·15

$$= 329 \cdot 85 + 0\ 15 = 330\ 00$$

Tangent correction at this point on the curve

$$\left(\frac{1}{6}\right)^2 \times AF = 0 \cdot 015 \text{ m}$$

∴ R. L. of the first point on the curve = R. L. of the first point on the tangent − tangent correction

$$= 330 \cdot 00 - 0 \cdot 015 = 329 \cdot 985.$$

Similarly, at chainage 380, R L of the second point on the tangent $= 329 \cdot 85 + 0 \cdot 30 = 330 \cdot 15$

$$\text{Tangent correction} = \left(\frac{2}{6}\right)^2 \times 0\ 54 = 0 \cdot 06$$

R L of the second point on the curve $= 330 \cdot 15 - 0\ 06$
$= 330 \cdot 09$, and so on.

The tangent corrections on the other side of the point of intersection are exactly the same

The results may be tabulated as under :

Station	Chainage	Grade Elevation	Tangent Correction (—ve)	Curve Elevation	Remarks
0	320	329 85	0	329 850	beginning of the curve
1	350	330 00	0 015	329 985	
2	380	330 15	0 060	330 090	
3	410	330 30	0 135	330 165	
4	440	330 45	0 240	330 210	
5	470	330 60	0 375	330 225	
6 (F)	500	330 75	0 540	330 210	vertex of the curve
7	530	330 54	0 375	330 165	
8	560	330 33	0 240	330 090	
9	590	330 12	0 135	329 985	
10	620	329 91	0 060	329 850	
11	650	329 70	0 015	329 685	
12 (B)	680	329 49	0	329 490	end of the curve

Example 3 —A down grade of 1·5% is followed by an up grade of 2 5% The reduced level of the point of intersection is 70 00, and its chainage 360 m A vertical parabolic curve 120 m long is to be introduced to connect the two grades The pegs are to be fixed at 15 m intervals Calculate the elevations

of the points on the curve by (a) tangent corrections, and (b) chord gradients, and (c) the staff readings required, if the pegs are to be driven with their tops at the formation of the curve, given that the height of collimation is 72.18

(See Fig 87)

(i) Tangent corrections —The tangent corrections may be calculated by $y = cx^2$

Now $g_1 = -1.5\%$, $g_2 = +2.5\%$, length of the curve on either side of the apex A = 60 m

No of stations on either side of the apex = 4

$$\text{Fall for 15 m} = \frac{15}{100} \quad 1.5 = 0.225 \text{ m}$$

$$\text{Rise for 15 m} = \frac{1.5 \times 2.5}{100} = 0.375 \text{ m}$$

$$c = \frac{g_1 - g_2}{400\, l} = \frac{-1.5 - 2.5}{400 \times 60} = -\frac{1}{6000}$$

$$\begin{aligned}\text{Tangent correction for the first point} &= -\frac{1}{6000} \times 15^2 = -3.80 \\ &= -0.038 \text{ m}\end{aligned}$$

Tangent corrections for the other station points can be obtained by putting 30, 45, 60, 75 90 105 and 120 in place of 15 in the above expression

Chainage of the intersection point A = 360 m

Chainage of the beginning of the curve (O) = 300 m

Chainage of the end point (B) of the curve = 420 m

Now R L of A = 70.00

$$\text{R. L of O} = 70 + \frac{1.5}{100} \times 60 = 70.90$$

$$\text{R L of B} = 70 + \frac{2.5}{100} \times 60 = 71.50$$

Station	Chainage	Grade elevation	Tangent correction + ve	Curve elevation	Height of collimation.	Staff reading	Remarks
O	300	70 900	0	70 90	72 180	1 280	beginning of the curve
	315	70 675	0 034	70 709		1 471	
	330	70 450	0 150	70 600		1 580	
	345	70 225	0 338	70 563		1 467	
4 (F)	360	70 000	0 600	70 60		1 580	vertex of the curve.
	375	69 775	0 938	70 713		1 467	
	390	69 550	1 350	70 900		1 280	
	405	69 325	1 838	70 163		1 017	
	420	69 100	2 400	71 500		0 680	end of the curve

Note the above staff readings will be corrected to 0 005 m the smallest reading that can be obtained on the metric staff

Check —Elevation of E $= \frac{1}{2}$ (70 90 + 71 50) = 71 20
Elevation of F $= \frac{1}{2}$ (71 20 + 70 00) = 70 60

(b) **By Chord Gradients** —The successive chord gradients are

$$\frac{g_1 I}{100} - Cg \quad \frac{g_1 I}{100} \quad 3Cg \quad \frac{g_1 I}{100} - 5Cg \quad \text{etc}$$

$$\text{Now } \frac{g_1 I}{100} = \frac{-1\ 5 \times 15}{100} = -0\ 225$$

$$Cg = \frac{(-1\ 5 - 2\ 5) \times 15^2}{400 \times 60} = -\frac{3}{80} = -\ 034$$

The calculations of the elevations of the points may be made thus

O chainage 300 $\frac{g_1 I}{100} = -0\ 225$ R L of O = 70 90

$$Cg = -\ 034$$

$$\frac{g_1 I}{100} - Cg = -\ 0\ 191 \quad \frac{g_1 I}{100} - Cg = -0\ 191$$

1 chainage 315 R L of 1 = 70 709

$\frac{g_1 l}{100} - 3Cg = -0\ 113$ $\frac{g_1 l}{100} - 3Cg = -0\ 113$

2 „ 330 R L of 2, = 70 596

$\frac{g_1 l}{100} - 5Cg = -0\ 037$ $\frac{g_1 l}{100} - 5Cg = -0\ 037$

3 „ 45 R L of 3, 70 559

$\frac{g_1 l}{100} - 7\,Cg = -0\ 038$ $\frac{g_1 l}{100} - 7\,Cg = -0\ 038$

4 „ 360 R L of 4 70 597

$\frac{g_1 l}{100} - 9\,Cg = +0\ 113$ $\frac{g_1 l}{100} - 9\,Cg = +0\ 113$

5 „ 375 R L of 5, 70 710

$\frac{g_1 l}{100} - 11\,Cg = +0\ 188$ $\frac{g_1 l}{100} - 11\,Cg = +0\ 188$

6 „ 390 R L of 6, 70 898

$\frac{g_1 l}{100} - 13\,Cg = +0\ 263$ $\frac{g_1 l}{100} - 13\,Cg = -0\ 263$

7 „ 405 R L of 7 71 161

$\frac{g_1 l}{100} - 15Cg = +0\ 338$ $\frac{g_1 l}{100} - 15\,Cg = -0\ 338$

8 „ 420 R L of 8, 71 499

Example 4 – Calculate the length of vertical curve for the data given below

(i) Sight distance = twice the length of vertical curve

$g_1 = 1\%$

$g_2 = -1\ 5\%$

$h = 1{\cdot}12$ m

(ii) Sight distance = half the length of the vertical curve

$$g_1 = 1.5\%$$
$$g_2 = -2.5\%$$
$$h = 1.12\ m$$

$$\text{(i)}\quad 1.1^{\circ} = \frac{(4l - l)}{400} \times (1 + 1.5)$$

$$440 = 3l \times 2.5 = 7.5\, l$$

$$l = 59\ \text{m say } 60\ \text{m.}$$

$$L = 2\, l = 120\ \text{m}$$

$$\text{(ii)}\quad l = \frac{(1.5 + 2.5)\, l^2}{1600 \times 1.12}$$

$$l = \frac{1600 \times 1.12}{4} = 448$$

$$L = 2l = 896\ \text{say } 900\ \text{m}$$

Lemniscate Curve – The form of the transition curve usually used in modern road work is the Bernoulli's Lemniscate (Fig 97). It is symmetrical curve and well adapted when the deflection angle between the tangents is large. It is used in preference to the spiral for the following reasons

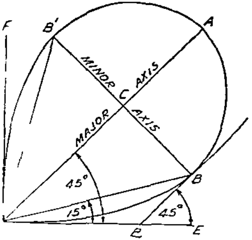


Fig 97

(1) The radius of curvature decreases more gradually

(2) It fulfils the condition that the rate of increase of curvature should diminish towards the end of the transition curve.

(3) It most fully corresponds to what is known as "autogenous curve" of an automobile, i e the path actually traced by an automobile when turning freely

Fig 97 shows the shape of the curve in the first quadrant

OE and OF = the tangents at the origin O

OA = the major axis of the curve (the polar ray making an angle of 45° with OE)

BB′ = the minor axis of the curve

In Fig. 98, let M = any point on the curve

MM_1 = the tangent at M

ϕ = the deflection angle MM_1E between the tangents MM_1 and OE

ρ = OM = the polar ray of M, i e the line joining the origin to the point M

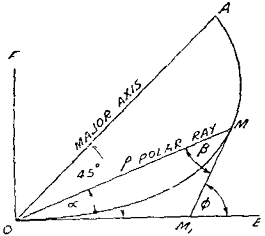


Fig 98

α = MOE = the polar deflection angle of M, i e the angle which the polar ray OM makes with the tangent OE

β = OMM_1 = the angle between the polar ray OM and the tangent MM_1

The polar equation of the curve is $\rho = K\sqrt{\sin 2\alpha}$ (88)

From the properties of polar co-ordinates, we have

$$\tan\beta = \rho\frac{d\alpha}{d\rho} \quad \text{Now} \quad \frac{d\rho}{d\alpha} = \frac{K\cos 2\alpha}{\sqrt{\sin 2\alpha}}$$

$$\tan\beta = \frac{K\sqrt{\sin 2\alpha} \times \sqrt{\sin 2\alpha}}{K\cos 2\alpha} = \tan 2\alpha$$

or $\beta = 2\alpha$ Now $\phi = \alpha + \beta$ $\quad \phi = 3\alpha$ (exact) (89)

The radius of curvature (r) at any point by the usual formula

for polar co-ordinates $$r = \frac{\left\{\rho^2 + \left(\frac{d\rho}{d\alpha}\right)^2\right\}^{\frac{3}{2}}}{\left\{\rho^2 + 2\left(\frac{d\rho}{d\alpha}\right)^2 - \rho\frac{d^2\rho}{d\alpha^2}\right\}}$$

Substituting the values of $\frac{d\rho}{d\alpha}$ and $\frac{d^2\rho}{d\alpha^2}$, we get

$$r = \frac{K}{3\sqrt{\sin 2\alpha}} \tag{90}$$

Substituting the value of $K\left(=\frac{\rho}{\sqrt{\sin 2\alpha}}\right)$ in formula (90),

we have $$r = \frac{\rho}{3\sin 2\alpha} \tag{90a}$$

From equation (90) $K = 3r\sqrt{\sin 2\alpha}$, and from equation (88),

$$\sqrt{\sin 2\alpha} = \frac{\rho}{K}$$

$$K = 3r\frac{\rho}{K} \text{ i.e. } K^2 = 3\rho r \text{ or } K = \sqrt{3\rho r} \tag{91}$$

For small deflection angles (4° or 5°), the length of the curve OM $= 6r\alpha$ (approximate), in which α is in radians (92)

or $OM = \frac{r\alpha}{9.55}$ in which α is in degrees (92a)

At the end of the curve $r = R$; $l = L$; $\phi = \phi_1 = 3 \measuredangle_{max}$. To determine the position of the minor axis, draw the polar ray OB making an angle of 15° with the tangent OE (Fig. 92). Draw BB′ perpendicular to OA, meeting it in C and the other side of the curve in B′. If a tangent be drawn at B, the angle between the tangent BB_1 and OE is 45°. BB_1 is, therefore, parallel to the major axis OA. The triangle OBB′ being equilateral,

$$BB' = OB = K\sqrt{\sin 30^\circ} = \frac{K}{\sqrt{2}}. \text{ Now } OA = K\sqrt{\sin 90^\circ} = K.$$

$$\therefore \quad BB' = \frac{OA}{\sqrt{2}} \quad \text{or} \quad \frac{BB'}{AO} = \frac{1}{\sqrt{2}} = \frac{1}{1\cdot4142}.$$

Radius of curvature at A from equation 90, $= \frac{K}{3} = \frac{1}{3}OA.$

Thus the radius of curvature decreases gradually from infinity at the origin O to a minimum ($\frac{1}{3}$OA) at A (or at 45°)

Length of the curve OBA $= 1\cdot31115$ OA $= 1\cdot31115$ K

Two cases will now be considered.

Case 1:—When the curve between the tangents is transitional throughout.

In Fig. 99 let AO and OB = the tangents intersecting at O

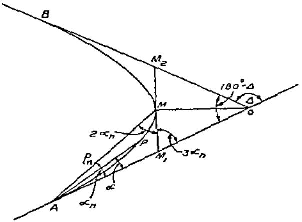


Fig. 99

A and B = the tangent points
M = the apex of the curve
OM = the apex distance
M_1MM_2 = the common tangent at M.
$\phi_1 = MM_1O$ = the angle between MM_1 and AO
$\phi_1 = MM_2O$ = the angle between MM_2 and OB.
$\propto_n = MAO$ = the polar deflection angle of AM
Δ = the deflection angle between AO and OB

The two lemniscates AM and BM joining at M are symmetrical about OM which is the bisector of the angle AOB They are so arranged at M that OM is the common normal

$$M_1MO = 90° = M_2MO$$

Now $MM_1O = \phi_1 \; = MM_2O$ and $MAO = \propto_n = MBO$

Also, $M_1OM = \frac{1}{2}\ AOB = \frac{1}{2}\ (180° - \Delta) = 90° - \frac{\Delta}{2}$

$$\phi_1 = \frac{\Delta}{2},\ \text{ but } \ \phi_1 = 3\propto_n$$

Hence $\propto_n = \frac{1}{3}\phi_1 = \frac{1}{6}\ \Delta$

which is the condition for the curve to be transitional throughout.

In the $\triangle$ OAM, $OAM = \propto_n$, $AMO = 90° + 2\propto_n$

$$AOM = 90° - \frac{\Delta}{2}$$

If the apex distance OM and the deflection angle Δ be given, the three angles and one side of the $\triangle$ AOM being known, the other two sides AM and OA can be found by the Sine rule Knowing the tangent distance OA, the tangent point A may be located Since OB = OA, the other tangent point B may also be located If, *on the other hand* the radius at the end (M) of the curve and the deflection angle Δ be given, AM can be calculated from equations (90) and (88) Knowing AM and the deflection angle Δ, OA and OM can be calculated Having located the tangent points A and B, the first lemniscate AM may be set out from A and the other from B by the method

of deflection angles as already explained, for which a table giving the various values of α and ρ may be prepared by assuming the successive values of α and then calculating the corresponding lengths of the polar rays to the successive point from $\rho = K\sqrt{\sin 2\alpha}$

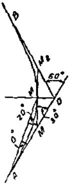


Fig 100

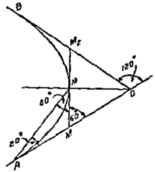


Fig 101

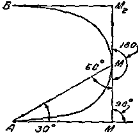


Fig 102

Figures 100, 101, and 102 illustrate the curve which is transitional throughout for deflection angles of 60°, 120°, and 180° respectively

Case II —When the value of α_n is less than $\frac{1}{6}\Delta$, it is necessary to insert a circular curve between the two lemniscates the central angle 2θ subtended by the circular arc being $(\Delta - 2\phi_1) = \Delta - 6\alpha_n$ as shown in Fig 103

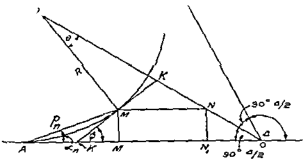


Fig 103

At M draw K_1MK, the tangent common to both the transition curve and the circular arc Through M draw MN parallel to AO, and draw MM_1 and NN_1 perpendicular to AO

Then the tangent length $AO = AM_1 + M_1N_1 + N_1O$

Now $AM_1 = \rho_n \cos \alpha_n$, $MM_1 = \rho_n \sin \alpha_n = NN_1$

In the $\triangle\, O_1MN$, $MO_1N = \theta$, $MNO_1 = AON = 90^\circ - \dfrac{\Delta}{2}$, $O_1M = R$

Now $MKO_1 = KK_1O + KOK_1 = \phi_1 + 90^\circ - \dfrac{\Delta}{2}$,

Also $\quad$, $\quad = 90^\circ - \theta$, $\qquad \theta = \dfrac{\Delta}{2} - \phi_1$,

$$\frac{MN}{O_1M} = \frac{\sin\ MO_1N}{\sin\ MNO_1}$$

$$MN = \frac{R \sin\theta}{\sin\left(90^\circ - \dfrac{\Delta}{2}\right)} = \frac{R\sin\left(\dfrac{\Delta}{2} - \phi_1\right)}{\cos\dfrac{\Delta}{2}}$$

$$= R\left(\cos\phi_1 \tan\frac{\Delta}{2} - \sin\phi_1\right)$$

$M_1N_1 = MN = R\,(\cos\phi_1 \tan\dfrac{\Delta}{2} - \sin\phi_1)$ R being the radius of the circular arc

$$N_1O = NN_1 \cot\left(90^\circ - \frac{\Delta}{2}\right) = \rho_n \sin\ \alpha_n \tan\frac{\Delta}{2}$$

$$AO = \rho_n \cos\alpha_n + R\,(\cos\phi_1 \tan\frac{\Delta}{2} - \sin\phi_1) + \rho_n \sin\alpha_n \tan\frac{\Delta}{2}$$

Example 1 —Given that the deflection angle (Δ) between the two tangents is 120° and the apex distance 16 m The curve is transitional throughout Make the necessary calculations for setting out the curve (Fig 99)

In the $\triangle AOM$ $OM = 16$ m

$$MOA = 90^\circ - \frac{\Delta}{2} = 90^\circ - 60^\circ = 30^\circ, \; OAM = \frac{\Delta}{6} = 20^\circ$$

$$AMM_1 = 2 \propto_n = 40^\circ \quad AMO = 90^\circ + 2 \propto_n = 130^\circ$$

$$OA = OM \frac{\sin AMO}{\sin OAM} = 16 \frac{\sin 130^\circ}{\sin \;20^\circ} = 35\ 84 \text{ m}$$

$$AM = OM \frac{\sin MOA}{\sin OAM} = 16 \frac{\sin \;30^\circ}{\sin \;20^\circ} = 23\ 38 \text{ m}$$

Now from equation (88) $K = \dfrac{\rho}{\sqrt{\sin 2 \propto}} = \dfrac{23\ 38}{\sqrt{\sin 40^\circ}} = 29\ 16\,\text{m}$

The polar equation of the curve is $\rho = 29\ 16 \sqrt{\sin 2 \propto}$

Now calculate ρ for different values of $\propto$ and tabulate the results as under

$\propto$	K	ρ in m	Formula
2° 30	29 16	8 61	$\rho = 29\ 16\sqrt{\sin 2 \propto}$
5° 0		12 15	
7° 30		14 84	
10° 0		17 06	
12° 30		18 96	
15° 0		20 62	
17° 30		22 09	
20° 0		23 38	

Example 2 —Two straights intersect at 66 4 m the deflection angle being 120° The curve is transitional throughout and the minimum radius of curvature is 60 m Make the necessary calculations for setting out the curve (Fig 99)

As before $\propto_n = \dfrac{1}{6} \Delta = 20^\circ$

Now from formula (90) $K = 3R\sqrt{\sin 2 \propto_n}$

$$= 3 \times 60\sqrt{\sin 40^\circ} = 144\ 33$$

Now by formula (88) $\rho_n = 144\ 33\sqrt{\sin 40^\circ} = 115\ 71 = AM$

$$\text{Tangent length OA} = \text{AM}\,\frac{\sin \text{AMO}}{\sin \text{AOM}} = \rho_s\,\frac{\sin 130^\circ}{\sin\ 30^\circ}$$

$$= 115\ 77\ \frac{\sin 50^\circ}{\sin 30^\circ} = 17{\sim}\ 39 \text{ m}$$

The values of ρ for the various values of α may be calculated as above from equation $\rho = 144\ 33\ \sqrt{\sin 2\alpha}$

PROBLEMS

1 What is meant by a transition curve ? What are the different forms of a transition curve ? Give reasons for introducing a transition curve between a tangent and a circular curve on a road or railway

2 What is mean by an easement curve ' ? Derive an expression for the ideal trans ion curve What are the modifications of the ideal transition curve ?

3 What is meant by shift of a curve ? Derive an expression for the same Explain how you would set out a transition curve

4 Explain the various methods of determining the length of a trans tion curve

5 Explain how you would set out a transition curve (a) by deflection angle and (b) by tangent offsets

6. *Explain clearly the procedure adopted in setting out a comb ned* (or composite) curve (a) by means of a theodolite and (b) with a chain and tape only

7 What are the difficulties in setting out simple curves ? Describe briefly the methods employed in overcoming them

8 Write brief notes on the following —

(i) Cub c sp ral (ii) Verti al c irve (iii) Reverse curve, and (iv) Compo ind curve

9 Two straights on the centre line of a proposed railway curve intersect at ?610 0 m the deflection angle being 46° A circular curve with 400 m radius and transi ion curves are to be inserted the latter being 90 m in length Make all necessary calculations to set out this curve by d fle ction angles Pegs are to be set out at every 30 m of continuous chainage

[1st transition curve deflection angles 2?″ 19 40 1° 7 34 2° 8 54″

(Ans Circular curve deflection angles 22 10″ 2 31 ?″ 4°40 0″ 6°48 ?5″ 8°?7 51″ 11°6 46″ 13°15 41″ 15°?4 36″, 16° 33 ?8″ [(½ 33° 11)]

2nd transition curve deflection angles 2° 8 54″ 1°31 58″ 33 42″ 4 05″

10 Two tangents intersect at 2865 m the deflection angle being 52° 30. It is proposed to put in a circular curve of 480 m radius with a cubic parabolic transition curve 60 m in length at each end. The circular curve is to be set out with pegs at every 30 m and the transition curve at every 15 m of through chainage. Tabulate the data relative to stations at ch 2640 and ch 2700 and the junctions of the transition curves with the circular arc.

(Ans deflection angle @ ch 2640 = 34 54"
deflection angle @ ch 2588 11 = 1°11 36"
deflection angle @ ch 2700 = 2°30
deflection angle @ ch 3037 99 = 22 42)

11 A reverse curve AB is to be set out between two parallel railway tangents, 12 m apart. If the two arcs of the curve are to have the same radius, and the distance between the tangent points A and B is 96 m, calculate the radius. The curve is to be set out from A B at 8 m intervals along that line. Calculate the tangent offsets.

{Ans for each branch 0m 0 848 m, 1 328 m, 1 488 m
1 328 m 0 848 m 0m }

12 A vertical parabola 120 m long, is to be put in between a 2 per cent up grade and 1 per cent down grade which meet at a chainage 600 m. The reduced level of the point of intersection of the two gradients being 150 00 m. Calculate the reduced levels of the tangent points and at every 15 m along the parabola.

{Ans R L 148 8 149 073 149 289, 149 448 149 550,
149 595 149 589 149 523 149 4 }

13 CD and EF are two straights such that C and F are on opposite sides of a common tangent DE. It is required to connect CD and EF with a reverse curve given that the angles CDE and DEF are 151° 40 and 142° 20 respectively, and that DE 372 9 m and the chainage of D 2534 4 m. Calculate (a) the common radius and the chainage of the points of tangency and the point of reverse curvature and (b) the total tangential angle of the point of reverse curvature.

(Ans 23 5 4 m 2687 1 m 3101 1 m 14°10 32")

14 On a proposed railway two straights intersect at chainge 1753 5 m with a right deflection angle of 36°24. It is proposed to put in a circular curve of 450 m radius with a cubic parabolic transitions at each end. The maximum speed on this part of the railway 65 km per hour and the rate of acceleration is 0 3 m per sec^3. Find a suitable length for the transition curves and calculate the chainge at the begining and at the end of the combined curve.

Describe how you would set out the combined curve by the method of deflection angles. After setting out six pegs (peg interval being 30 m) on the circular arc of the combined curve, it is discovered that further angles cannot be turned from the existing position of the instrument

owing to intervening obstacle to the line of sight Describe briefly the method you would employ in setting out the remaining portion of the circular curve

[Ans 45 m 1553 1 1884 0 m]

15 Design a vertical curve connecting two gradients 2% to 1.5% at a summit (R L. 70 50 chainge 850 m) The curve is to be such that two points 300 m apart and 1 25 m above the curve are intervisible

(Ans Length of vertical curve — 315 m
offset at the intersections point = 1 3 8 m.)

16. Two roads having a deviaton of 5°30 are to be joined by a simple 18° curve Chainage at the intersection point is 870 m. Calculate necessary data and explain in detail how to set out the curve

(a) by chain and offsets only
(b) if a theodolite is available

(Note —A degree of the curve is the angle subtended at the centre by a chord of 30 m length)

[Ans. offsets from chord produced O_1 1 5 m O_2=3 908 m O_3 to O_6=4 158 m O_7=2 4 5 m
Deflection angles Δ_1 = 3 4 4", Δ_2 = 7°4, 4 "
Δ — 0 15]

17 Two tangents meet at chainage 1022 m the deflection angle being 36° A circular curve o radius 300 m is to be introduced in between the two tangents. calculate the following

(1) Tangent length.
( ) Length of circular curve
(3) Chainages of the tangent points.
4 Deflection angles for setting out the first three pegs and the last peg on the curve by the Rankine Method. Pegs are to be fixed at 0 m interval

Describe briefly how you would set out the curve

(Ans. (1) 9 4 m (2) 188 5 m (3) 9 4 35 m 1113 05 m (4) 1°18 40" 3°23 20" 5°17 40" 18°

18. A compound curve is to consist of an arc of 900 m radius followed by one of 1 00 m radius and is to connect two straights intersecting at an angle of 84°3' At the intersection point the chainage if continued along the first tangent would be 23 9 0 m and the starting point of the curve is selected at chainage 1354 90 m. Calculate the chainages at *the point of junction of two branches and at the end of the curve*

Describe briefly the steps involved in setting out the curve

[Ans T_1=975 0 m ϕ_1 =23°11 ϕ_2 — 61°21
L_1—363 30 m, L_2 — 1°86 10 m,
Chainage of junction pt. = 1718 30 m
Chainage of end pt. = 3004 40 m]

+ + +

CHAPTER VI

FIELD ASTRONOMY

Spherical Trigonometry

A Sphere is a solid bounded by a surface and is such that every point on the surface is equidistant from a certain point called the centre of a sphere. It is a solid formed by the revolution of a circle or a semi circle about its diameter.

A radius of a sphere is a straight line joining the centre to any point on the surface of a sphere.

A section of a sphere by any plane is a circle.

A section of a sphere is called a great circle when the cutting plane passes through the centre of the sphere (Fig 104).

A section of a sphere is called a small circle when the plane cutting the surface does not pass through the centre of the sphere (Fig 99).

The *shortest* distance between any two points on the surface of a sphere is along *an arc of a great circle* passing through them.

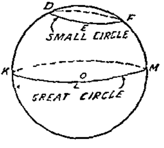


Fig 104

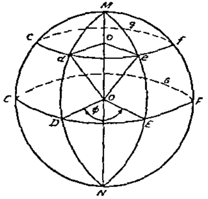


Fig 105

Length of a Great Circle Arc —

In Fig 105 let CDEFG = a great circle

MON = the diameter of the sphere perpendicular to the plane of the great circle, the extremities of the perpendicular being called the *poles* of the great circle

M and N = the poles of the great circle CDEFG

R = the radius of the sphere

ϕ = the angle subtended by the great circle arc DE at the centre O of the sphere, expressed in circular measure

Then arc DE $= R\phi = \phi$ when R = unity

The length of a great circle arc is, therefore, equal to the angle in radians or circular measure which it subtends at the centre of a sphere of a unit radius In practice, the length of a great circle arc is expressed in degrees, minutes, and seconds

Now consider the semi circle MDN of the great circle in which the plane passing through MON and D cuts the sphere Since OM is perpendicular to the plane of the great circle CDEFG, it is perpendicular to any line such as OD in this plane Therefore, the angle MOD = 90° Hence the great circle arc MD subtends 90° at the centre of the sphere, i e MD = 90° In other words, the angular distance from the pole of a great circle to any point on that great circle is 90°

Length of a small Circle Arc —

In Fig 105 let O_1 = the centre of the small circle *cdefg*

M and N = the poles of the small circle

$R_1 = O_1d$ = the radius of the small circle

de = the arc of the small circle

R = the radius of the great circle CDEFG

Let the great circle passing through M and *d* cut the great circle CDEFG in D, similarly, let the great circle passing through M and *e* cut the great circle CDEFG in E

Now arc $de = R_1 \times \angle dO_1e$

arc DE $= R \times \angle DOE$

But $\angle dO_1e = \angle DOE$, since O_1d and OD are parallel

and O_1e and OE are parallel $\dfrac{\text{arc } de}{\text{arc DE}} = \dfrac{R_1}{R}$

The angle dO_1O being a right angle,

$$\frac{R_1}{R} = \frac{O_1d}{Od} = \sin dOO_1 = \sin Md$$

Hence $\dfrac{\text{arc } de}{\text{arc DE}} = \sin Md = \cos dD$, since $Md + dD = 90°$

or $\qquad$ arc de = arc DE cos dD

A spherical triangle (Fig 106) is a triangle bounded by three arcs of great circles

A spherical angle is an angle between two great circles It is defined by the plane angle between the tangents to the circles at their point of intersection

Thus in Fig 106 ABC is a spherical triangle and the angle BAC is the spherical angle A between the great circles AB and AC It is measured by the plane angle A_1AA_2 between the tangents AA_1 and AA_2 to the great circles AB and AC

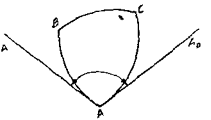


Fig 106

Properties of a Spherical Triangle —The following are the properties of any spherical triangle —

(i) Any angle is less than two right angles or π

(ii) The sum of the three angles is less than six right angles or 3π and greater than two right angles or π

(iii) The sum of any two sides is greater than the third

(iv) If the sum of any two sides, is equal to two right angles or π the sum of the angles opposite them is equal to two right angles or π

(v) The smaller angle is opposite the smaller side, and vice versa

Formulæ in Spherical Trigonometry —When three of the six parts (three sides and three angles) of a spherical triangle

are known, the remaining three may be computed by the following formula

Let A, B, and C be the spherical angles, and a, b, and c the sides opposite them in any spherical triangle ABC

Sine formula $$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} \qquad (1)$$

Cosine formula $$\cos a = \cos b \cos c + \sin b \sin c \cos A \qquad (2)$$

$\cos c = \cos a \cos b + \sin a \sin b \cos C$

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c} \qquad (3)$$

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a \qquad (3a)$$

with similar expressions for $\cos b$, $\cos c$, $\cos B$, and $\cos C$

$$\sin \frac{A}{2} = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin b \sin c}} \qquad (4)$$

$$\cos \frac{A}{2} = \sqrt{\frac{\sin s \sin (s-a)}{\sin b \sin c}} \qquad (5)$$

$$\tan \frac{A}{2} = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin s \sin (s-a)}} \qquad (6)$$

in which $s = \frac{1}{2}(a+b+c)$, with similar expressions for

$$\sin \frac{B}{2},\ \sin \frac{C}{2},\ \cos \frac{B}{2},\ \cos \frac{C}{2},\ \tan \frac{B}{2}, \text{ and } \tan \frac{C}{2}$$

$$\tan \frac{1}{2}(a+b) = \frac{\cos \frac{1}{2}(A-B)}{\cos \frac{1}{2}(A+B)} \tan \frac{1}{2} c \qquad (7)$$

$$\tan \frac{1}{2}(a-b) = \frac{\sin \frac{1}{2}(A-B)}{\sin \frac{1}{2}(A+B)} \tan \frac{1}{2} c \qquad (8)$$

$$\tan \frac{1}{2}(A+B) = \frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} \cot \frac{1}{2} C \qquad (9)$$

$$\tan \frac{1}{2}(A-B) = \frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}(a+b)} \cot \frac{1}{2} C \qquad (10)$$

The formulæ for a right angled spherical triangle may be obtained from 'Napier's Rules of Circular Parts'

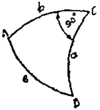


Fig 107a

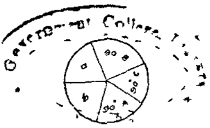


Fig 107b

In Fig 107a, ABC is spherical triangle right-angled at C Draw a circle and divide it into five parts (Fig. 107b) Write down in *order* the two sides containing the right angle and the complements of the remaining three parts A, c, and B as shown in the figure Then if any part is considered as 'middle part', the two parts *adjacent* to it are called 'adjacent parts', and the remaining two '*opposite* parts'. Then we have the following rules

(1) Sine of the Middle part = product of the *tangents* of the two *adjacent* parts

(2) ,, ,, ,, = product of the *cosines* of the two *opposite* parts

e g $\sin b = \tan a \tan (90°-A) = \tan a \cot A$
and $\sin b = \cos (90° - c) \cos (90° - B) = \sin c \sin B$

Area of a Spherical Triangle —The area of a spherical triangle may be obtained from the following formula —

$$\text{Area of a spherical triangle} = \Delta = \pi R^2 \left(\frac{A + B + C - 180°}{180°} \right)$$

$$= \pi R^2 \frac{E}{180°} \qquad (11)$$

where R = the radius of the sphere

E = A + B + C − 180°, the quantity E by which the sum of the three angles A, B, and C exceeds 180°, being called the spherical excess of the triangle

Useful Data —

1 radian = 57° 17′ 45″ = 3437 75′ = 206265″.

$1'' = \frac{1}{206265}$ radian, $1' = \frac{1}{3437 \cdot 75}$ radian

Log $\pi = 0 \cdot 4971499$ or $0 \cdot 4972$

When the angle α is very small, $\sin \alpha = \alpha$ in radians $= \tan \alpha$, $\cos \alpha = 1$.

$$\therefore \quad \sin 1'' = \frac{1}{206265} = \tan 1'' \text{ and } \sin 1' = \frac{1}{3437 \cdot 75} = \tan 1'.$$

$$\sin \alpha'' = \frac{\alpha''}{206265} = \alpha'' \sin 1'', \quad \sin \alpha' = \frac{\alpha'}{3437 \cdot 75} = \alpha' \sin 1'$$

Similarly, $\tan \alpha'' = \alpha'' \tan 1''$ and $\tan \alpha' = \alpha' \tan 1'$

$$\text{Log} \sin 1'' = \bar{6} \cdot 6855749 = \log \tan 1''$$

$$\text{Log} \sin 1' = \bar{4} \cdot 4637261 = \log \tan 1'$$

When degrees, minutes, and seconds are to be converted to hours, minutes, and seconds of time, the following relations may be used

$360^\circ = 24$ hours, $15^\circ = 1$ h, $15' = 1$ m, $15'' = 1$ sec

Latitude and Longitude

The position of a place on the earth's surface is specified by means of latitude and longitude

In Fig 108, let O = the centre of the earth
P = the north pole
P_1 = the south pole
POP_1 = the polar axis or the polar diameter about which the earth rotates
K = any point on the earth's surface

The great circle AK_1G_1B, the plane of which is perpendicular to the axis of rotation POP_1 is called the *terrestrial equator*

The semi circle (PKP_1) passing through K and terminated by the poles P and P_1 is called the *meridian* of the place

Latitude of a place is the angular distance measured from the equator towards the nearer pole along the meridian of the place

The sphere being divided into two hemispheres by the equator, the upper one containing the north pole is called the northern hemisphere while the lower one containing the south pole is called the southern hemisphere The place is said to have a north latitude if it is in the northern hemisphere, and

south latitude if in the southern hemisphere In this case the arc K_1K or the angle K_1OK is the latitude of the place K

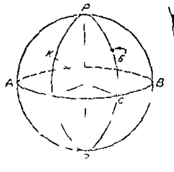


Fig 108

Latitude north of the equator is considered positive and that south of the equator negative

Longitude of a place is the angular distance between the meridian of the place and the standard or prime meridian The meridian (PGP_1) passing through Greenwich (Fig 108) has been adopted internationally as the standard meridian This meridian divides the sphere into two hemispheres one the eastern and the other the western The longitude is measured from 0° to 180° either towards the west or towards the east The west longitudes are considered as positive and east longitudes as negative Thus the longitude of the place K is the equatorial arc G_1K_1 or the spherical angle GPK Hence it will be seen that the position of the place K is completely specified by the latitude K_1K and the longitude G_1K_1

Parallel of Latitude —A parallel of latitude is a small circle of which P is the pole All places having the same latitude lie on the parallel of latitude

In Fig 109 let K M and N be the points on the parallel of latitude KMN so that they have the same latitude say θ PGG_1P_1 is the Greenwich meridian

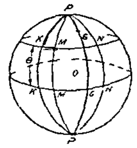


Fig 109

The angular radius (PK) of the parallel of latitude is equal to $90° - \theta$, since the great circle arc $PK_1 = 90°$ and $K_1K = \theta$

Now arc KM = arc $K_1M_1 \cos KK_1$

But $K_1M_1 = G_1K_1 - G_1M_1$

= difference of longitude

arc KM = difference of longitude × cos latitude (12)

This distance is called the *departure* and is measured in nautical miles

From Fig 109 it is evident that a degree of longitude has the greatest value at the equator and becomes less and less as the poles are approached ($KM \angle K_1M_1$) At the equator a degree of longitude is equivalent to about 111 km (69 miles) On the other hand a degree of latitude has approximately the same value no matter where it is measured (i e at the equator or near the poles) since it is measured along the meridians which are the great circles of the same diameter A degree of latitude is equivalent to about 111 km (69 miles)

Nautical Mile —A nautical mile is the distance measured along the great circle joining two points which subtend one minute of arc at the centre of the earth Taking the radius of the earth to be equal to 3960 miles we have

$$\text{one nautical mile} = \frac{\text{circumference of the great circle}}{360^\circ \times 60}$$

$$= \frac{2\pi \times 3960 \times 5280}{360^\circ \times 60} = 6080 \text{ feet}$$

$$= 1853\ 109 \text{ m}$$

The distance between two points in nautical miles measured along the parallel of latitude is called the *departure*

$$\text{Departure} = \text{difference of longitude in minutes} \times \cos \text{latitude} \qquad (10a)$$

If the two points are in the same hemisphere either western or eastern the difference between their longitudes may be obtained by subtracting one longitude from the other If however they are in different hemispheres the required difference of longitude may be obtained by the sum of their longitudes In case the sum exceeds 180° it should be subtracted from 360° to obtain the required difference of longitude of the two points

The shortest distance measured along the surface of the earth between two places is the length of the arc of the great circle joining them

Example 1 —Determine the difference of latitude between two places A and B given that their latitudes are

(a) A, 35° 42 N, B, 62° 55 N, (b) A, 42° 32 S, B, 53° 43 S, and (c) A 28° 16′ N B, = 46° 33 S

(a) The difference of latitude between A and B
$= 62° 55 - 35° 42 = 27° 13$

(b) The difference of latitude between A and B
$= 53° 43 - 42° 32 = 11° 11$

(c) The difference of latitude between A and B
$= 28° 16 - (-46° 33) = 74° 49$

Example 2 —Find the difference of longitude between two places C and D from their following longitudes

(i) long of C = 46° W (ii) long of C = 34° 24 E
of D = 64° W „ of D = 162° 10 E

(iii) long of C = 37° 44 W (iv) long of C = 58° 27 E
of D = 63° 18 E , of D = 138° 36 W

(i) The difference of longitude between C and D
$= 64° - 46° = 18°$

(ii) The difference of longitude between C and D
$= 162° 10 - 34° 24 = 127° 46$

(iii) The sum of the two longitudes
$= 37° 44 + 63° 18 = 101° 2$

Since the sum is less than 180° the difference of longitude between C and D = sum of the two longitudes = 101° 2

(iv) The sum of the two longitudes
$= 58° 27 + 138° 36 = 197° 3$

Since the sum of the two longitudes is greater than 180° the difference of longitude between C and D = 360° − the sum = 360° − 197° 3 = 162° 57

Example 3 —Calculate the distance in nautical miles between E and F along the parallel of latitude given that

(a) latitude of E, 48° 24 N longitude of E 37° 32 W
of F, 48° 24 N , , of F, 15° 24 W

(b) latitude of E 23° 12 S , longitude of E 120° 22′ W
of F 23° 12 S , „ of F, 162° 35 E

Distance in nautical miles between E and F along the parallel of latitude = departure = diff of long in minutes × cos latitude

(a) The difference of long between E and
$= 37° 32' - 15° 24' = 22° 8' = 1328$ minutes

Departure $= 1328 \cos 48° 24' = 881{\cdot}6$ n m

(b) The sum of the two longitudes $= 120° 22' + 162° 35'$
$= 282° 57'$

The diff of long between E and F $= 360° - 282° 57'$
$= 77° 3 = 4623$ minutes

Departure $= 4623 \cos 23° 12 = 4249$ n m

Example 4 —Find the shortest distance between two places K and L, given that the latitudes of K and L are 19° 0′ N and 13° 4 N and their longitudes 72° 30 E and 80° 12′ E respectively

In the spherical triangle PKL, PK $= 90° - $ lat of K $= 90° - 19° = 71°$ PL $= 90° - $ lat of L $= 90° - 13° 4 = 76° 56'$; the spherical angle KPL = the diff of long $= 80° 12 - 72° 30'$
$= 7° 42'$

Using the cosine rule, we have

$$\cos KL = \cos PK \cos PL + \sin PK \sin PL \cos KPL$$
$$= \cos 71° \cos 76° 56' + \sin 71° \sin 76° 56 \cos 7° 42'$$
$$= 0\ 9863382$$
$$KL = 9° 29 = 9°\ 483$$

Now arc = R × central angle, where R = the radius of the earth = 6372 km 6372

$$KL = \frac{\cancel{3958} \times 9°\ 483 \times \pi}{180°} = 1054\ 71 \text{ km}$$

$= \frac{6372 \times 9°\ 483 \times 3.14}{180} =$

radian $\frac{\pi}{180}$

The Celestial Sphere —When we view the heavens on any clear night we see a large number of stars of different degrees of brilliancy and consider them as situated on the surface of an imaginary sphere of infinite radius, the centre of which is the position of the observer or the earth This sphere is known as the *celestial sphere* Of the celestial (or heavenly) bodies, viz. the sun, stars, moon and planets, we are concerned only with the sun and the fixed stars for surveying purposes The stars which always maintain the same relative positions are commonly known as the fixed stars to distinguish them from the planets whose positions among the others are continually changing In practical astronomy, we are not concerned with the distances

of the celestial bodies from us (or from the earth), but only with the directions in which they are viewed Their directions are conveniently defined in terms of the positions on the surface of a celestial sphere in which the lines joining the heavenly bodies to the observer cut this surface The stars being infinitely distant from the earth the lines joining any particular fixed star to different points on the earth s surface are considered as parallel and its apparent direction remains unaltered when viewed from different places

As a result of the daily rotation of the earth on its axis from west to east all celestial bodies (the sun and fixed stars) appear to revolve from east to west round a point called the celestial pole However it is found more convenient to consider the earth as fixed and the celestial sphere as revolving from east to west about the axis of the earth prolonged Also due to the annual revolution of the earth around the sun the sun appears to move relatively to the stars from west to east

For field observations the instruments required are (1) a transit or a sextant and (2) a good watch or chronometer for recording the time of observation For computations the seven figure logarithmic tables and the Nautical Almanac (N.A) are required

Astronomical Terms

The *Celestial Sphere* is an imaginary sphere upon the surface of which all the stars in the sky appear to be studded to an observer stationed at its centre

The *Zenith* (Z) is the intersection of a vertical line through the observer's station with the upper portion of the celestial sphere It is the point on the celestial sphere immediately above the observer s station

The *Nadir* (Z, or N) is the intersection of a vertical line through the observer s station with the lower portion of the celestial sphere It is the point on the celestial sphere vertically below the observer s station

The *Celestial Horizon* (also called *True or Rational Horizon*) is the great circle in which a plane at right angles to the Zenith and Nadir line and passing through the centre of the earth

intersects the celestial sphere The Zenith and Nadir are the poles of the celestial horizon

The *Sensible Horizon* is the circle in which a plane tangent to the earth's surface (or at right angles to the Zenith and Nadir line) and passing through the point of observation intersects the celestial sphere The line of sight of an accurately levelled telescope lies in this plane

The *Visible Horizon* is the circle of contact of the earth and the cone of visual rays passing through the point of observation

The *Terrestrial Equator* (or simply, *Equator*) is the great circle of the earth, the plane of which is perpendicular to the axis of rotation (polar axis)

The *Polar Axis* (or *Polar Diameter*) is the diameter about which the earth spins The extremities of the axis of rotation (polar axis) of the earth are known as the Poles They are distinguished as the *North Pole* and the *South Pole*

The *Celestial Equator* is the great circle in which the plane of the equator cuts the celestial sphere

The *Celestial Poles* are the points of intersection of the axis of the earth (or the polar axis) when produced with the celestial sphere

The *Ecliptic* is the great circle which the sun appears to trace on the celestial sphere with the earth as a centre in the course of a year The plane of the ecliptic is not coincident with the plane of the equator the angle between them being known as the *Obliquity of the Ecliptic* Its value is about 23° 27

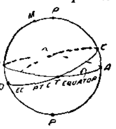


Fig 110

The points of intersection of the ecliptic with the equator are called the *Equinoctial Points* The point at which the sun's declination changes from south to north (i e the sun passing from south to north of the equator) is known as the *Vernal Equinox* or the *First Point of Aries* (♈) (Fig 110) while the other is called the *Autumnal Equinox* or the *First Point of Libra* (♎) The Vernal Equinox marks the beginning of spring while the Autumnal Equinox marks the commencement of autumn

The points on the ecliptic at which the north or south declination of the sun is maximum are known as the Solstices. The point C at which the north declination of the sun is maximum, is called the *Summer Solstice*, while the point D at which the south declination of the sun is maximum is known as the *Winter Solstice*. In the southern hemisphere, the reverse is the case.

The sun is at the Vernal Equinox (♈) on March 21, and its declination and right ascension are each equal to zero. On June 21 the sun is at C on the ecliptic and 90° from ♈, and its declination is maximum and equals 23° 27′ N., and its right ascension is 6h (or 90°). The sun is at Autumnal Equinox on September 21 (or 22) and its declination is zero and its right ascension is 12h (or 180°). The sun is at D on December 22 (or 21) and its declination is again maximum and is equal to 23° 27′ S. and its right ascension is 18h (or 270°). It will thus be seen that the sun's declination is north from March 21 to Sept. 22, while it is south from Sept. 22 to March 21. On March 21 and Sept. 22, the days and nights are of equal length all over the world.

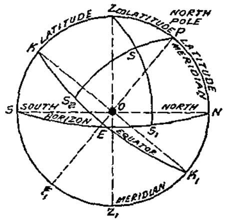


Fig. 111.

The *Celestial Meridian* is the great circle in which the plane passing through the celestial poles intersects the celestial sphere.

The *Meridian* of a place or an observer is the great circle passing through the zenith, and nadir and the poles.

The *Vertical Circle* is the great circle passing through the zenith and nadir The meridian of a place is, therefore also a vertical circle

The *Prime Vertical* is the vertical circle which passes through the east and west points of the horizon It is at right angles to the meridian ofthe place

The *Latitude* (θ) of a place or station is the angular distance measured from the equator towards the nearer pole, along the meridian of the place The latitude is the declination of the zenith

The *Co latitude* of a place is the angular distance from the zenith to the pole It is the complement of the latitude and is, therefore equal to 90° − latitude

The *Longitude* (ϕ) of a place is the angular measure of the arc of the equator between some primary meridian and the meridian of the place

The *Altitude* (α) of a heavenly body is its angular distance above the horizon measured on the vertical circle passing through the body

The *Co altitude* also called the *Zenith Distance* (z) is the angular distance of a heavenly body from the zenith It is the complement of the altitude and equals 90°− altitude

The *Azimuth* (A) of a heavenly body is the angle between the observer s meridian and the vertical circle passing through the body

The *Declination* (δ) of a heavenly body is its angular distance from the equator measured along the meridian generally called the declination circle i e the great circle passing through the body and the celestial poles

The *Co declination* also termed as the *Polar Distance* (p), is the angular distance of the heavenly body from the pole It is the complement of the declination, and is equal to 90°− declination

The *Hour Angle* (H) of a heavenly body is the angle between the observer's meridian and the declination circle passing through the body

The *Right Ascension* (R. A.) of a heavenly body is its equatorial angular distance measured eastward from the *First Point of Aries*.

The Altitude of the Celestial Pole is Equal to the Latitude of the Place of Observation —In Fig. 112, let O be the centre of the earth, C the position of the observer The vertical at C (as determined by the plumb line held by the observer at C), i e OC when produced defines the direction of the observer's zenith which is denoted by CZ. POQ is the earth's axis about which the earth rotates This axis when prolonged cuts the celestial sphere in P_1 which is called the north celestial pole. NCS drawn at right angles to CZ defines the plane of the observer's horizon Since the celestial pole is at infinite distance from the earth, it is seen by the observer at C in the direction CP_2 drawn parallel to the earth's axis The angle P_2CN is called the altitude of the north celestial pole EOE′ drawn at right angles to the earth's axis marks the plane of the equator so that the angle POE is a right angle Now the angle COE or the arc EC measures the latitude (θ) of the observer

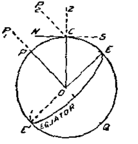


Fig. 112

$$\angle POC = 90° - \theta \quad \text{Since } CP_2 \text{ is parallel to } OP_1,$$

$$\angle P_2CZ = \angle POC = 90° - \theta \quad \text{Now} \angle P_2CN + \angle P_2CZ = 90°$$

$$\therefore \quad \angle P_2CN = 90 - \angle P_2CZ = 90° - (90° - \theta) = \theta$$

$\therefore$ Altitude of the celestial pole = latitude of the observer

Co-ordinate Systems

There are three systems of co ordinates by means of which the position of a heavenly body a star or the sun) on the celestial sphere can be completely specified

(1) **The Altitude and Azimuth System**:—In this system the co ordinates of a heavenly body are (i) the *altitude*, and (ii) the *azimuth*, the horizon being the plane of reference.

In Fig 113 let O be the centre of the celestial sphere Z, the zenith, P, the pole, NWS the horizon, ZPN the principal vertical circle (meridian) passing through Z and P and cutting the horizon in N, Y, the position of a heavenly body on the celestial sphere ZYY_1, the vertical circle passing through Y and cutting the horizon in Y_1, KYK_1 the parallel of altitude (a small circle passing through Y parallel to the horizon)

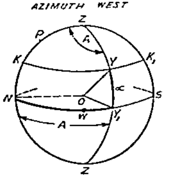


Fig 113

Then the position Y of the heavenly body is specified by (1) the angle Y_1OY or the great circle arc Y_1Y called the *altitude* and (2) the spherical angle PZY or the great circle arc NY_1 or the angle NOY_1 called the *azimuth* (west) the altitude and azimuth being denoted by α and A respectively Here the azimuth of a heavenly body is measured from the north point towards west and its value lies between 0° and 180° The great circle arc ZY is called the zenith distance (z d) or *co altitude* and is denoted by z OZ being perpendicular to the plane of the horizon the great circle arc $ZY_1 = 90°$

$$\text{Zenith distance of } Y = \text{arc } ZY = ZY_1 - YY_1 = 90° - \alpha$$
$$= 90° - \text{altitude}$$

When the star is in the eastern part of the celestial sphere the azimuth is measured from the north point towards east as shown in Fig 114 The spherical angle PZY or the great circle arc NY_1 or the angle NOY_1 is the azimuth (east) its value lying between 0° and 180° As before, the zenith distance $(z) = 90° - \alpha$ When the star s azimuth is 90° E or 90° W, the star is on the prime vertical (i e a vertical circle passing through the east and west points of the horizon) (Fig 115) Alternatively the position of a heavenly body is specified in terms of the zenith distance and the azimuth Owing to the diurnal motion of the stars these co ordinates are continually changing

The following rules should be observed in drawing diagrams of the heavenly body

Observer in north latitude —(i) Draw a circle and mark Z at the top of the circle

(ii) Draw the horizon and mark W (Fig 113), if the heavenly body is in the western hemisphere If the heavenly body is in the eastern hemisphere mark E as in Fig 114

(iii) Mark the cardinal points N and S according to the usual convention

(iv) In both cases, mark the celestial pole P on the vertical ZN at the given latitude (NP = θ)

(v) Draw the vertical ZYZ_1 through the heavenly body Y In both cases, the angle PZY is known as the azimuth of the heavenly body, and its value lies between 0° and 180°

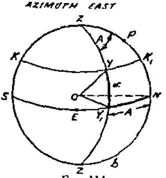


Fig 114

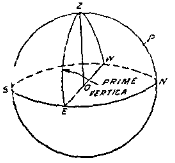


Fig 115

Let θ be the latitude of the observer ZP = co latitude $= 90° - \theta$ Now PN = altitude of the pole P = ZN − ZP = 90° − (90° − θ) = θ Whence, it follows that the altitude of the pole is equal to the latitude of the observer or the place of observation

(2) The Declination and Hour Angle System —In this system the co ordinates of a heavenly body are (i) the declination, and (ii) the hour angle the celestial equator being the reference plane

In Fig 116, let O be the centre of the celestial sphere, Z, the zenith, P, the north pole, KWK_1, the celestial equator NWS, the horizon, W and E, the points of intersection of the celestial equator with the horizon, Y, the position of a heavenly body, DYD_1, the parallel of declination (i e a small circle

parallel to the celestial equator); $PZKSP_1$, the observer's

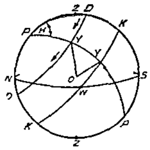


Fig 116

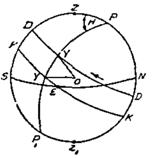


Fig 117

meridian PYP_1 the meridian through Y cutting the celestial equator in Y_1

Then the position of Y is specified by (1) the angle Y_1OY or the great circle arc Y_1Y called the *declination* and (2) *the angle* ZPY (H) or the great circle arc KY_1 called the *hour angle*

The declination (δ) of a heavenly body i e its angular distance *from the celestial equator is measured* along the great circle PYY_1P_1 termed as the declination circle When the body is north of the equator i e between the celestial equator and the north pole P its declination is north or positive (δ N or $+\delta$) while it is south or negative (δ S or $-\delta$) when the body is south of the equator i e between the celestial equator and the south pole P_1 The arc PY is known as the north *polar distance or co declination* Since $PY_1 = 90°$ and $Y_1Y = \delta$ $PY = PY_1 - YY_1 = 90° - \delta = 90° -$ declination $=$ co declination When the declination is south (δ S or $-\delta$) the north polar distance $= 90° - (-\delta) = 90° + \delta = 90° +$ declination

The angular distance between the body and the south pole P_1 is called the south polar distance and is equal to $90° - \delta$ or $90° + \delta$ according as the declination is south ($-\delta$) or north ($+\delta$)

(3) **The Declination and Right Ascension System** —In this system the co-ordinates of a heavenly body are (i) the *declination* and (ii) the *right ascension* (R A) the celestial equator being the reference plane and the vernal equinox or the First Point of Aries ♈ being chosen as a reference point

In Fig 118, let O be the centre of the celestial sphere, P, the north pole, Y, the position of a heavenly body, PYY,

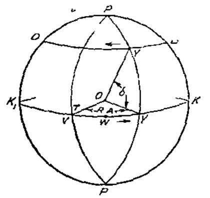


Fig 118

the meridian or the declination circle through Y, Pϒ, the declination circle through ϒ KWK$_1$ the celestial equator Then the position of a heavenly body Y is completely defined by (1) the angle Y$_1$OY or the great circle arc Y$_1$Y called the *declination* as before and (2) the angle ϒOY$_1$ (or ϒPY) or the arc ϒY$_1$ of the equator called the *right ascension* (R A) The right ascension of a celestial body (or the angle between the meridians through ϒ and Y) is measured eastwards from ϒ along the equator from 0° to 360° or in time units from 0h to 24h Now the angle KPY (or ZPY) or the arc KY$_1$ is the hour angle (H) of the heavenly body It is evident from the figure that Kϒ equals KY$_1$ + ϒY$_1$ But Kϒ is the hour angle of ϒ which is called the sidereal time (S T) Hence we can write

S T = westerly hour angle of Y + right ascension of Y

Here it may be noted that the direction in which the right ascension is measured is opposite to that in which the hour angle is measured In all cases the hour angle is measured westward from the observer s meridian

When ϒ is on the observer s meridian, the hour angle of ϒ is 0h, i e S T is 0h This instant is known as sidereal noon

The right ascension and declination of a star are constant This system of co-ordinates is therefore, the most convenient

for specifying the relative positions of the stars on the celestial sphere

The following rules regarding the azimuth and hour angle may be noted

(1) *When an observer is in the northern hemisphere* —

The azimuth of a star is measured from the *north* point to the east or to the west

(2) *When an observer is in the southern hemisphere* —

The azimuth of a star is measured from the *south* point to the east or to the west

(3) When a star is in the *western* hemisphere, its azimuth is west and its hour angle is between 0h and 12h and is given by the spherical angle ZPY (H) of the astronomical triangle ZPY (Fig 116) and vice versa

(4) When a star is in the *eastern* hemisphere, its azimuth is east and its hour angle is between *12h* and *24h*, and is given by 360° — the spherical angle ZPY (H_1) of the astronomical *triangle* ZPY (Fig 117) and *vice versa*

Circumpolar Stars

In Fig 119 let O be the position of the observer in latitude θ N Z and P the zenith and the north pole respectively, NWS, the horizon, EWE_1, the celestial equator, KK_1 and LL_1 the parallels of declination for the two stars Y and Y_1 respectively

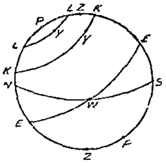


Fig 119

The stars which are always above the horizon and which do not therefore set are *called* the *circumpolar* stars They will appear to the observer to describe the circles about the pole P In order that a star should not set i e it should be circumpolar, its distance from the pole must be less than the latitude of the place of observation If δ = the declination of a star, and θ = the latitude of the place

Then the polar distance of the star Y $= PK_1 = 90° - \delta$.
PN = latitude $= \theta$.

$\therefore$ PK_1 must be less than PN, i e. $90° - \delta$ must be less than θ

or δ must be greater than $90° - \theta$ (co latitude)
or expressed in words, the declination of a star must be greater than the co-latitude

Culmination —When a star or other heavenly body crosses the observer s meridian, it is said to culminate or transit. In one revolution round the pole each star crosses the meridian twice, the two culminations or transits being distinguished as the upper culmination (or upper transit), and the lower culmination (or lower transit). A star is at the upper culmination as at K (Fig 119) when its altitude is maximum, and at the lower culmination as at K_1 when its altitude is minimum The upper culmination of a star may occur on the north side of the zenith as at L for the star Y_1, or on the south side of the zenith as at K for the star Y according to the following conditions.

Zenith distance of the star Y_1 at L (i e at upper culmination) $= ZL = ZP - PL = (90° - \theta) - (90° - \delta) = (\delta - \theta)$

Zenith distance of the star Y at K (i e at the upper culmination) $= ZK = PK - ZP = (90° - \delta) - (90 - \theta) = \theta - \delta$

Hence it follows that (i) when the declination of a star is greater than the latitude ($\delta > \theta$), the upper culmination occurs on the north side of the zenith i e between the pole (P) and the zenith (Z)

(ii) When the declination of a star is less than the latitude ($\delta < \theta$), the upper culmination occurs on the south side of the zenith (Z)

When $\delta = \theta$, the culmination of the star occurs in the zenith

The zenith distance of the star Y at the lower culmination $= ZK_1 = ZP + PK_1 = (90° - \theta) + (90 - \delta) = 180° - (\theta + \delta)$

When a star is at the upper culmination, its hour angle is $0h$,
" " " lower " " " is $12h$

Example 1.—Find the zenith distance at the upper culmination of the stars from the following data

(i) Lat $= 45° 30' N$
Declination $= 20° 15' N$

(ii) Lat. $58° 15' N$
Declination $= 18° 30' S$

(iii) Lat $= 35° 45' N$
Declination $= 64° 32' N$

(i) Since $\delta < \theta$, the upper culmination occurs on the south side of the zenith

Zenith distance of the star at the upper culmination
$= \theta - \delta = 45° 30 - 20° 15 = 25° 15'$

(ii) δ being less than θ, the upper culmination is on the south side of the zenith

Zenith distance of the star at the upper culmination
$= \theta - \delta = 58° 15 - (-18° 30) = 58° 15' + 18° 30 = 76° 45$

(iii) Since the star's declination (δ) is greater than $90° - \theta$, the star is circumpolar and as its declination is greater than latitude ($\delta > \theta$) its upper transit is on the north side of the zenith

Zenith distance of the star at the upper culmination
$= \delta - \theta = 64° 32 - 35° 45' = 28° 47'$

Example 2 —Find the zenith distance at the lower culmination of the following stars, given that (a) latitude $= 42° 15$ N and declination $= 50° 45$ N and (b) latitude $= 48° 17$ S and declination $= 62° 12'$ S

(a) Zenith distance at the lower culmination $= 180° - \theta - \delta$
$= 180° - 42° 15 - 50° 45$
$= 87°$

(b) „ „ „ culmination $= 180° - \theta - \delta$
$= 180° - 48° 17 - 62° 12$
$= 69° 31'$

Example 3 —The declination of a star is $48° 46'$ N and its upper transit is in the zenith of the place Find the altitude of the star at the lower transit

At the upper transit, the star is in the zenith

Polar distance of the star = co latitude

or $\quad 90 - \delta = 90° - \theta$

Hence $\quad \delta = \theta$

At the lower transit zenith distance of the star

$$= 180° - (\theta + \delta) = 180° - 2\delta$$
$$= 180° - 2(48° 46) = 82° 28'$$

Whence the altitude of the star $= 90° - z\,d = 90° - 82° 28$
$= 7° 32$

Example 4 —The altitudes of a star at the upper and lower culminations are 76° 23 and 20° 31 both culminations being on the north side of the zenith of the place Find the declination of the star and the latitude

Since the star s upper culmination is on the north side of zenith

the zenith distance of the star at the upper culmination
$= \delta - \theta = 90° -$ altitude

" , at the lower culmination
$= 180° - \theta - \delta - 90° -$ altitude

$$\delta - \theta - 90° - \text{alt} = 90° - 76° 23 = 13° 37 \quad (1)$$

and $180° - \theta - \delta = 90° \quad \text{alt} = 90° - 20° 31 - 69° 29$

or $\delta + \theta = 180° - 69° 29 = 110° 31 \quad (2)$

Whence $\delta = 62° 4$ N and $\theta - 48° 27$ N

The Astronomical Triangle

The spherical triangle ZPS (Fig 120) formed by joining the zenith (Z) the pole (P) and the heavenly body (S) (a star or the sun) by great circle arcs is called the *Astronomical triangle* Three of the six parts of the triangle being given the other three may be obtained by solving it

Let α = the altitude of the celestial body
δ = the declination of "
θ = the latitude of the observer

Then the side ZS = the co-altitude or zenith distance of the body $= 90° - \alpha$

the side ZP = the co-latitude of the observer $= 90° - \theta$

PS = the co declination or polar distance of the body $= 90° - \delta$

The angle at Z = SZP = the azimuth (A) of the body
The angle at P = ZPS = the hour angle (H) of the body
The angle at S = ZSP = the parallactic angle

The astronomical triangle is right angled at S i.e. the parallactic angle ZSP is a right angle when the heavenly body is at *elongation* i.e. at its greatest distance east or west of the meridian (Fig 121)

Fig 120

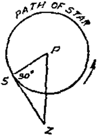


Fig 121

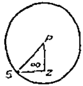


Fig 122

When the celestial body is on the *Prime vertical* of the observer the astronomical triangle is right angled at Z, i.e. the angle SZP is a right angle (Fig 122)

(1) The three sides of the astronomical triangle ZPS being known the angles A and H may be computed by means of the following formulæ

$$\cos A = \frac{\sin \delta}{\cos \alpha \cos \theta} - \tan \alpha \tan \theta \tag{13}$$

$$\text{or} \quad \tan \frac{A}{2} = \sqrt{\frac{\sin (s - ZS) \sin (s - ZP)}{\sin s \sin (s - PS)}} \tag{14}$$

in which $s = \frac{1}{2}(ZS + ZP + PS)$

$$\cos H = \frac{\sin \alpha}{\cos \delta \cos \theta} - \tan \delta \tan \theta \tag{15}$$

$$\tan \frac{H}{2} = \sqrt{\frac{\sin (s - ZP) \sin (s - PS)}{\sin s \sin (s - Z)}} \tag{16}$$

(2) When the astronomical triangle ZPS is right angled at S, the following formulæ may be used to calculate the angles A and H, and the altitude (α) of the heavenly body, when its declination and the latitude of the place of observation are known

$$\sin \alpha = \frac{\sin \theta}{\sin \delta} = \frac{\sin \text{latitude}}{\sin \text{declination}} \tag{17}$$

$$\sin A = \frac{\cos \delta}{\cos \theta} = \frac{\cos \text{declination}}{\cos \text{latitude}} \tag{18}$$

$$\cos H = \frac{\tan \theta}{\tan \delta} = \frac{\tan \text{latitude}}{\tan \text{declination}} \tag{19}$$

(3) When the astronomical triangle ZPS is right angled at Z, and the declination of the heavenly body and the latitude of the place of observation are known the altitude (α) and the hour angle (H) may be calculated from the following formulæ

$$\sin \alpha = \frac{\sin \delta}{\sin \theta} = \frac{\sin \text{declination}}{\sin \text{latitude}} \tag{20}$$

$$\cos H = \frac{\tan \delta}{\tan \theta} = \frac{\tan \text{declination}}{\tan \text{latitude}} \tag{21}$$

Example 1 —Determine the azimuth and altitude of a star from the following data —

(i) Latitude of the observer $= 46°$ N, (ii) hour angle $45° 45$ and (iii) declination $= +22°$

In the triangle ZPS
$$\begin{aligned} ZP &= 90° - \theta = 90° - 46° = 44° \\ PS &= 90° - \delta = 90° - 22° = 68° \\ ZPS = H &= 45° 45 \end{aligned}$$

Using the cosine rule we have

$$\begin{aligned} \cos ZS &= \cos ZP \cos PS + \sin ZP \sin PS \cos H \\ &= \cos 44° \cos 68° + \sin 44° \sin 68° \cos 45° 45 \\ &= 0.2695 + 0.4494 = 0.7198 \end{aligned}$$

$$ZS = 44° 2$$

Whence, the altitude of the star $= \alpha = 90° - ZS$
$$= 90° - (44° 2) = 45° 58$$

$$\cos A = \frac{\cos PS - \cos ZP \cos ZS}{\sin ZP \sin ZS} = \frac{\cos 68^\circ - \cos 44^\circ \cos 44^\circ 2'}{\sin 44^\circ \sin 44^\circ 2'}$$
$$= -0\ 2948.$$

Since $\cos A$ is negative, A lies between 90° and 180°

Hence $\cos (180° - A) = +0\ 2948.$

$$180° - A = 72° 51' \text{ or } A = 180° - (72° 51') = 107° 9' W$$

Example 2 —Find the azimuth and altitude of a star given the following —

(a) Latitude of the place = 48° N, (b) hour angle of the star = 21h 40 m, and (c) declination of the star = 18° 12' South

In the triangle ZPS, $ZP = 90° - \theta = 90° - 48° = 42°$
$$PS = 90° - \delta = 90° - (-18° 12') = 108° 12$$
$$\angle ZPS = 24h - (21h\ 40\,m) = 2h\ 20\,m = 35°$$

The angle SZP (A) and the side ZS may be computed by the cosine rule

$$\cos ZS = \cos ZP \cos PS + \sin ZP \sin PS \cos H$$
$$= \cos 42° \cos 108° 12' + \sin 42° \sin 108° 12' \cos 35°$$
$$= -0\ 2321 + 0\ 5207 = +0\ 2886$$

$$ZS = 73° 13$$

Whence, the altitude of the star $= \alpha = 90° - ZS$
$$= 90 - (73° 13') = 16° 47'$$

Now $\cos PS = \cos ZS \cos ZP + \sin ZS \sin ZP \cos A$.

$$\therefore \cos A = \frac{\cos 108° 12 - \cos 73° 13' \cos 42°}{\sin 73° 13 \sin 42°} = -0\ 8224$$

Since $\cos A$ is negative A lies between 90° and 180°

Hence $\cos (180° - A) = +0\ 8224$ or $180° - A = 34° 40$

$$A = 180° - 34° 40 = 145° 20$$

The azimuth of the star = A = 145° 20' E

Example 3 —Determine the hour angle and declination of a star from the following data ·

(i) Latitude of the place = 48° 30' N, (ii) azimuth of the star = 50° W, and (iii) altitude of the star = 28° 24'.

In the triangle ZPS, $ZP = 90° - \theta = 90° - 48° 80' = 41° 30$

$ZS = 90° - \alpha = 90° - 28° 24' = 61° 36'$

$SZP = 50°$

Using the cosine rule, we have

(i) $\cos PS = \cos 41° 30 \cos 61° 36 + \sin 41° 30 \sin 61° 36 \cos 50°$
$= 0\ 3563 + 0\ 3747 = 0\ 7310 \qquad PS = 43° 2'$

The declination of the star $= \delta = 90° - PS = 90° - 43° 2$
$= 46° 58$

(ii) $\cos H = \dfrac{\cos 61° 36 - \cos 41° 30 \cos 43° 2}{\sin 41° 30 \sin 43° 2} = -0\ 1590$

$\cos(180° - H) = +0\ 1590$ or $180° - H = 80° 51$

$H = 180° - 80° 51 = 99° 9$

Hence the hour angle of the star $= 99° 9 = 6h\ 36m\ 36s$

Example 4 —Find the declination and the hour angle of a star, given that the latitude of the place is 52° N the azimuth of the star, 135° 18 E and the zenith distance 65° 12

In the triangle ZPS $ZP = 90° - 52° = 38°$, $ZS = 65° 12$
$SZP = 135° 18$

(i) $\cos PS = \cos 65° 12 \cos 38° + \sin 65° 12 \sin 38° \cos 135° 18'$
$= 0\ 3306 - 0\ 3972 = -0\ 0666$

$180° - PS = 86° 11$ or $PS = 93° 49$

Hence the declination of the star $= -3° 49$ or $3° 49$ S

(ii) $\cos ZPS = \cos H_1 = \dfrac{\cos 65° 12 - \cos 38° \cos 93° 49}{\sin 38° \sin 93° 49}$

$= 0\ 7685$

$H_1 = 39° 47 = 2h\ 39\ m\ 8\ s$

Since the star s azimuth is east the hour angle of the star $= 360° - H_1$

$H = 360° - 39° 47 = 320° 13' = 21h\ 20m\ 52s$

or $= 24h - (2h\ 39\ m\ 8s) = 21h\ 20m\ 52s$

Example 5.—Calculate the sun's azimuth and hour angle at sunset at a place in latitude 55° N when its declination is (a) 22° N and (b) 16° S

(a) In the triangle ZPS, $ZP = 90° - 55° = 35°$; $ZS = 90°$
$PS = 90° - 22° = 68°$

The sun being on the horizon, $ZS = 90°$.

Using the cosine rule, we have

(i) $\cos PS = \cos ZP \cos ZS + \sin ZP \sin ZS \cos A$

But $ZS = 90°$ $\cos ZS = \cos 90° = 0$
$\sin ZS = \sin 90° = 1$

$$\cos A = \frac{\cos PS}{\sin ZP} = \frac{\cos 68°}{\sin 35°} = 0\ 6531 \quad . \quad A = 49°\ 14'$$

Azimuth of the sun at sunset $= 49°\ 14'$ West

(ii) $\cos ZS = \cos ZP \cos PS + \sin ZP \sin PS \cos H$

Since $\cos ZS = 0$, $\cos H = -\cot ZP \cot PS$
$= -\cot 35° \cot 68° = -0\ 5771$

or $\cos (180° - H) = +0\ 5771$

$180° - H = 54°\ 45'$ or $H = 180° - 54°\ 45' = 125°\ 15$

Whence the sun s hour angle at sunset $= 125°\ 15' = 8h\ \ 21m$

(b) In this case, $ZP = 35°$, $ZS = 90°$;
$PS = 90° - (-16°) = 106°$.

(i) As before $\cos A = \dfrac{\cos PS}{\sin ZP} = \dfrac{\cos 106°}{\sin 35°} = -\dfrac{\cos 74°}{\sin 35°}$
$= -0\ 4805$

or $\cos (180° - A) = +0\ 4805$

$\therefore$ $180° - A = 61°\ 17$ or $A = 180° - 61°\ 17 = 118°\ 43$

Azimuth of the sun at sunset $= 118°\ 43'$ West

(ii) $\cos H = -\cot ZP \cot PS = -\cot 35° \cot 106°$.
$= +\cot 35° \cot 74° = +0\ 4096$. $H = 65°\ 49$

The sun's hour angle at sunset $= 65°\ 49' = 4h\ \ 23m\ \ 16s$

Example 6—Calculate the sun's hour angle and azimuth at sun rise for a place in latitude 48° S when its declination is 18° N.

In the triangle SP_1Z, $ZP_1 = 90° - \theta = 90° - 48° = 42°$
The sun being on the horizon, $ZS = 90°$

$$P_1S = 90° + 18° - 108°$$

By the cosine rule we have

(i) $\cos ZS = \cos ZP_1 \cos P_1S + \sin ZP_1 \sin P_1S \cos H_1$
But $\cos ZS = \cos 90° = 0$

$$\cos H_1 = -\cot ZP_1 \cot P_1S = +\cot 42° \cot 72° = 0.3609$$

or $H_1 = 68° 51' = 4h\ 35m\ 24s$
Hour angle at sunrise $= 24 - (4h\ 35m\ 24s)$
$= 19h\ 24m\ 36s$

(ii) $\cos P_1S = \cos ZP_1 \cos ZS + \sin ZP_1 \sin ZS \cos A$
But $ZS = 90°$ $\quad \cos ZS = \cos 90° = 0,$
$\sin ZS = \sin 90° = 1$

$$\text{Whence, } \cos A = \frac{\cos P_1S}{\sin ZP_1} = \frac{\cos 108°}{\sin 42°} = -0.4618$$

or $\cos (180° - A) = +0.4618$
$180° - A = 62° 30$ or $A = 180° - 62° 30 = 117° 30$
Hence the sun's azimuth $= 117° 30'$ East

Example 7—A vertical wall, 4 m high, is built on level ground at a place in latitude 50° N and faces due East (a) How many hours will the face of the wall be exposed to the rays of the sun when the sun's declination is (i) 18°N and (ii) 18°S ?

(b) Find the width of the shadow normal to the wall at 11 a m in the first case

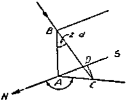


Fig 193

(a) (i) The time of exposure of the wall to the rays of the sun is given by the hour angle of the sun Now in the triangle ZPS,

$ZP = 90° - 50° = 40°$, $ZS = 90°$, and $PS = 90° - 18° = 72°$

Now $\cos ZS = \cos ZP \cos PS + \sin ZP \sin PS \cos H$

$$\cos 90° = \cos 40° \cos 72° + \sin 40° \sin 72° \cos H$$

or $\cos H = -\cot 40° \cot 72°$

Since cos H is negative, H lies between 90° and 180°

$$\cos (180° - H) = +\cot 40° \cot 72°$$

Hence $180° - H = 67° 13'$ or $H = 112° 47' = 7h\ 30m\ 48s$

(ii) The sun's declination being 18° S, $PS = 90° - (-18°) = 108°$, $ZS = 90°$, and $ZP = 90° - 50° = 40°$

$$\cos H = -\cot 40° \cot 108° = +\cot 40° \cot 72°$$

Hence $H = 67° 13' = 4h\ 28m\ 52s$

(b) In order to find the width of the shadow normal to the wall, we must know the sun's azimuth and its zenith distance i e A and ZS in the $\triangle ZPS$ At 11 a m the sun's hour angle is 15°

In the $\triangle ZPS$ $ZP = 40°$, $PS = 72°$, and $H = 15°$

$$\cos ZS = \cos 40° \cos 72° + \sin 40° \sin 72° \cos 15°$$

Hence $ZS = 34° 11'$

Now $\cos A = \dfrac{\cos PS - \cos ZP \cos ZS}{\sin ZP \sin ZS} = \dfrac{\cos 72° - \cos 40° \cos 34° 11'}{\sin 40° \sin 34° 11'}$,

or $\log \cos A = -1\ 9540$

$180° - A = 25° 54'$ or $A = 180° - 25° 54' = 154° 6'$

Now the length of the shadow AC (Fig 123)
= height of the wall × tan zenith distance = 4 tan 34°11′ = 2 717 m

Width of the shadow normal to the wall = CD = 2 717 sin A
= 2 717 sin 25° 54′ = 1 19 m

where A is the supplement of the azimuth A = 25° 54′

Time —The measurement of time is based upon the apparent motion of heavenly bodies caused by the earth's rotation on its axis

Since the earth rotates on its axis from *west* to *east*, all heavenly bodies (the fixed stars and the sun) appear to revolve from *east to west* (in a clock wise direction) around the earth and therefore they appear to cross the observer's meridian twice each day

The earth also moves in an elliptic orbit round the sun and makes one complete revolution in one year. Therefore, the sun appears to move relatively to the stars from west to east and to make a complete circuit of the heavens in one year.

There are four kinds of time viz. (1) sidereal time, (2) apparent solar time, (3) mean solar time and (4) standard time. The first two kinds of time are convenient to the astronomer, while the latter two are convenient for our everyday affairs.

(1) **Sidereal Time.**—Sidereal time is the time when its measurement is based upon the diurnal motion of a star or the vernal equinox. The time interval between two successive upper transits of the vernal equinox also called the First Point of Aries (♈) over the same meridian is called a sidereal day.

This unit of time is most convenient for astronomical purposes as the whole system of the stars revolves around the polar axis of the celestial sphere with absolute uniformity from east to west and is, therefore, much used by the astronomers. The sidereal day is divided into 24 hours, each hour subdivided into 60 minutes, and each minute into 60 seconds. The sidereal day begins at the instant of the upper transit of the First Point of Aries so that the sidereal time is $0h$ at its upper transit and $24h$ at the next upper transit.

Sidereal time at any instant is therefore equal to the hour angle of the First Point of Aries. The right ascension of the meridian of a place is known as local sidereal time (L.S.T.). It is the time interval which has elapsed since the transit of the First Point of Aries over the meridian of the place. Since the hour angle of a star is the sidereal time that has elapsed since its transit, we have

Local sidereal time (L.S.T.) = R. A. of a star + westerly hour angle of a star (22)

If this sum is greater than 24 hours, deduct 24 hours, while if it is negative, add 24 hours.

Also, L. S. T. = R.A. of mean sun (R.A.M.S.) ± 12 h + mean time at the place (22a)

When the star is at its upper transit or culmination, its hour angle is zero and the sidereal time is equal to its right ascension.

sidereal time of transit of a star = R A of a star (23)

(2) **Apparent Solar Time** —Apparent solar time is the time when its measurement is based on daily motion of the sun The time interval between two successive lower transits of the centre of the sun over the same meridian is called an apparent solar day It is divided into 24 hours, each hour into 60 minutes, and each minute into 60 seconds The apparent solar time is given by the sun dial Since the sun's apparent daily path is in the ecliptic, (a great circle inclined to the equator at an angle of 23° 27') and the sun does not move at a uniform rate along the ecliptic the apparent solar day is not of uniform length and consequently it cannot be recorded by a clock having a uniform rate

(3) **Mean Solar Time** —In order to obviate the variation in apparent solar time a fictitious body called the mean sun is introduced by the astronomers The mean sun is an imaginary point and is assumed to move at a uniform rate along the equator so as to make a solar day of uniform length the motion of the mean sun being the average of that of the true sun in right ascension It is supposed to start from the vernal equinox at the same time as the true sun and to return to the vernal equinox with the true sun Time when measured by the diurnal motion of the mean sun is called the mean solar time or simply mean time The mean solar day is the average of all the apparent solar days of the year The time which is in common use by the people is the mean solar time or civil time It is the time kept by our clocks and watches The time interval between two successive lower transits of the mean sun over the same meridian is called a mean solar day, which is also known as a civil day It is divided into 24 hours each hour into 60 minutes and each minute into 60 seconds

There are two systems of reckoning mean solar time, viz (i) civil time and (ii) astronomical time Prior to Dec 31 1924 the astronomical day was considered to begin at noon Since Jan 1 1925 both the civil day and the astronomical day begin at 0h midnight But the civil day is divided into two periods one from midnight to noon and the other from noon to midnight

so that the time of an event occurring before mean noon is denoted by the letters A M (antemeridian), and the time of event occurring after mean noon by the letters P M (post meridian), while the astronomical day is divided continuously from 0h to 24h Civil time may be converted into astronomical time and vice versa by the following rules —

(1) (a) If the civil time is A M the astronomical time is the same as the civil time

e g civil time 6 A M corresponds to astronomical time 6h

(b) If the civil time is P M, the astronomical time = civil time + 12h

e g civil time 8 P M is equivalent to astronomical time 20h In both cases the date remains unchanged

(2) (a) If the astronomical time is less than 12h, the civil time is the same as the astronomical time and is denoted by the letters A M

e g astronomical time 9 h corresponds to civil time 9 A M

(b) If the astronomical time is greater than 12h, the civil time = astronomical time − 12h and is denoted by the letters P M

e g astronomical time 22h is equivalent to civil time 10 P M

It is well to note the following relations between the hour angle, right ascension, and time

At any instant,

the apparent solar time = the hour angle of the sun + 12h (24)

the mean solar time = „ „ „ of the mean sun + 12h (25)

Local sidereal time (L S T) = R A of the sun + hour angle of the sun (26)

„ „ „ = R A of the mean sun + hour angle of the mean sun (27)

The instant at which the sun crosses the meridian of any place is called the local apparent noon (L A N) while the instant

at which the mean sun crosses the meridian of any place is called the local mean noon (L M N)

The hour angle of the sun being zero at its upper transit

Sidereal time of Apparent Noon = R A of the sun (28)

Similarly the hour angle of the mean sun being zero at its upper transit

Sidereal time of Mean Noon = R A of the mean sun (29)

By local mean time is meant the mean time at the place of observation (for the meridian of the observer) All places on the same meridian have the same local time The mean time for any other meridian is denoted by the name e g Greenwich Mean Time

The earth moves uniformly on its axis from west to east and this causes the sun to appear to move from east to west and to cross the meridians in succession Consequently, the farther *east* a place is situated the sooner will the sun cross the meridian and the later will be the local time

Equation of Time —The difference between apparent solar time and mean solar time at any instant is known as the *equation of time* (E T) Formerly apparent time was determined by solar observation and was reduced to mean time by means of the equation of time But now mean time is obtained by first determining sidereal time by stellar observations and then converting it or directly from wireless signals Hence the values of equation of time at 0h (midnight) at Greenwich are tabulated in the Nautical Almanac for every day of the year in the sense apparent—mean and are to be added algebraically to mean time to give apparent time and vice versa The Greenwich mean time (G M T) of apparent noon i e the instant at which the sun transits at Greenwich is also given

The values of equation of time are sometimes prefixed with the plus sign (or specified as the sun after clock) or with the minus sign (or specified as the sun before clock) indicating that they are to be added to or subtracted from apparent time to give mean time

Equation of time = R A of the mean sun − R A of the sun

The value of the equation of time (Fig 124) varies from 0 to about 16 minutes at different seasons of the year It vanishes four times during the year, on or about April 15, June 14, September 1, and December 25 On these dates the true sun and the mean sun are on the same meridian and apparent time and mean time are the same

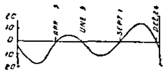


Fig 124 Variation of Equation of Time

Note —The difference between mean time and apparent time is due to two causes —(1) The earth moves round the sun in an ellipse and not in a circle Consequently the motion of the earth is not uniform and varies with its distance from the sun.

(2) Since the real sun moves along the ecliptic, uniform motion along the ecliptic does not represent uniform motion in the right ascension, and hence does not correspond to uniform motion of the mean sun along the equator

At apparent noon, i e when the real sun is on the meridian, the apparent time is zero, and, therefore, equation of time = mean time of apparent noon

Now L S T = R A of the mean sun + hour angle of the mean sun (1)

" = R A of the sun + hour angle of the sun (2)

Subtracting the second equation from the first, we get

R A of the mean sun − R A of the sun = hour angle of the sun − hour angle of the mean sun

E T = hour angle of the sun − hour angle of the mean sun (24)

= apparent time − mean time (24a)

Whence, apparent time = mean time + E T (25)

Summary —The following points may be noted in connection with different kinds of time —

(1) Apparent solar time (or apparent time) is measured from 0h to 24h from the lower transit of the real sun The instant when the real sun crosses the meridian at lower transit

is known as Apparent Midnight while the instant when it crosses the meridian at upper transit is known as Apparent Noon

(2) Mean solar time (or mean time) is measured from 0h to24 h from the lower transit of the mean sun The instant when *the mean sun crosses the meridian at lower transit is called* Mean Midnight while the instant when it crosses the meridian at upper transit is called Mean Noon

(3) The apparent time for the meridian of the place of observation is known local apparent time (L A T) Similarly the apparent time for Greenwich meridian is known as Greenwich apparent time (G A T)

(4) The mean time for the meridian of the place of observation is called local mean time (L M T) while the mean time for Greenwich meridian is called Greenwich mean time (G M T)

(5) The difference in local time between two places is equal to the difference in longitude between the two places expressed in hours minutes and seconds This relation applies to all kinds of time (sidereal apparent solar or mean solar)

(6) Sidereal time is measured from 0 h. to 24 h from the upper transit of the First Point of Aries But mean solar time is measured from the lower transit of the mean sun

(7) Westerly hour angle of a star or the sun is considered as positive while its easterly hour angle as negative

(8) It is well to note here the relation between the hour angle of a heavenly body for the Greenwich meridian and any other meridian

Hour angle of a heavenly body for the Greenwich meridian = hour angle of a heavenly body for any other meridian ± longitude the hour angle and longitude being expressed in the hour system Use plus sign if the longitude is west and minus sign if the longitude is east

This relation is true whether the heavenly body is the mean sun the sun the vernal equinox or the star

Since the local time is 0h at the instant of local midnight Greenwich Time of Local Midnight — Longitude in time

(4) Standard Time —In order to avoid confusion arising from the use of different local mean times by the people it is

necessary to adopt the mean time on a particular meridian as the standard time for the whole of a country This meridian is known as the *Standard Meridian* and usually lies an exact number of hours from Greenwich The mean time associated with this meridian is called the *Standard Time* which is kept by all watches and clocks throughout the country The longitude of the standard meridian adopted in India is 82° 30 E or 5h 30 m L Greenwich meridian is the standard meridian for Great Britain

It is evident that the difference between the local mean time at any place and the standard time is due to the difference of longitude between the given place and the standard meridian The standard time may, therefore be converted to the local mean time and *vice versa* by the relation

Standard Time = L M T ± difference of longitude in time between the given place and the standard meridian Use *plus* sign, if the place is to the *west* of the *standard meridian*, and *minus* sign if it is to the *east*

Note —If the place is to the *east* of standard meridian, local mean time is *later* than standard time and if it is to the *west* of standard meridian local mean time is *earlier*

Example 1 —Find the local mean time at a place in longitude 90° 40 E when the standard time is 10h 32 m 34 s. and the standard meridian 82° 30 E

Difference of longitude = 90° 40 − 82° 30 = 8° 10′ = 32 m 30 s
Since the place is to the east of the standard meridian we have
10h 32 m 34 s = L M T − 32 m 40 s

L M T = 11 h 5 m 14 s

Example 2 —In India the standard meridian is 82° 30 E Find the standard time corresponding to local mean time 7 h. 23 m 32 s at a place in longitude 68° 36 E

Difference of longitude = 82°30 − 68°36 = 13°54 = 55 m 36 s

Te place being west of the standard meridian

Standard time = L M T + difference of longitude
= 7 h 23 m 42 s + 55 m 36 s
= 8 h 19 m 18 s

Alternative Method —(i) Find the G M T corresponding to the given local mean time from the relation

G M T = L M T ∓ longitude in time $\begin{bmatrix}\text{east}\\ \text{west}\end{bmatrix}$

(ii) Convert the G M T so obtained to the standard time by the relation

Standard mean time = G M T ± longitude in time of standard meridian $\begin{bmatrix}\text{east}\\ \hline \text{west}\end{bmatrix}$

Example 3 —Data The same as in example 1

10 h 32 m 34 s = G M T + 5 h 30 m

G M T = 5 h 2 m 34 s

The longitude of the place being 6 h 2 m 40 s E we have

5 h 2 m 34 s = L M T − 6 h 2 m 40 s

L M T = 11 h 5 m 14 s

Example 4 —Data the same as in example 2

G M T = 7 h 23 m 42 s — (4 h 34 m 24 s)

= 2 h 49 m 18 s

Standard mean time = 2 h 49 m 18 s + 5 h 30 m

= 8 h 19 m 18 s

The time in which the earth makes a complete revolution in its orbit is called the year

A *Tropical year* which is the time interval between two successive vernal equinoxes contains 366 2422 sidereal days The earth actually makes 366 2422 revolutions on its axis in one tropical year and during this period it makes one complete revolution round the sun Therefore the sun appears to make one upper transit less than the First Point of Aries (♈) There are therefore 365 2422 mean solar days in one tropical year

Hence we have

365 2422 mean solar days = 366 2422 sidereal days

Conversion of Mean Solar Time into Sidereal Time and vice versa —When the mean time is to be converted into sidereal time the following relation may be used

365 2422 mean solar days = 366 2422 sidereal days

$$1 \text{ mean solar day} = 1 + \frac{1}{365\ 2422} \text{ sidereal day}$$

$$= 24 \text{ h } 3 \text{ m } 56\ 56 \text{ s sidereal time}$$

(The mean solar day is 3 m 56 56 s longer than the sidereal day)

Whence,	1 h mean solar time	= 1 h + 9 8565 s	sidereal time
	1 m „ „	= 1 m + 0 1643 s	„ „
	1 s „ „	= 1 s + 0 0027 s	„ „

The correction 9 8565 s per hour of mean time which is to be added to the mean time to obtain the corresponding sidereal time is known as *acceleration*

The calculation work is facilitated by the use of Conversion tables given in the Nautical Almanac

Example 5 —To find the sidereal time interval corresponding to 6 h 12 m 30 s M T

Acceleration at 9 8565 s per hour		h m s
for 6 h = 6 × 9 8565 = 59 139 s	M T =	6 12 30
12 m = 12 × 0 1643 = 1 972 s	Acceleration =	1 1 192
30 s = 30 × 0 0027 = 0 081 s		
Total = 61 192 s	Sidereal =	6 13 31 192
= 1 m 1 192 s	time interval	

The following relation may be used to convert sidereal time into mean time

$$1 \text{ sidereal day} = \frac{365\ 2422}{366\ 2422} \text{ mean solar day}$$

$$= 1 - \frac{1}{366\ 2422} \text{ mean solar day}$$

1 sidereal day	= 23 h 56 m 4 09 s	mean solar time	
1 h sidereal time	= 1 h − 9 8296 s	„	„
1 m „ „	= 1 m − 0 1638 s	„	„
1 s „ „	= 1 s − 0 0027 s	„	„

The correction 9 8296 s per hour of sidereal time, which is to be subtracted from sidereal time to obtain the corresponding mean time is called *retardation*

Example 6 —To find the mean time interval corresponding to the sidereal time interval of 8 h 24 m. 36 s

Retardation at 9 8296 s per hour,		h m s
for 8h = 8×9 8296 — 78 637 s	Sidereal Interval	=8 24 36
24 m = 24×0 1638 = 3 931 s		
36 s = 36×0 0027 = 0 097 s	Retardation	= — 1 22 665
Total = 82 665 s		
= 1 m 22 665 s	M T Interval	= 8 23 13 335

Abbreviations and Symbols

The following abbreviations and symbols are used in the following discussion

G M T = Greenwich Mean Time (sometimes called Universal Time (U T)
L M T = Local Mean Time
G A T = Greenwich Apparent Time
L A T = Local Apparent Time
G A N = Greenwich Apparent Noon
L A N = Local Apparent Noon
G M N = Greenwich Mean Noon.
L M N = Local Mean Noon
G M M = Greenwich Mean Midnight, i e 0h
L M M = Local Mean Midnight, i e 0h
G S T = Greenwich Sidereal Time
L S T = Local Sidereal Time

S I = Sidereal Interval	E T = Equation of Time.
R A = Right Ascension	♈ = First Point of Aries
R O = Referring object	H = Hour angle.
R M = Reference mark	δ = Declination
α = Altitude	θ = Latitude
A = Azimuth	ϕ = Longitude
z = Zenith distance (= d)	N A = Nautical Almanac

The astronomical method of writing dates is year, month, day, and hour, minute, and second e g 1957 Jan 7d 8h 10m 42 s

Conversion of Degrees into Hours —Longitudes and hour angles are expressed in degrees as well as in hours The degrees may be converted into hours or vice versa by the following relation

Since the earth rotates through 360° in 24 hours, 360 degrees correspond to 24 hours Hence we have

$$\begin{array}{lll} 15^\circ = 1\ h & 1' = 4\ s & 1\ h = 15^\circ \\ 1^\circ = 4\ m & 15'' = 1\ s & 1\ m = 15' \\ 15' = 1\ m & & 1\ s = 15'' \end{array}$$

Example 1 —Express the following angles in hours, minutes, and seconds

(a) 78° 43′ 45″, (b) 10° 16′ 14″, (c) 25° 38′ 48″

$$\begin{array}{llll} & \text{h m s} & & \text{h m s} \\ \text{(a) } 78^\circ = \frac{78}{15}\text{h.} & = 5\ 12\ 0 & \text{(b) } 10^\circ = \frac{10}{15}\text{h} & = 0\ 40\ 0 \\ 43' = \frac{43}{15}\text{m} & = 0\ 2\ 52 & 16' = \frac{16}{15}\text{m} & = 0\ 1\ 4 \\ 45'' = \frac{45}{15}\text{s} & = 0\ 0\ 3 & 14'' = \frac{14}{15}\text{s} & = 0\ 0\ 0.93 \\ \hline \text{Time corresponding to the given angle} & = 5\ 14\ 55 & & = 0\ 41\ 4.93 \end{array}$$

$$\begin{array}{ll} & \text{h m s} \\ \text{(c) } 25^\circ = \frac{25}{15}\text{h} & = 1\ 40\ 0 \\ 38' = \frac{38}{15}\text{m} & = 0\ 2\ 32 \\ 48'' = \frac{48}{15}\text{s} & = 0\ 0\ 3.2 \\ \hline \text{Time corresponding to the given angle} & = 1\ 42\ 35.2 \end{array}$$

Example 2 —Express the following hours etc into degrees minutes, and seconds

(i) 6 h 42 m 34 s, (ii) 14 h 24 m 22 s

$$\begin{array}{lll} \text{(i) } 6\ h & = 6 \times 15^\circ & = 90^\circ\ 0\ 0'' \\ 42\ m & = 42 \times 15' & = 10\ 30\ 0 \\ 34\ s & = 34 \times 15'' & = 0\ 8\ 30 \\ \hline \text{Angle corresponding to the given time} & & = 100^\circ\ 38'\ 30'' \end{array}$$

(ii)	14 h $= 14 \times 15° =$	210°	0′	0″
	24 m $= 24 \times 15' =$	6	0	0
	22 s $= 22 \times 15'' =$	0	5	30
Angle corresponding to the given time	$=$	216°	5	30″.

Conversion of Local Time into Greenwich Time and vice versa —The following relationships may be used in converting Local time into Greenwich time, and vice versa If the civil time is given, it should be expressed in astronomical reckoning

$$\text{L M T} = \text{G M T} \pm \text{longitude of the place in time} \begin{Bmatrix} \text{E} \\ \overline{\text{W}} \end{Bmatrix} \quad (30)$$

$$\text{L A T} = \text{G A T} \pm \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \quad (31)$$

$$\text{L S T} = \text{G S T} \pm \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \quad (32)$$

Use *plus* sign if the longitude of the place is *East* and *minus* sign if it is *West*

$$\text{G M T} = \text{L M T} \mp \text{longitude of the place in time} \begin{Bmatrix} \text{E} \\ \overline{\text{W}} \end{Bmatrix}$$

$$\text{G A T} = \text{L. A T} \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"}$$

$$\text{G S T} = \text{L S T} \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"}$$

Use *minus* sign if the longitude of the place is East and *plus* sign if it is West

Note —If the sum is greater than 24 hours, deduct 24 hours and add one day to the date, if the sum is negative, add 24 hours and deduct one day from the date

Example 1 —Find the G M T corresponding to the following L M T

(a) 8 h 45 m 15 s A M at a place in longitude 48° 32′ W

(b) 3 h 32 m 20 s P M " " , 62° 45′ 20″ E

(i) G M T = L M T + long in time, since the longitude is west

Longitude 48° 32 W

	h m s.		h m s
48° =	3 12 0	L M T =	8 45 15
32′ =	0 2 8	Add long in time =	3 14 8 (+ve)
Long. in time =	3 14 8	Corresponding G M T =	11 59 23

(b) G M T = L M T − long in time since the longitude is east

L M T = 3 h 32 m 20 s P M − 15 h 32 m 20 s astronomical time

Long = 62° 45′ 20″ E			h m s
h m s		L M T	= 15 32 20
62° = 4 8 0		Deduct long in time	= 4 11 1 33 (−ve)
45′ = 0 3 0		Corresponding G M T	= 11 21 18 67
20″ = 0 0 1 33			
long in time = 4 11 1 33			

Example 2 —G C T is 6 h 45 m p m on April 15 1924 Find the L M T at the places the longitudes of which are (i) 77° 20′ E (ii) 85° 35′ 36″ W and (iii) 105° 45′ 20″ E

(i) L M T = G M T + long in time

Now G C T 6h 45 m p m = 18 h 45 m in astronomical reckoning = G M T

Long = 77° 20′ E		h m s	
= 5 h 9 m 20 s	G M T	= 18 45 0	
	Add long in time	= 5 9 20	(+ve)
	L M T	= 23 54 20	April 15
or	L C T	= 11 54 20 p m	

(ii) L M T = G M T − long in time

Long = 85° 35′ 36″ W		h m s	
= 5 h 42 m 22 4 s	G M T	= 18 45 0	
	Deduct long in time	= 5 42 22 4	(−ve)
	L M T	= 13 2 37 6	April 15
	L C T	= 1 2 37 6 p m	

(iii) L M T = G M T + long in time

Long = 105° 45′ 20″ E		h m s	
= 7 h 3 m 1 33 s	G M T	= 18 45 0	
	Add long in time	= 7 3 1 33	
	L M T	= 25 48 1 33	
		= 1 48 1 33	April 16
	L C T	= 1 48 1 33 a m	

Example 3 —Find the local apparent time of an observation at a place in longitude 80° 12′ E corresponding to local mean time 11 h 25 m 40 s, the equation of time at G M N being 3 m 6 52 s subtractive from apparent time and decreasing 0 27 s per hour

Long = 80° 12 E
= 5 h 20 m 48 s

		h m s
	L M T of observation	= 11 25 40
	Deduct long in time	= 5 20 48 (−ve)
	G M T of observation	= 6 4 52

Mean time interval before G M N = 12 h − (6h 4 m 52 s)
= 5 h 55 m 8 s
= 5 919 h

Increase for 5 919 h at 0 27 s per hour = (0 27 × 5 919)
= 1 598 s

	m s	
E T at G M N	= 3 6 52	
Add increase	= 1 60	(+ve)
E T at observation	= 3 8 12	

	h m s	
G M T of observation	= 6 4 52	
Add E T	= 3 8 12	(+ve)
G A T of observation	= 6 8 0 12	
Add long in time	= 5 20 48	(+ve)
L A T of observation	= 11 28 48 12	

Example 4 —Find the L M T of observation at a place from the following data

L A T of observation = 14 h 20 m 42 s on July 21, 1939

E T at G M N on that date = 6 m 12 32 s additive to apparent time and increasing at 0 14 s per hour

Longitude of the place = 17° 15′ W

Longitude 17° 15′ W = 1 h 9 m

		h m s
	L A T of observation	= 14 20 42
	Add long in time	= 1 9 (+ve)
E T at G M N = 6 m 12 32 s	G A T of obs	= 15 29 42
M T interval from G M N	Add E T	= + 6 12 81
= 3 h 29 m 42 s = 3 495 h		
Increase for 3 495 h at 0 14 s	G M T of obs	= 15 35 54 81

per hour $= 0\ 14 \times 3\ 495$
$= 0\ 49$ s

E T at observation
$= 6$ m $12\ 81$ s

Deduct long
in time $= 1\ \ 9$

L M T of obs $= 14\ 26\ 54\ 81$

Conversion of L S T to L M T and vice versa —

In converting local sidereal time to local mean time and local mean time to sidereal time the following rules may be used

Rule 1 —

L S T of L M N (or L M M) = G S T of G M N (or G M M) $\pm$ 9 86 s per hour of longitude $\left[\frac{W}{E}\right]$

Use *plus* sign when the longitude is *west* and *minus* sign when it is *east* The quantity 9 86 s per hour of longitude is called retardation if minus and acceleration if plus

Rule 2 —

L S T = L S T of L M N (or L M M) + S I from L M N (or L M M)

Case I —Conversion of L S T to L M T

In converting local sidereal time to local mean time, the following procedure may be adopted

Data —(i) L S T (ii) G S T of G M N (or G M M) and (iii) longitude

(1) Express the longitude in degrees etc in hours etc

(2) Find L S T of L M N (or L M M) by the rule 1

(3) Using the rule 2 obtain the S I since L M N or L M M by subtracting L S T of L M N (or L M M) from the given L S T

(4) Convert S I into mean time units by deducting 9 8296 s per hour of S I thus obtaining the local mean time (L M T)

Case II —Conversion of L M T to L S T

The procedure in converting the given local mean time into local sidereal time is as follows the data consisting of (i) L M T, (ii) G S T of G M N (or G M M) and (iii) longitude

(1) Convert the longitude in degrees, etc into hours, etc

(2) Using the rule 1, obtain L S T of L M N (or L M M)

(3) Convert the given L M T to sidereal units by adding 9 8565 s per hour of L M T, thus obtaining the sidereal interval (S I) from L M N (or L M M)

(4) Find the L S T by adding the S I thus obtained to the L S T of L M N (or L M M) (Rule 2)

Note —If the standard time is given, find the corresponding local mean time and then convert it to local sidereal time

Alternative Method of Conversion of L S T into L M T and vice versa —When the G M T of transit of the First Point of Aries (♈), i e the *mean time of* 0 h sidereal time at Greenwich is given the mean time being reckoned from the previous midnight, the following procedure may be adopted in converting L S T into L M T, and vice versa

Rule —L M T of transit of ♈ = M T of transit of ♈ at Greenwich $\pm$ 9 83 sec per hour of longitude $\left\{\begin{matrix}\text{East}\\ \overline{\text{West}}\end{matrix}\right\}$.

Use *plus* sign when the longitude is *east*, and *minus* sign when it is *west*

Case 1 —Conversion of L S T into L M T

Data —(i) L S T (ii) G M T of transit of ♈, and (iii) longitude

(1) Express the given longitude in hours, minutes, and seconds

(2) Find the local mean time (L M T) of transit of ♈ by the above rule

(3) Find the mean time interval corresponding to the given local sidereal time by deducting 9 83 s per hour of the sidereal time

The result represents the mean time interval since transit of ♈.

(4) Find the local mean time (L M T) by adding the two results as obtained in steps 2 and 3

Case 2 —Conversion of L. M T into L S T

Data —(i) L M T (ii) G M T of transit of ♈, and (iii) longitude

(1) Express the given longitude in hours minutes and seconds

(2) Find the local mean time (L M T) of transit of ♈ by the above rule

(3) Deduct the local mean time of transit of ♈ from the given local mean time (L M T) The difference gives the mean time interval since the transit of ♈

(4) Convert this mean time interval into sidereal time by adding 9 8565 sec per hour The result gives the L S T corresponding to the given L M T

Conversion of L M T into L A T

Rule —(1) Find the G M T corresponding to the given instant of L M T (2) Interpolate the value of equation of time (E T) for the G M T found in (1) from the values given for 0 h G M T (mean midnight) in the N A (3) Add the value so obtained algebraically to the given L M T The result gives the required L A T

Note —Apparent time − mean time $= \pm$ E T

Conversion of L A T into L M T

Rule —(1) Find the G A T corresponding to the given instant of L A T (2) Obtain the G M T by subtracting E T at 0 h from the G A T (3) Interpolate the value of E T for the G M T found in (2) from the values given for 0 h G M T (mean midnight) in the N A (4) Add the value so obtained algebraically to the given L A T The result gives the required L M T

Alternative method —The N A now gives the values of the G M T of transit of the sun at Greenwich (i e the G M T of apparent noon) The difference between the G M T of apparent noon and 12 h gives the value of the equation of time at apparent noon

Hence we use the following equation to find the L M T
L M T = L A T + G M T of transit at Greenwich corrected or G A T instant − 12 h

Rule —(1) Find the G A T corresponding to the given L A T (2) Interpolate the value of G M T of transit at Greenwich to the G A T found in (1) (3) Add the value so obtained to the given L A T and subtract 12 h from the sum The result gives the required L M T

To Find the L M T of Local Apparent Noon —

Rule —(1) Obtain the G M T of transit at Greenwich from the N A (2) Interpolate it to the given longitude in time The result gives the required L M T of local apparent noon

To Find the L M T of Local Sidereal 0 h —

Rule —(1) Obtain the G M T of transit of the First Point of Aries (♈) for the date from the N A (2) Calculate the correction at the rate of 9 8296 s per hour of longitude (3) Add this correction to or subtract it from the G M T obtained in (1) according as the longitude is east or west The result represents the L M T of local sidereal 0 h

To Find the L M T of Transit of a Star —

Since the hour angle of a star is zero at its upper transit L S T is equal to its right ascension (R A)

Rule —(1) Obtain the R A of the star from the N A (2) Convert L S T to L M T by any of the two methods described on pages 279 and 280

The hour angle of the star at its lower transit is 12 sidereal hours

To Find the L M T of Elongation of a Star —

Rule —(1) First compute the hour angle of the star at elongation (2) Find the L S T of elongation by the relation L S T = R A ± hour angle, using plus sign for west elongation and minus sign for east elongation the result being increased or decreased by 24 h if required
(3) Convert this L S T to L M T as already explained

Exmaple 1 —Find the L M T from the following data — (i) L S T = 18 h 46 m 12 s, (ii) G S T of G M N = 7 h 32 m 28 s (iii) longitude = 88° 54 30" W

		h m. s
Longitude 88° 54′ 30″ W.	G. S. T of G M N	= 7 32 28
= 5 h 55 m 38 s	Acceleration	= + 58 42
Acceleration for long W	L S T of L. M N	= 7 33 26 42
at 9 856 s per hour of long	L S T	= 18 40 12
= 58 42 s	Deduct L S T of L M N	= 7 33 26 42
Converting to M. T at	S I from L M N	= 11 12 45 58
9 8296 s per hour of S I	Retardation	= − 1 50 22
for 11 h, 11 × 9 8296 = 108 126 s	M T	= 11 10 55 36

for 12m, $\frac{12}{60}$ × 8 9296 = 1 966 s Interval from L M N L M T

for 45 58 s, $\frac{45\ 58}{3600}$ × 9 8296 = 0 124 s = 11 h 10m 55 36 s p m

Total = 110 22 s.

Retardation 1 m 50 22 s

Example 2 —Find the L S T at a place in longitude 72° 10′ E at 8 h. 40 m p m, the G S T of G M N being 6 h 42 m 32 s

Long 72° 10 E

h m s		h m s
72° = 4 48 0	G S T of G M N	= 6 42 32
10 = 40	Retardation	= − 47 42
Long 4 48 40 E.	L S T of L M N	= 6 41 44· 58

Retardation at 9 856 s per hour of long

for 4 h, 4 × 9 856 = 39 424 s

,, 48 m, $\frac{48}{60}$ × 9 856 = 7 885 s

,, 40 s, $\frac{40}{3600}$ × 9 856 = 0 110 s

sum = 47 419 = 47 42 s

C M T = 8 h. 40 m p m

M. T. Interval from L. M N. = 8 h 40 m.

Difference of two longitudes = (5 h 30 m) − (4 h 32 m) = 58 m

Converting L M T to sidereal time

Acceleration at 9 857 s per hour of L M T = 1 m 43 82 s

since L M M

Correcting for longitude 4 h 32 m E

Retardation at 9 857 s per hour of longitude = 44 68 s

	h m s
Standard time	= 11 30 0
Deduct difference	= − 58
L M T	= 10 32 0
Acceleration	= + 1 43 82
S I	= 10 33 43 82
G S T of G M M	= 9 38 48 66
Retardation	= − 44 68
L S T of L M M	= 9 38 3 98
Add S I	= 10 33 43 82
L S T	= 20 11 47 8

We shall check the result by the following alternative method

Converting G M T to sidereal units

Acceleration at 9 857 s per hour of G M T = 59 14 s

	h m s
Standard time	= 11 30 0
Deduct longitude (E) of standard meridian	= 5 30 0
G M T	= 6 0 0
Acceleration	= + 59 14
S I	6 0 59 14
G S T of G M M	= 9 38 48 66
G S T	= 15 39 47 80
Add east long of the place	= 4 32 0
L S T	= 20 11 47 8

Example 6 —Find the R A of the mean sun at 4 45 a m on August 10 1953 in a place in longitude 78° 30 W and also the R A of the meridian of the place given that S T at G M M on the given date is 21 h 12 m 42 51 s

		h m s
Longitude 78° 30′ W	L M T	= 4 45 0
= 5 h 14 m W	Add long	= 5 14 0
	Corresponding G M T	= 9 59 0
	S T of G M M	= 21 12 42.51
Acceleration at 9.8565 s per hour for 9 h 59 m	Acceleration	= + 1 38.4
= 98.4 s	R A M S ± 12	= 21 14 20.91
	Deduct 12 h	= 12
	R A M S at the given L M T	= 9 14 20.91

R A of the meridian of the place = local sidereal time = R A M S at the given instant + mean time at the place ± 12 h

		h m s
	R A M S	= 9 14 20.91
	L M T	= 4 45 0
	Sum	= 13 59 20.91
	Deduct 12 h	= 12
	L S T	= 1 59 20.91
Alternative method —		h m s
Correction at 9.8565 s	S T of G M M	= 21 12 42.51
for long 5 h 14 m W	Add correction	= 51.58
= 51.58 s	S T of L M M	= 21 13 34.09
S I corresponding to	Add S I	= 4 45 46.82
4 h 45 m	Sum	= 25 59 20.91
= 4 h 45 m 46.82 s	Deduct 24 h	= 24
	L S T	= 1 59 20.91

Example 6 —Determine the local apparent time at a place in longitude 72° 30′ 24″ E from the following data

(i) L S T = 10 h 20 m 18 s (ii) G S T of G M N = 3 h 14 m 28 s (iii) E T at G M N on April 8, 1939, = 2 m 15.18 s subtractive from M T and decreasing at 0.7075 s per hour

Long , 72° 30 24″ E
− 4 h 50 m 1 6 s
Retardation at 9 856 s per hour
for 4 h 50 m 1 6 s = 17 64 s

Retardation at 9 8296 s per
hour of S I
for 7 h 7 × 9 8296 − 68 80″s

6 m $\frac{6}{60}$ × 9 8296 0 983 s

3″ 64 s $\frac{3'' \ 64}{3600}$ × 9 8296 0 102 s

sum = 69 892 s
= 69 89 s

L T at G M N = 2 m 15 18 s
Decrease for 2 h 15 m 26 16 s
(= 2 2573 h)
at 0 7075 s per hour
= 0 7075 × 2 2573
= 1 597 − 1 60 s
E T at observation
= 2 m 15 18 s − 1 60 s
= 2 m 13 58 s

		h m s
G S T of G M N	=	3 14 28
Retardation	= −	47 64
L S T of L M N	=	3 13 40 36
L S T	=	10 20 18
Deduct L S T of L M N	= −	3 13 40 36
S I since L M N	=	6 37 64
Retardation	= −	1 9 89
L C T of obs	=	5 27 75
Deduct long E	= −	4 50 1 60
G C T	=	2 15 26 15 p m.
Deduct E T	= −	2 13 58
G A T	=	2 13 12 57 + 12 h
Add long	=	4 50 1 60
L A T	=	7 3 14 17 + 12 h

Example 7 —The following notes refer to an observation for time made on a star on Jan 10 1939

Latitude of the place = 45° 28 40″ N , mean observed altitude of the star = 32° 12 30″, R A of the star = 4 h 25 m 3 03 s declination of the star = 19° 2 48″ 45

The star east of the meridian mean sidereal time of observation — 0 h 12 m 53 s by sidereal chronometer Find the error of the chronometer

(i) Hour angle (H) of the star

In the astronomical triangle ZPS $ZP = 90° - \theta$, $ZS = 90° - \alpha$ and $PS = 90° - \delta$ where θ = latitude, α = altitude, and δ = declination. Using the cosine rule we get

$$\cos ZS = \cos ZP \cos PS + \sin ZP \sin PS \cos H_1$$

i.e. $\sin \alpha = \sin \theta \sin \delta + \cos \theta \cos \delta \cos H_1$

$$\text{or } \cos H_1 = \frac{\sin \alpha}{\cos \theta \cos \delta} - \tan \theta \tan \delta$$

$$= \frac{\sin 32° 12' 30''}{\cos 45° 28' 40'' \cos 19° 2' 48''.45}$$

$$- \tan 45° 28' 40'' \tan 19° 2' 48''.45$$

$$= 0.8041642 - 0.3510477 = 0.4531165$$

$$H_1 = 63° 3' 22''.28 = 4\text{ h } 12\text{ m } 13.49\text{ s}$$

Since the star is east of the meridian

Hour angle of the star $= 360° - H_1 = 360° - (63° 3' 22''.28)$
$= 296° 56' 37''.72$
$= 19\text{ h } 47\text{ m } 46.51\text{ s}$

or $= 24\text{ h} - (4\text{ h } 12\text{ m } 13.49\text{ s})$

		h	m	s
R A of the star	=	4	25	3.03
H A of	=	19	47	46.51
L S T of obs	=	24	12	49.54
	=	0	12	49.54
Chronometer time	=	0	12	53
Chronometer fast				3.46 s

To Determine the Sun's Declination at any Instant of Local Time.—In the N A the values of the sun's declination are given for both (i) 0 h G M T (midnight) and (ii) transit at Greenwich which are for G A T = 12 h for every day of the year and also variations per day

Rule.—(1) First obtain the corresponding G M T or G A T Express the hours minutes and seconds as a fraction of a day This fraction (n) is to be used in the interpolation

(2) If the time so obtained is G M T, obtain the sun's declination at 0 h G M T and its variation per day for the given date If the time obtained is G A T obtain the sun's declination at transit at Greenwich and its variation per day

(3) Compute the change in declination for time interval since Greenwich midnight or Greenwich apparent noon as found in (1) from the known variation per day

There are two ways of computing the change in declination

First method —(i) Convert hours minutes, and seconds into decimals of a day (ii) Multiply the daily variation by decimals of a day (iii) Add this change in declination algebraically to the sun's declination as found in (2) The result gives the sun's declination at the given instant of local time This method does not give quite accurate results since the hourly variation itself is not constant but changes from hour to hour

Second Method (Bessel's Method) —This is a most accurate method In this method the required value is obtained by using Bessel's formula

Let f_{-1} f_0, f_1, f_2 be the successive values of the function which is to be interpolated Let the first differences between these values be denoted by Δ with appropriate suffixes and the second differences by Δ'' with appropriate suffixes Then we have

$$f_{-1} - f_0 = \Delta_{-\frac{1}{2}},\quad f_1 \quad f_0 = \Delta'_{\frac{1}{2}},\quad f_2 - f_1 = \Delta_{\frac{3}{2}} \quad \text{and}$$

$$\Delta_{\frac{1}{2}} - \Delta_{\frac{1}{2}} = \Delta''_0 \quad \Delta_{\frac{3}{2}} - \Delta_{\frac{1}{2}} = \Delta''_1$$

Bessel's formula —$$f_n = f_0 + n\Delta_{\frac{1}{2}} + \frac{n(n-1)}{4}(\Delta''_0 + \Delta''_1)$$

in which f_n = the value of the function which we want to find and which lies between f_0 and f_1, n the fractional value of the interval between two tabular values

The method is best illustrated by the following example

Example —Find the sun's declination at 10 a m on August 5 1953 in longitude 30° E

$$\begin{array}{llll} & & & \text{h.} \\ \text{August 5:} & \text{L. M. T.} & = & 10 \\ & \text{Deduct Long. E.} & = & \underline{2} \\ \end{array}$$

Corresponding G. M. T. $= 8$ h. $= 0 \cdot 3333$ d.

$\therefore \quad n = 0 \cdot 3333.$

From N. A. the following data are extracted :

Date	Sun's declination at 0 h. G. M. T.	Variation per day.
4		
		$-957'' \cdot 5$
5	$+17° 7' 21'' \cdot 3 \ (f_o)$	
		$-974'' \cdot 3$
6		
		$-990'' \cdot 7$
7		

Here $f_{-1} - f_o = \Delta'_{-\frac{1}{2}} = -957'' \cdot 5$; $f_1 - f_o = \Delta'_{\frac{1}{2}} = -974'' \cdot 3$; $f_2 - f_1 = \Delta'_{\frac{3}{2}} = -990'' \cdot 7$

$$\Delta'_{\frac{1}{2}} - \Delta'_{-\frac{1}{2}} = \Delta''_o = -974'' \cdot 3 - (-957'' \cdot 5) = -16'' \cdot 8$$

$$\Delta'_{\frac{3}{2}} - \Delta'_{\frac{1}{2}} = \Delta''_1 = -990'' \cdot 7 - (-974'' \cdot 3) = -16'' \cdot 4.$$

Note :—The four dates on which the tabular differences are obtained should be so selected that the instant lies between the two middle dates.

$\therefore$ By Bessel's rule, we have

$$\text{Change in declination} = n \Delta'_{\frac{1}{2}} + \frac{n(n-1)}{4} (\Delta''_o + \Delta''_1)$$

$$= 0 \cdot 3333 (-974 \cdot 3) + \frac{0 \cdot 3333 (0 \cdot 3333 - 1)}{4} (-16'' \cdot 8 - 16'' \cdot 4)$$

$$= -324'' \cdot 77 + 1'' \cdot 84 = -322'' \cdot 93 = -5' \, 22'' \cdot 93.$$

Hence the Sun's declination on 1953 Aug. 5, 10 h. $= f_n$

$$= +17° \, 7' \, 21'' \cdot 3 - 5' \, 22'' \cdot 93$$
$$= +17° \, 1' \, 58'' \cdot 37$$

In the approximate (first) method, the second term in the formula is omitted. $\therefore$ change in declination $= -324'' \cdot 77 = -5' \, 24'' \cdot 77$ When great accuracy is required, Bessel's method may be used for finding the right ascension of the sun, and the equation of time at a given instant of local time. For fair accuracy, the first method is used to compute them.

Corrections to the Observed Altitude of a Celestial body, (a star or the sun):—

When determining the true altitude of a star or the sun it is necessary to apply the following corrections to its observed altitude.

I Instrumental Corrections —

(1) Correction for Index Error —The index error is the small vertical angle between the line of collimation and the horizontal bubble line of the azimuthal or altitude bubble It may be determined as follows —

(i) Bisect a well defined object such as a church spire with the telescope normal (Face Left) and observe the vertical angle α_1

(ii) Change face and with the telescope reversed (Face Right) bisect the same object again Observe the vertical angle α_2

Let $\alpha_1 = 8° \; 20 \; 40"$ and $\alpha_2 = 8° \; 21 \; 10"$

Mean vertical angle $= 8° \; 20 \; 55"$

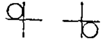
Whence the index correction for face left observation $= +15$

" " right " $= -15"$

It may be noted that the index error is said to be $+E$ or $-E$ according as this amount is to be added to or subtracted from the observed altitude

When the altitude of a star or the sun is to be observed it may sometimes happen that it is not practicable to take observations on both faces and therefore only one face observation is taken In such a case the correction for the index error is necessary The index error is eliminated by taking Face left and Face right observations

When the Face left and Face right observations are made on the sun the upper and left hand limbs should be made to touch the horizontal and vertical hairs respectively in the N W quadrant for one face observation as in Fig 125 a and the lower and right hand limbs should be made to touch the horizontal and vertical hairs respectively



(a) Fig 125 (b)

in the S E quadrant for the other face observation as shown in Fig 125b, or vice versa

(2) **Correction for Bubble Error** —The correction for the bubble error is necessary when the altitude bubble does not remain central while the observations are being taken

$$\text{Correction for bubble error} = + \frac{\Sigma O - \Sigma E}{n} \times v''$$

in which ΣO = the sum of the readings of the object glass end of the bubble

ΣE = the sum of the readings of the eye piece end of the bubble

n = the number of the bubble ends read

v = the angular value of one division of the bubble in seconds

The sign of the correction is plus or minus according as ΣO is greater or less than ΣE

The observed altitude when corrected for index error and bubble error (and semi diameter) is called the apparent altitude

II **Observational Corrections** —

(1) **Correction for Refraction** —(Fig 126) Rays of light passing through layers of air of different densities are bent Consequently, the objects appear higher than they really are, causing an error in altitude Thus in Fig 126a, the path of a ray of light from the celestial body S to an observer at B is curved, and when the ray finally reaches the observer,

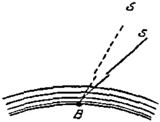


Fig 126 (a)

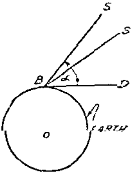


Fig 126 (b)

it appears to him to come in a straight line from S (i e BS) Thus the celestial body S appears to the observer at B to b

situated at S′, whereas in reality it is at S. Due to refraction, the observed altitude of a heavenly body appears greater than it really is The correction for refraction is, therefore, always to be *subtracted* from the observed altitude (always negative).

For instance, in Fig. 120 b, B is the position of the observer, S the position of the heavenly body; S′ its apparent position; the angle S′BD the apparent altitude ($\propto$) The angle S′BS is known as the correction for refraction.

. The true altitude = SBD = S′BD − S′BS.

It may be calculated from the formula

Correction for refraction $= 58'' \cot \propto$, (in seconds)

or " " $= 58'' \tan z$,

at a pressure of 75 cm of murcury and a temperature of 10°C .. .. (32)

in which $\propto$ = the apparent altitude of the heavenly body.

z = the apparent zenith distance of the heavenly body

This formula should not be used for small altitudes (less than 20°)

The amount of the refraction correction depends upon (i) the altitude, (ii) the barometric pressure, and (iii) the atmospheric temperature It is zero when the heavenly body is in the zenith, and about 33 when it is on the horizon. It may be noted that the refraction correction does not depend upon the distance, but is the same for all bodies The value of the correction for refraction for a particular altitude, atmospheric pressure, and temperature may be obtained from refraction tables given in Chamber's Mathematical Tables For more accurate work, Bessel s Refraction Tables may be used

(2) **Correction for Parallax** —Since the sun is comparatively nearer, the altitude of the sun when measured at a point on the surface of the earth differs from that when measured at the centre of the earth, this difference of altitude being known as the sun's *parallax in altitude.* The sun's parallax in altitude is the angle subtended at the centre of the sun by the line joining the place of observation to the centre of the earth.

When the sun is on the horizon, the angle subtended at the centre of the sun by that line is known as the sun's *horizontal*

parallax The horizontal parallax varies inversely with the sun's distance from the earth It varies from 8″ 95 (on 31st Dec) to 8″ 66 (on 1st July) on account of the varying distance of the sun from the earth, the average value being 8″ 8 Its value is given daily in the Nautical Almanac The formula for the sun's parallax in altitude may be derived as follows

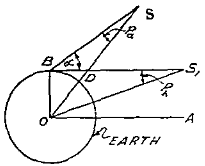


Fig 127

In Fig 127 let B = the place of observation O = the centre of the earth
S = the position of the sun
S_1 = the position of the sun when on the horizon

OA = the true horizon BD = the sensible horizon
SOA = the true altitude, SBD = the apparent altitude(α)
BSO = parallax in altitude BS_1O = horizontal parallax
$SOA = SDS_1 = SBD + BSD$

True altitude = observed altitude + parallax in altitude (33)

$$OBS = OBD + SBD = 90° + \alpha.$$

Now from the triangle OBS, $\sin BSO = \frac{OB}{OS} \sin OBS = \frac{OB}{OS} \cos \alpha$

But $\frac{OB}{OS} = \frac{OB}{OS_1} = \sin BS_1O$

$$\sin BSO = \sin BS_1O \cos \alpha$$

Since the angles BSO and BS_1O are very small, we have
Parallax in altitude = horizontal parallax $\times \cos \alpha$ (34)

Correction for parallax = + parallax in altitude
= + horizontal parallax $\times$ cos apparent altitude
= + 8″ 8 $\cos \alpha$ (34a)

The sign of the correction is *always plus* The correction for parallax is required in the case of observations upon the sun only When observations are made upon the fixed stars the correction for parallax being very small owing to their infinite distance, is ignored

III. Correction for Semi-diameter —(Fig 128) Semi-diameter of the sun is one-half of the angle D, subtended at the centre of the earth, by the diameter of the sun Since the sun's distance from the earth is not constant, semi-diameter varies throughout the year, its value lying between 15′ 45″ and 16′ 18″ It is given for every day of the year in the Nautical Almanac

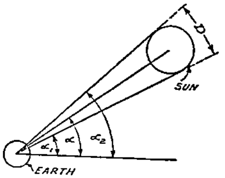


Fig 128

When observing the sun, it is the usual practice to observe its upper or lower limb (edge) as the cross hairs cannot be accurately set on its centre, the diameter being considerable The altitude of the sun's centre may then be obtained by applying the correction for semi diameter algebraically to the observed altitude It is additive when the lower limb is observed, and subtractive when the upper limb is observed The correction for semi-diameter is eliminated by taking observations on both faces (F L and F R) When the stars are observed, observations can be made upon their centres, since they appear as points of light There is therefore no need to apply this correction

Thus in Fig 128, α_1 = the observed altitude of the lower limb α_2 = the observed altitude of the upper limb, α = the altitude of the centre, and $\frac{D}{2}$ = the correction for semi diameter

Then $\alpha = \alpha_1 + \frac{D}{2} = \alpha_2 - \frac{D}{2}$.

When the horizontal angle to the sun is to be measured, the east or west limb of the sun is observed A correction for

semi diameter in azimuth must, therefore be applied to the observed horizontal angle in order to obtain that to the sun s centre It is given by

Correction for semi diameter in azimuth
= semi diameter × secant altitude

IV Correction for Dip —It is necessary to apply the correction for dip when observations are taken upon a star or the sun with a sextant at sea its altitude being observed from the visible or sea horizon The angle between the sensible horizon and the visible horizon is called the angle of dip

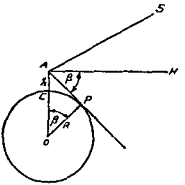


Fig 129

In Fig 129 let

S = the position of the sun
AH = the sensible horizon
AP = the visible horizon
HAP = the angle of dip (β) = the correction for dip
SAP = the observed altitude corrected for refraction
SAH = the altitude corrected for dip
AC=h= the height of the observer above sea level in m
R = the radius of the earth in m

HAP = AOP = β OA = R + h OP = R,

$$AP = \sqrt{(R+h)^2 - R^2}$$

$$\text{Now } \tan\beta = \frac{AP}{OP} = \frac{\sqrt{(R+h)^2 - R^2}}{R} = \frac{\sqrt{h(2R+h)}}{R} \quad \text{(exact)} \qquad (35)$$

$$= \sqrt{\frac{2h}{R}} \quad \text{(approximate)} \qquad (35a)$$

SAH = SP − HAP = the observed altitude corrected for refraction − the correction for dip

The correction for dip is *always negative* Its amount varies with the height of the observer above sea level and its values for various heights may be obtained from the tables given in Chamber s Mathematical Tables

Correction to Horizontal Angles

In making observations to determine azimuth, it is necessary to measure the horizontal angle between a reference mark and the heavenly body The instrument must, therefore, be in perfect adjustment Even though the adjustments are made with great care there remain certain residual errors which affect the accuracy of observations The effect of these errors is eliminated by taking double face observations

As the sights to the heavenly bodies are usually highly inclined it is most important that (1) the instrument must be accurately levelled (i e the vertical axis must be truly vertical) and (2) the trunnion axis (horizontal or transverse axis) must be exactly perpendicular to the vertical axis If the vertical axis is not truly vertical the trunnion axis will be inclined even though it is in perfect adjustment, and this error (i e due to the inclination of the trunnion axis) cannot be eliminated The inclination of the trunnion axis may be ascertained by means of a striding level

Trunnion Axis Dislevelment —When the trunnion axis is not truly horizontal as shown by the observations with the striding level, and when inclined sights are taken, it is necessary to correct each observed horizontal direction The amount of this correction is given by

C in seconds $= e \tan \alpha$

where e = the inclination of the trunnion axis, in seconds

α = the vertical angle (+ or − angle)

The value of e may be determined from the formula

$$e \text{ in seconds} = \frac{\Sigma l - \Sigma r}{4} \times \text{angular value of a bubble division,}$$

in which Σl = the sum of the readings of the *left hand end* of the bubble in the direct and reversed positions of the striding level on the trunnion axis

Σr = the sum of the readings of the *right-hand end* of the bubble in the direct and reversed positions of the striding level on the trunnion axis

If Σl is greater than Σr, the left-hand end of the axis is higher, while if Σr is greater than Σl, the right hand end of the axis higher. If the left-hand end of the axis is higher, the correction to the observed direction (i e to the average of the two vernier readings) is positive, while if the right-hand end of the axis is higher, the correction to the observed direction is negative The horizontal circle reading for each direction should be corrected separately and then the horizontal angle should be obtained by subtraction

The error of the level $= \frac{1}{2}\left\{\frac{(l_1 - r_1)}{2} - \frac{(l_2 - r_2)}{2}\right\} \times$ angular value of a bubble division

The observer should face the direction in which the instrument is pointed while the ends of the bubble are being read

Due to the inclination of the trunnion axis there is an apparent displacement of the objects sighted It is towards the side of the higher end of the trunnion axis for angles of elevation, and towards the side of the lower end of the trunnion axis for angles of depression The sign of the correction is, therefore, given by the following rule

(a) When the vertical angle is an angle of *elevation* —

The sign of the correction to the observed direction is *plus* or *minus* according as the *left hand* end of the trunnion axis is higher or the *right-hand* end of the trunnion axis is higher

(b) When the vertical angle is an angle of *depression* —

The sign of the correction to the observed direction is plus if the right-hand end of the trunnion axis is higher, and minus, if the left-hand end of the trunnion axis is higher

The error is eliminated when the objects sighted are at the same elevation, and also when Face left and Face right observations are taken

Example :—Given (i) the inclination of the horizontal axis is 8″; (ii) left-hand end of the horizontal axis is higher, (iii)

when the left hand station was bisected the vertical angle was $+ 10° 12$ and when the right hand station was bisected it was $- 5° 6$ Find the correction for dislevelment to be applied to the horizontal angle

The correction to the observed direction when the left hand station was bisected

$= + 8'' \tan 10° 12 = + 1'' 44$ since the vertical angle is plus

The correction to the observed direction when the right hand station was bisected

$= - 8'' \tan 5° 6 = - 0\ 714$ since the vertical angle is minus

If x and y be the respective horizontal circle readings when the two stations were bisected the corrected horizontal circle readings are $x + 1''\ 44$ and $y - 0''\ 714$

The horizontal angle between the two stations

$$= (y - 0\ 714) - (x + 1\ 44)$$
$$= (y - x) - (0\ 714 + 1\ 44)$$

Whence the correction to the horizontal angle =

$$- (0\ 714 + 1\ 44) = - 2''\ 154$$

Determination of Azimuth

Reference Mark —In determining the azimuth of a star or other heavenly body it is necessary to have a reference mark (R M) or a referring object (R O) The reference mark may be a triangulation station or it may consist of a lantern or an electric light placed in a box or behind a screen in which a small circular aperture is cut to admit light to the observer Sometimes a narrow vertical slit is cut instead of a circular hole For daytime observations the face of the screen is painted with stripes or a target is painted on the side of the box towards the observer The size of the aperture depends upon the distance of the mark from the instrument (0 9 cm at a distance of $1\frac{1}{2}$ km) The reference mark should be wherever possible about $1\frac{1}{2}$ km away in order to obviate the necessity of refocussing the telescope in bisecting the mark after bisecting the star It should be so situated that the line of sight is well above the ground to minimise the error due to lateral refraction

I Observations for Azimuth —When determining the azimuth of a survey line, the process consists in (i) measuring the horizontal angle between the reference mark and the heavenly (or celestial) body, and (ii) determining the azimuth of the celestial body The azimuth of the reference mark may then be calculated from the measured angle and the calculated azimuth of the celestial body The azimuth of a survey line may then be obtained by measuring the horizontal angle between the mark and the line, and combining it with the azimuth of the mark Alternatively, if the reference mark is the other end of the survey line, the azimuth of the line may be determined by measuring the horizontal angle between the line and the celestial body and combining it with the azimuth of the celestial body There are several methods of determining the azimuth of a line But in practice, preference is given to such methods which will permit face left and face right observations to be taken in order to eliminate instrumental errors and which will allow observations to be made at any time and with the required precision It is advisable to select a close circumpolar star, since such a star changes very slowly in azimuth in a given length of time

Determination of Approximate Position of a Star —Prior to making observation on any star, its approximate position at a given time should be known so that it can be easily picked out and readily brought into the field of view of the telescope If a star chart is available a given star may be readily indentified with the eye and may be brought into the field of view of the telescope without any difficulty But if the star chart is not available, the approximate altitude and azimuth of a given star must be computed The right ascension and declination of the star may be obtained from N A Knowing the local mean time, the hour angle of the star may be calculated as already explained Knowing the latitude of the place, and the hour angle and declination of the star the altitude and azimuth of the star may be found by solving the astronomical triangle ZPS The vertical vernier of the theodolite is then set at the value of the altitude Knowing roughly the direction of the meridian the star may be readily brought into field of view by turning the vernier plate through an angle equal to the azimuth of the star

Polaris — In the northern hemisphere, there is a bright star called Polaris (∝ Ursa Minoris) also known as the Pole Star of

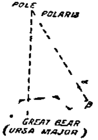


Fig 130

the North Star It is bright enough to be seen with the naked eye Owing to its proximity to the pole (within about 1° from the pole) it is most favourably situated and is most commonly used for the determination of the azimuth and latitude It describes a small circle of about 1° radius round the pole Its azimuth therefore changes very slowly It can be easily identified by means of the constellation (or group) of stars called the "Great Bear or 'Ursa Major' This constellation consists of seven most brilliant stars as shown in Fig 130 and can be readily identified on any clear night The stars ∝ and β are commonly known as the pointers — so called as they point almost directly to the north pole The line joining these two stars passes very nearly through the pole and Polaris and by following this line, Polaris can be readily located

Determination of the True Meridian —When determining the true meridian it is necessary to know the latitude of the place of observation It may be taken from a map or may be obtained with sufficient accuracy by observing the altitude of Polaris which is very nearly equal to the latitude of the place There are two most common methods of determining the true meridian viz (1) by observation on Polaris at culmination and (2) by observation on Polaris at elongation The latter is the most accurate method In both the methods the theodolite should be set up in such a position that a clear and unobstructed view can be had for a distance of about 120 to 150 m

I Observation on Polaris at Culmination —In this method a date on which Polaris is at upper or lower culmination during

the night or during the early part of the evening is selected The exact time of upper culmination of Polaris is then obtained from special tables which give local times of upper culmination of Polaris for different dates The time of lower culmination of Polaris can be obtained by the relation, viz time of lower culmination = time of upper culmination $\pm$ 11h 58 m Use plus sign when the time of upper culmination is less than 11 h 58 m and minus sign when it is greater than 11 h 58 m

In the absence of special tables the time of culmination may be computed as already explained It may be noted here that at the instant of upper culmination local sidereal time is equal to the right ascension of the star and at the instant of lower culmination it equals its right ascension plus 12 hours

Procedure —(1) About 15 minutes before the time of culmination set up the theodolite over a given station and level it carefully The vertical circle vernier should be set to the latitude of the place to facilitate finding Polaris

(2) About 5 minutes before the time of culmination, direct the telescope to the star The cross wires should be illuminated by holding a lamp in front and towards one side of the object glass The modern instrument is electrically illuminated With both motions clamped follow the star continuously with the vertical hair by means of either tangent screw until the exact time of culmination

It may be noted that when the star is approaching upper culmination it will appear to be moving towards the left, while when it is approaching lower culmination, it will appear to be moving towards the right

(3) Depress the telescope and set a stake on the line of sight at a distance of about 120 to 150 m from the instrument and fix a tack in it exactly in line with the vertical cross hair The line joining the two stakes defines the true meridian

II Observation on Polaris at Elongation (Western or Eastern) —In this method it is necessary to know the approximate time of elongation to enable the observer to know about when to make an observation It may be readily obtained from special tables or it may be calculated as already explained

Polaris is at western elongation about 5 h 55 m after upper culmination, while it is at eastern elongation about 5 h 55 m before upper culmination.

Procedure —(1) About 20 or 30 minutes before the time of elongation, set up the instrument over a given station and level it carefully Set the vertical vernier to the latitude of the place to facilitate finding Polaris

(2) Set the vernier to zero and direct the telescope to the star Having clamped both motions follow it continually with the vertical hair by means of the lower tangent screw It may be noted that if the star is approaching western elongation it will be moving to the left while if the star is approaching eastern elongation, it will be moving to the right Just about the time of elongation the star stops moving horizontally but appears to move vertically along the vertical hair downward for western elongation and upward for eastern elongation

Follow the star with the vertical cross hair until the time of elongation.

(3) Depress the telescope and set a mark on a stake in line with the vertical cross hair at a distance of about 120 m

(4) Transit the telescope and relevel the instrument if necessary, and bisect the star again Depress the telescope and set a second mark on the stake beside the first mark. The point exactly midway between these two marks gives the exact position of Polaris at elongation and the line joining the instrument station to the point so established gives the direction of Polaris at elongation

(5) Calculate the azimuth of Polaris by the equation $\sin \text{azimuth} = \dfrac{\cos \text{declination}}{\cos \text{latitude}}$ and lay off this azimuth to the east or to the west according as the star is at western elongation or eastern elongation The line of sight is then directed along the true meridian.

(6) Drive a stake on the line of sight at a distance of about 120 to 150 m and set a mark on the stake exactly in line with the vertical cross hair The line joining the two stakes gives the direction of the true meridian.

Alternatively, the true meridian may be established by a perpendicular offset from the established point (in step 4), which may be calculated by the relation offset = distance from the instrument station to the point established × tangent of azimuth of Polaris

Determination of Azimuth —The azimuth of a survey line may be determined by (1) extra-meridian observation of the sun, (2) extra meridian observation of a circumpolar star, or of a star near the Prime Vertical, (3) observation of a circumpolar star at elongation, and (4) equal altitudes of the sun, or of a circumpolar star

(1) By Extra-Meridian Observation of the Sun —In this method the altitude of the sun is observed with a transit set up at one end of the line, and the horizontal angle between the survey line and the sun is measured in the usual way Whenever possible, Face left and Face right observations should be taken and the mean of the results adopted The local mean time of observation must also be noted Knowing the altitude of the sun the latitude of the place and the sun s declination at the instant of observation, the azimuth of the sun may be computed by solving the astronomical triangle ZPS (Fig 131) The azimuth of the line may then be determined from the azimuth of the sun and the angle between the line and the sun It is advisable to draw a sketch showing the relative positions of the meridian, the sun, and the line, from which it may readily be seen if the angles are to be added or subtracted. In Fig 131, let

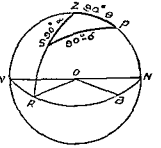


Fig 131

O = the position of the observer (Instrument station)
B = the referring object
OB = the survey line whose azimuth is to be determined.
S = the position of the sun at the instant of observation.
BOR = the mean horizontal angle between the line and the sun.
PZS = NOR = A = the azimuth of the sun.
α = the corrected altitude of the sun.

δ = the declination of the sun at the instant of observation

θ = the latitude of the place

Knowing the local mean time of observation and the longitude of the place, the declination of the sun at the instant of observation may be obtained from its declination at G M M or G. M N as taken from the Nautical Almanac (N A) The true altitude of the sun is found by applying the proper corrections to the observed altitude The three sides of the triangle ZPS being known, the angle PZS (A) may be computed from

$$\tan\frac{A}{2} = \sqrt{\frac{\sin(s-ZP)\sin(s-ZS)}{\sin s \sin(s-PS)}}$$

where $s = \frac{1}{2}(ZP + ZS + PS)$

or $$\cos A = \frac{\sin\delta - \sin\alpha\sin\theta}{\cos\alpha\cos\theta}$$

Then the azimuth of OB = NOB = NOR − BOR = A − BOR

In order to obtain best results, observations should be made between 8 a m to 10 a m or between 2 p. m to 4 p m *local mean time*

For greater accuracy, the sun should be observed when it is near the prime vertical, since it then moves slowly in azimuth

(2) **By Extra-Meridian Observation of a Circumpolar Star, or of a Star near the Prime Vertical** —In this case the procedure is similar to that described in the preceding case The L M T of observation need not be known very accurately, since the declination of the star varies very slowly

In order to minimise the effect of errors of the observed altitude, the star should be observed when it is on or near the prime vertical since it then moves slowly in azimuth Consequently, there is sufficient time to take circle left (F L) and circle right (F R) observations The star at the time of observation should not be too low, otherwise refraction will be great

(3) **By Observation of a Circumpolar Star at Elongation** —A star is said to be at elongation when it is farthest from the pole. If it is east of the meridian, it is said to be at eastern elongation, while if it is west of the meridian, it is

said to be at western elongation When the star is at elongation it appears to move vertically and the vertical circle through the observer's zenith (swept out by the telescope) is tangent to the circular path of the star (Fig 132) In this position the angle between the plane of the declination circle and the plane of the vertical circle is a right angle In other words the parallatic angle ZSP of the astronomical triangle ZPS is a right angle This position is the most favourable for observations to determine azimuth

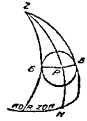


Fig 132

Prior to making observations it is necessary to ascertain the time at which the star will elongate It may be determined as follows —(i) Knowing the latitude of the place and the declination of the star calculate the hour angle of the star by the equation $\cos \text{hour angle} = \dfrac{\tan \text{latitude}}{\tan \text{declination}}$

(ii) Convert it into time and add it to the R A of the star for west elongation and subtract it from R A for east elongation The result gives the local sidereal time (L S T) of elongation

(iii) Convert this time into mean time

The horizontal angle between the line and the circumpolar star when at its eastern or western elongation is measured with a transit on both faces Knowing the declination of the star and the latitude of the place the sides PS and ZP of the triangle ZPS are known The azimuth of the star i e the angle PZS may then be calculated from

$$\sin A = \sin PZS = \frac{\cos \text{declination}}{\cos \text{latitude}} = \frac{\cos \delta}{\cos \theta} \text{ since the triangle ZPS}$$

is right angled at S The azimuth of the line may then be determined from these angles as explained above The star observed may be Polaris (Pole Star) or any other circumpolar star Polaris is a bright star and being near the pole it changes its position slowly Of all the bright stars it is most favourably situated for accurate determination of azimuth and latitude and is therefore most commonly used It is not convenient for time observation as it moves slowly in azimuth

(4) (a) By Equal Altitudes of a Circumpolar Star — Let OB be the line whose azimuth is to be determined

The instrument is set up at O and the horizontal angle between the reference mark or referring object at B and the star is observed The vertical circle is then clamped When the star reaches the same altitude on the other side of the meridian, it is bisected with the cross hairs by turning the telescope in azimuth and using the tangent screw of the vernier plate, with the reading on the vertical circle remaining unaltered The mean of the two vernier readings gives the horizontal angle between the line OB and the new position of the star

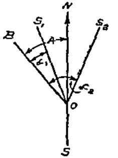


Fig 133 a

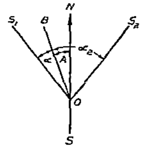


Fig 133 b

In Fig 127a, let NS = the direction of the meridian at O
S_1 and S_2 = the two positions of the star
OB = the line whose azimuth is to be determined
$BOS_1 = \alpha_1$ = the horizontal angle between OB and the first position (S_1) of the star
$BOS_2 = \alpha_2$ = the horizontal angle between OB and the second position (S_2) of the star
A = the azimuth of the line OB

Since the direction of the meridian is midway between the two positions of the star, the azimuth of the line may be determined as follows

Case I —When the two positions of the star are on the same side of the line (Fig 133a)

Azimuth of the line OB = NOB = A

$$= \alpha_1 + \frac{\alpha_2 - \alpha_1}{2} = \frac{\alpha_1 + \alpha_2}{2}$$

i e half the sum of the observed horizontal angles

Case II —When the two positions of the star are on opposite sides of the line (Fig 127b)

Azimuth of the line OB = NOB = A

$$\frac{\alpha_1 + \alpha_2}{2} - \alpha_1 - \frac{\alpha_2 - \alpha_1}{2}$$

i e half the difference of the observed horizontal angles

The above procedure is followed when the instrument is in perfect adjustment If the instrument is not in perfect adjustment it is necessary to take a set of at least four observations in order to eliminate instrumental errors The procedure is as follows —

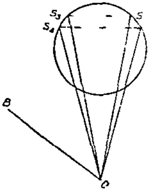


Fig 133 c

(1) The instrument is set up at O as before With both plates clamped at zero and with the verticle circle say on the left the reference mark B (Fig 133 c) is bisected The star in the position S_1 is then bisected and the horizontal angle BOS_1 (H_1) and the vertical angle (V_1) are recorded

(2) The face of the instrument is then changed With the vertical circle on the right the star which is now at S_2 is again bisected and the horizontal angle BOS_2 (H_2) and the vertical angle (V_2) are noted The mean of the two angles H_1 and H_2 gives the value of the horizontal angle between the line OB and the star at observation (3) The instrument is left undisturbed with the telescope kept clamped at the vertical angle (V_2) When the star approaches the same altitude (i e the position S_3) on the other side of the meridian the vernier plate is unclamped When the star is seen in the field of view of the telescope the vernier plate is clamped and the star is

bisected by means of the tangent screw, noting the horizontal BOS_3 (H_3) (4) The face of the instrument is changed and the vertical circle vernier is set to read the vertical angle (V_1) With the vertical circle on the left, the star is again bisected when it is in the position S_4 and the horizontal angle BOS_4 (H_4) is recorded. The mean of the two angles H_3 and H_4 gives the value of the horizontal angle between the line OB and the star at observation when it is on the other side of the meridian

When both positions of the star are on the same side of the line OB (Fig 133 a), the azimuth $OB = \frac{H_1 + H_2 + H_3 + H_4}{4}$ as in the preceding method On the other hand, if both positions of the star are on opposite sides of the line OB as in Fig 133 b the azimuth of $OB = \frac{(H_1 - H_2) - (H_3 + H_4)}{4}$.

(4) (b) By equal Altitudes of the Sun —The procedure in this method is similar to that described above except the following —

(1) Observations are made during the daytime (2) Since the sun's centre cannot be bisected, observations should be made on the right hand and left hand limbs of the sun with the telescope normal and inverted in both the morning and afternoon observations (3) Since the actual altitude of the sun is not required the upper or the lower limb should be observed throughout. (4) The number of forenoon observations should equal the number of afternoon observations (5) The time for each observation must be noted (6) Owing to the change in the sun's declination during the interval between the morning and afternoon observations, the azimuth of the line so obtained requires a correction

Determination of Time

When determining azimuth or latitude by astronomical observations, an accurate knowledge of time is required It is, therefore, necessary to determine time by observation in order to find the error of the watch, the chronometer, or other time keeper which the observer may be using

The local time may be determined by (1) extra meridian observation of a star, or the sun, (2) meridian transit of a star or the sun, and (3) equal altitudes of a star or the sun

(1) (a) **By Extra-Meridian Observation (Altitude) of a Star** —In this method the altitude of a star is observed with a theodolite on both faces and the chronometer time at the instant of each observation recorded For accurate results the star should be observed when it is on or near the prime vertical To minimise errors of observation, several altitudes of the star are observed in quick succession, half of the observations being taken with Face left and half with Face right and the chronometer time of each observation recorded The mean of the altitudes is taken as the mean observed altitude and the mean of the chronometer times as the time at the instant of observation More accurate results are obtained when two stars are observed one east and the other west of the meridian, thereby eliminating the instrumental and other errors The proper corrections are then applied to the mean observed altitude of the star Knowing the altitude, the declination of the star, and the latitude of the place of observation the sides of the astronomical triangle ZPS are known The hour angle of the star may then be computed from

$$\tan \tfrac{1}{2}\, ZPS = \tan \frac{H}{2} = \sqrt{\frac{\sin(s - ZP)\sin(s - PS)}{\sin s \sin(s - ZS)}}$$

or $\sin \alpha = \sin \theta \sin \delta + \cos \theta \cos \delta \cos H$

When the star is on the prime vertical, its hour angle is given by $\cos H = \dfrac{\tan \text{declination}}{\tan \text{latitude}} = \dfrac{\tan \delta}{\tan \theta}$

The calculated hour angle in arc is then converted into time by the relation $15° = 1$ h Knowing the R A of the star, the local sidereal time of observation is found from

$L\ S\ T = R\ A \pm \dfrac{H}{15}$, the plus sing being used when the star is to the west of the meridian, and the minus sign when it is to the east of the meridian Knowing the longitude of the place and the sidereal time of G M M (or G M N), the L M T of observation can be computed as already explained The

difference between the calculated time and chronometer reading gives the error of the chronometer

(b) By Extra-Meridian Observation (Altitude) of the Sun - In this case, the procedure is the same as above The altitude of the sun's lower (or upper limb) is observed with the telescope normal and then the altitude of the other limb is observed with the telescope inverted and the watch time at the instant of each observation noted The true altitude is then found by applying the necessary corrections to the mean observed altitude To obtain more accurate results, several altitudes are measured in quick succession, the watch time for each observation being noted To eliminate the instrumental errors and correction for semi diameter half the number of observations should be made on the upper limb with Face left and half on the lower limb with Face right

Note —The best position for observation on a star or the sun is on or near the prime vertical since in this position the change of altitude is most rapid and, therefore, the least will be the influence of the error of observation on the computed time

The mean observed altitude is then corrected for refraction and parallax and the hour angle (H) of the sun computed from formula (16) or (15)

If the sun is observed when on the prime vertical, the hour angle is calculated from formula (21) The hour angle in are when converted into time gives the local apparent time (L A T) of observation

When the sun is to the west of the meridian,

$$\text{L A T of observation} = \frac{H}{15} \text{ since local apparent noon,}$$

When the sun is to the east of the meridian.

$$\text{L A T of observation} = 24\text{ h} - \frac{H}{15} \text{ since local apparent noon.}$$

$$\text{or L A T of forenoon observation} = 12\text{ h} - \frac{H}{15}.$$

$$\text{,, of afternoon ,,} = 12\text{ h} + \frac{H}{15}$$

Knowing the equation of time, the L M T of observation can be found The difference between the calculated time and the time recorded on the watch gives the error of the watch

(2) (a) By Meridian Transit of a Star —To use this method, the direction of the meridian must be known accurately The star is observed with a theodolite when it crosses the meridian and the chronometer time at the instant of its transit noted Since the hour angle of the star at transit is zero the local sidereal time at the instant of observation is given by the R A of the star L S T is then converted into L M T as already explained.

(b) By Meridian Transit of the Sun —In this case, the transit of the sun is observed with a theodolite and the times at which the east and west limbs of the sun cross the meridian (i e pass the vertical hair) are noted by means of a watch The mean of these two times gives the time of transit of the sun s centre, i e the watch time of the local apparent noon Knowing the equation of time, the L M T at L A N can be determined

(3) (a) By Equal Altitudes of a Star —This is a very simple and accurate method and is used when the direction of the meridian is not accurately known Prior to making observation, the approximate altitude of the star is computed The instrument is then set up and accurately levelled The computed altiude of the star is then set on the vertical circle and the motion of the star is followed in azimuth with the vertical cross hair by means of the horizontal tangent screw The time (T_1) at which the star crosses the horizontal hair near vertical hair is recorded When the star approaches the same altitude on the other side of the meridian the instrument is turned in azimuth and the star is again followed in azimuth with the vertical cross hair by means of the horizontal tangent screw until it is bisected by the cross hairs when the time (T_2) is again recorded The mean of these two times $\left(\frac{T_1+T_2}{2}\right)$ gives the time of transit of the star It is then compared with the local mean time computed from the R. A of the star to determine the error of the watch or chronometer For better results, the star should be obser-

ved when it is near the prime vertical Very accurate results are obtained if a series of observations in pairs are made on the same star It may be noted that the face of the instrument must remain unchanged and the telescope undisturbed for the second observation, but the altitude bubble must be centred prior to each observation by means of clip screws The advantages of this method are (1) Errors of graduation collimation error, index error are not involved as the actual altitude of the star is not required (ii) No knowledge of the declination of the star and the latitude of the place is required The disadvantages are (1) A long interval of time must elapse between the two observations and (2) It is liable to error due to change in refraction between the two observations The interval between the two observations can however be reduced by selecting a star whose declination differs from the latitude by a small amount It will be observed that the essential feature of this method is the equality of the altitudes on either side of the meridian and the mean of the two times at which the star attains equal altitudes east and west of the meridian gives the watch or chronometer time of transit of the star

(b) By Equal Altitudes of the Sun —The procedure is similar to that described above except that the same limb (upper or lower) is observed on each occasion to eliminate the semi diameter correction and that the time of transit so found needs a correction on account of the change in the declination of the sun

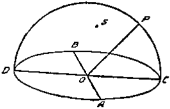


Fig 134

Sun Dial —The sun dial gives apparent solar time from which mean time may be obtained (though not precisely) for checking the watch or clock times It is useful particularly in places where there are no means available for checking watch or clock times It consists of (1) a sharp straight edge called

the *stile* or *gnomon* of the dial, and (2) the graduated circle When a sun dial is illuminated by the rays of the sun, the shadow of the stile is cast upon the plane containing the graduated circle. The reading at the point of intersection of the shadow with the graduated circle gives the apparent solar time. The stile is always set parallel to the earth's axis (always pointing to north) The sun dial is classified as (1) Horizontal dial, (2) Prime Vertical Dial, and (3) Oblique Dial, according as the plane of the dial containing the graduated circle is horizontal, or lies in the prime vertical, or is inclined to the horizontal

Thus in Fig 134, BDAC represents the plane of the dial, DPC is the plane of the meridian, OP is the direction of the stile; OA is the direction of the shadow of the stile, S is the position of the sun

Example 1 —To determine the index error of a theodolite a church spire was sighted and the Face Left and Face Right observations were 15° 48′ 50″ and 15° 47′ 20″ respectively A Face Right observation on the sun's lower limb was then made and the altitude was found to be 36° 40′ 25″ The semi diameter of the sun at the time of observation was 15′ 59″·78 Find the true altitude of the sun

The observed altitude of the sun is to be corrected for (i) index error (ii) semi diameter, (iii) refraction, and (iv) parallax

(i) Index correction —

Mean of the vertical circle readings $= \frac{1}{2}(15° 48' 50'' + 15° 47' 20'')$
$= 15° 48' 5''$.

Index correction for Face Right altitude

$= +(15° 48' 5'' - 15° 47' 20'')$
$= + 45''$

The observed altitude of the sun	=	36° 40′ 25″
Add index correction		45″ (+ve)
Altitude of the sun corrected for index error	=	36° 41′ 10″

(ii) Since the lower limb of the sun was observed, the correction for semi diameter is + 15′ 59″·78

(iii) Correction for refraction

$= -57'' \cot 36°41' 10'' = -1' 16''\cdot51$

(iv) " for parallax

$= +8\ 8'' \cos 36°41' 10'' = +\ 7''\ 06$

$\therefore$ Net observational correction $= -1'\ 9''\cdot45$

Altitude of the sun corrected for index error $= 36°\ 41'\ 10''$

Add correction for semi diameter	= 15 59 78	
Sum	=36 57 9 78	
Deduct net observational correction	= 1 9 45	(−ve)
True altitude of the sun	=36° 56 0″ 33	

Example 2 —Determine the altitude of the star, the azimuth of the line AB, and the local mean time of observation from the following data, the star being observed at western elongation

Latitude of station A	= 50° 45′ N
Longitude of "	= 54° 10′ W
Mean horizontal angle of the star to the right of the referring object B	= 95° 20 47″
Declination of the star	= 62° 4′ 51″ N.
Right ascension of the star	= 10 h 59 m 59 s
G S T of G M N	= 6 h 12 m 20 s

Since the star was observed at its elongation, the angle ZSP of the astronomical triangle ZPS is a right angle

Let α = the altitude of the star, H = the hour angle of the star

A = the azimuth of the star, θ = the latitude of the place

δ = the declination of the star

Then $\sin\alpha = \dfrac{\sin\theta}{\sin\delta}$, $\sin A = \dfrac{\cos\delta}{\cos\theta}$, $\cos H = \dfrac{\tan\theta}{\tan\delta}$

$\sin\alpha = \dfrac{\sin 50°\ 45}{\sin 62°\ 4'\ 51''}$, $\log\sin\alpha = \bar{1}\ 9427010$

$\alpha = 61°\ 12\ 38''\ 82$

$\sin A = \dfrac{\cos 62°\ 4'\ 51''}{\cos\ 50°\ 45'}$, $\log\sin A = \bar{1}\ 8692535$

$A = 47°\ 44'\ 4''\ 49$ W

$$\cos H = \frac{\tan 50° 45}{\tan 62° 4\ 51''},\quad \log \cos H = \bar{1}\ 8119545$$

$H \quad = 49° 33\ 59'' 02 = 3\text{ h } 18\text{ m } 15\ 93\text{ s}$

(1) Altitude of the star $= 61° 12\ 38'' 82$

(2) Azimuth of the line AB —

Since the star was at its western elongation it is to the west of the meridian

The azimuth of the line AB = azimuth of the star + mean horizontal angle between the line and the star
= 47° 44 4″ 49 + 95° 20 4″
= 143° 4 51″ 49 W
or = 360° − (143° 4 51″ 49)
= 216° 55 8″ 51 clockwise from north

(3) Local mean time of observation —

	h	m	s	
G S T of G M N	= 6	12	20	
Longitude West 54° 10 = 3 h 36 m 40s				
Acceleration at 9 86 seconds per hour of longitude = 35 61 seconds			35 61	(+ve)
L S T of L M N	= 6	12	55·61	

Now local sidereal time (L S T) = right ascension of the star + hour angle of the star

R A of the star	= 10	59	59		
H A of	= 3	18	15	93	
L S T	= 14	18	14	93	
Deduct L S T of L M N	= 6	12	55	61	(−ve)
S I from L M N	= 8	5	19	32	

Retardation at 9 8296 seconds per hour of S I for
8 h = 78 64 s
5 m = 0 82 s
19 32 s = 0 05 s

Total = 79 51 s = 1 m 19 51s	1	19	51	(−ve)
M T interval from L M N	= 8	3	59 81	
L M T of observation	8 h	3 m	59 81 s p m	

Example 3.—To determine the azimuth of a line AB, a star was observed at its eastern elongation, and the following results were obtained:

Latitude of the place $= 48° 20' 30'' S$
Longitude of the place $= 46° 20' 15'' E$
Declination of the star $= 74° 25' 34''\cdot72 S$
R. A. of the star $= 3$ h 48 m. 9.62 s
G. S. T. of G. M. M. $= 4$ h. 26 m. 12 s

Clockwise horizontal angle from the line AB to the star $= 125° 42' 30''$.

Find the azimuth of the line AB and the local mean time of elongation.

Since the star was observed at its elongation, the astronomical triangle ZPS is right-angled at S. The following formulæ may, therefore, be used to determine the azimuth and the hour angle of the star.

$$\sin A = \frac{\cos\delta}{\cos\theta}\ ;\ \cos H = \frac{\tan\theta}{\tan\delta}$$

in which A = the azimuth of the star,
H = the hour angle of the star;
δ = the declination of the star; θ = the latitude of the place.

$$\sin A = \frac{\cos 74° 25' 34''\cdot72}{\cos 48° 20' 30''} \quad \text{or} \quad \log \sin A = \bar{1}\cdot6062906$$

$A = 23° 49' 23''\cdot48$ E.

$$\cos H_1 = \frac{\tan 48° 20' 30''}{\tan 74° 25' 34''\cdot72} \quad \text{or} \quad \log \cos H_1 = \bar{1}\cdot4959267.$$

$\therefore H_1 = 71° 44' 35''\cdot7 = 4$ h. 46 m. 58·38 s

(1) Azimuth of the line AB.—

Azimuth of the star from south	$= 23° 49' 23''\cdot48$ east
Clockwise horizontal angle between the line and the star	$= 125° 42' 30''$
Azimuth of AB from south = sum	$= 149° 31' 53''\cdot48$
Azimuth of AB clockwise from north	$= 180° - (149° 31' 53''\cdot48)$ $= 30° 28' 6''\cdot52$

(2) Local mean time of elongation :—

Since the star is to the east of the meridian its hour angle = $360° - H_1$ = 24 h. − (4 h 46 m 58 38 s)

		h	m	s	
	Hour angle (H) of the star	= 19	13	1 62	
	R A of the star	= 3	48	9 62	
	L S T	= 23	1	11 24	
	G S T of G M M	= 4	26	12	
Long east, 46° 20 15″ = 3 h 5 m 21 s					
Retardation for 3 h 5 m 21 s at 9 86 s	Retardation	=		30 46	(−ve)
per hour of long = 30 46 seconds	L S T of L M M	= 4	25	41 54	

Deducting L S T of L M M from L S T, we get

		h	m	s	
	S I since L M M.	= 18	35	29 70	
Retardation for 18 h etc		=	3	2 75	(−ve)
at 9 8296 s per h = 182 75 s = 3 m 2 75 s	L M T of elongation from midnight	= 18 h	32 m	26 95 s	
	or	= 6 h	32 m	26 95 p m	

Example 4 —The following notes refer to an observation made on a star on a certain day

The mean observed altitude of the star	= 40° 18 24″
Latitude of station A	= 48° 30 20″ N
Declination of the star	= + 26° 17′ 45″
Mean horizontal angle between the star and the reference mark B (the line AB lying to the west and between the star and the elevated pole)	= 65° 24 36″

Barometer	= 75 cm
Temperature	= 7° C

Find the azimuth of the line AB

From Chambers Mathematical Tables, we have

Mean refraction for the observed altitude 40° 18′ 24″ = 1′ 7″ 4.

Correction for temperature $7° c$ $= + \ 1''$

" " barometer 75 cm $= + \ 2''$

Total $= + \ 3''$

Correction for refraction $= \ 1' \ 10'' \ 4$ $(-ve)$

Observed altitude $= 40° \ 18' \ 24''$

Deduct correction for refraction $= \ \ 1 \ \ 10 \ 4 \ (-ve)$

True altitude $= 40 \ \ 17 \ \ 13 \ 6$

Now in the astronomical triangle ZPS,

$$ZP = \text{co-latitude} = 90° - \theta = 90° - (48° 30' 20'') = 41° 29' 40''$$

$$PS = \text{co declination} = 90° - \delta = 90° - (26° 17' 43'') = 63 \ \ 42 \ \ 17$$

$$ZS = \text{co altitude} = 90° - \alpha = 90° - (40° 17' 13'' \ 6) = 49 \ 42 \ 46 \ 4$$

$$2s = \text{sum} = 154 \ 54 \ 43 \ 4$$

$$s = 77 \ 27 \ 21 \ 7$$

Whence, $s - ZP = 35° \ \ 57' \ \ 41'' \ 7$

$s - PS = 13 \ \ 45 \ \ 4 \ \ 7$

$s - ZS = 27 \ \ 44 \ \ 35 \ \ 3$

$$\text{Now} \quad \tan \frac{A}{2} = \left\{ \frac{\sin (s - ZS) \sin (s - ZP)}{\sin s \sin (s - PS)} \right\}^{\frac{1}{2}}$$

$$= \left\{ \frac{\sin 27° 44' \ 35'' \ 3 \ \sin 35° \ 57' \ 41'' \ 7}{\sin 77° 27 \ \ 21'' \ 7 \ \sin 13° \ 45' \ \ 4'' \ 7} \right\}^{\frac{1}{2}}$$

$$\log \tan \frac{A}{2} = 0 \ 0355970 \qquad \frac{A}{2} = 47° 20' \ 43''{\cdot}81$$

$A = 94° \ 41' \ 27'' \ 62$ or W = azimuth of the star

Mean horizontal angle between the star and the line AB

$= 65° \ 24' \ 36''$

Azimuth of the line AB $= 94° \ 41' \ 27'' \ 62 - (65° \ 24' 36'')$

$= 29° \ 16 \ \ 51'' \ 62$ W from north

or " " " " $= 360° - (29° \ 16' \ 51'' \ 62)$

$= 330° 43' \ \ 8'' \ 38$ clockwise from north

Example 5 —On May 11, 1939, an observation for azimuth was made on the sun at a station A in latitude $52° \ 26' \ 40''$ N and longitude $50° \ 22 \ \ 15''$ W, the sun's upper limb being observed and the following results were obtained :

(i) Horizontal angle between the referring object B and the sun $= 48° 12' 13''$

(ii) Observed altitude of the sun $= 46° 32'$

(iii) L M T at observation $=$ 9 h 30 m a m

(iv) Sun's declination at G M N on May 11, 1939 $= 17° 35' 17''.5$ N increasing $31''.18$ per hour

(v) Correction for horizontal parallax $= 8''.72$

(vi) Correction for semi diameter $= 15' 51''.66$

(vii) Correction for refraction $= 57''$ cot (apparent altitude)

Find the azimuth of the line AB

		h	m	s	
Local mean time of observation	$=$	9	30	0	
Longitude $50° 22' 15''$ W $= 3$ h 21 m 29 s W Add long	$=$	3	21	29	$(+ve)$
Corresponding G M T	$=$	12	51	29	
Sun's declination at G M N	$=$	17°	35'	17''.5	
Variation $= + 31''.18$ per hour					
Variation for 51 m 29 s $(= 0.8581$ h $) = 31.18 \times 0.8581 = + 26''.76$	$=$			26.76	$(+ve)$
Sun's declination at the time of observation	$=$	17	35	44.26	

Refraction correction $= 57''$ cot $46° 32'$	$=$		54''.028		$(-ve)$
Correction for parallax $= 8''.72$ cos $46° 32'$	$=$		5''.999		$(+ve)$
Net observational correction	$=$		48''.03		
The observed altitude of the sun	$=$	46°	32'	0''	$(-ve)$
Deduct net observational correction	$= -$			48''.03	
	$=$	46	31	11.97	
Deduct correction for semi diameter	$= -$		15	51.66	
True altitude of the sun	$=$	46	15	20.31	

Using the cosine formula we get

$$\cos PS = \cos ZS \cos ZP + \sin ZS \sin ZP \cos A$$

where $PS = 90° - \delta$, $ZS = 90° - \alpha$, $ZP = 90° - \theta$

Substituting the values of PS, ZS, and ZP, we have

$$\sin \delta = \sin \alpha \sin \theta + \cos \alpha \cos \theta \cos A$$

or $\cos A = \dfrac{\sin \delta}{\cos \alpha \cos \theta} - \tan \alpha \tan \theta.$

$= \dfrac{\sin 17° 35' 44''\cdot26}{\cos 46° 15' 20''\cdot31 \cos 52° 26' 40''}$

$- \tan 46° 15' 20''\cdot31 \tan 52° 26' 40''$

$= 0\cdot7172705 - 1\ 3589053 \quad = -0\cdot6416348$

Since cos A is negative, $\cos 180° - A = +0\cdot6416348$

$180° - A = 50° \ 5' \ 10''\cdot22$

or $A = 180° - (50° 5' 10'' 22)$ $= 129° \ 54' \ 49\cdot78$ E

= azimuth of the sun

Deduct mean horizontal angle between the line and the sun $= 48 \ \ 12 \ \ 13$ (−ve)

Azimuth of AB $= 81° \ 42' \ 36''\cdot78$ clockwise from north

Example 6 —Find the azimuth of the line PQ from the following extra meridian observation for azimuth.

Object	Face	Altitude level. O	E	Horizontal circle. Vernier A	Vernier B
Q	L			35° 25′ 30″	215° 25′ 20″
Sun	”	5 2	4·8	115° 55 50	295 55 40
Sun	R	5·4	4·6	296 13 10	116 13 0
Q	R	.		215 53 40	35 53 30

Vertical circle

Vernier A	Vernier B
26° 35′ 20″	26° 35′ 40″
27 5 30	27 5 50

(i) Latitude of station P = 55° 30′ 30″ N.

(ii) Longitude of ” ” = 3 h. 15 m 10 s E.

(iii) Declination of the sun at G. M. N. = 1° 32′ 12″·1 N decreasing 58″ 24 per hour.

(iv) Mean of L M T s of two observations = 4 h 13 m. 20 s p m by watch; watch 5 seconds slow at noon, gaining 1 2 seconds per day

(v) The value of a level division = 15″.

(vi) Correction for horizontal parallax = 8″·77.

(vii) ” ” refraction = 57″ cot (apparent altitude)

Mean horizontal angle $= \dfrac{80°30\ 20'' + 80°19\ 30''}{2} = 80°\ 24'\ 55''.$

Mean observed altitude = mean of the four vernier readings

$= \dfrac{106°\ 80\ \ 140''}{4} = 26°\ 50\ \ 35''.$

Level correction $= + \dfrac{\Sigma O - \Sigma E}{4} \times$ value of a level division

$= + \dfrac{10\ 6 - 9\ 4}{4} \times 15'' = +\ 4''\ 5$

Apparent altitude $= 26°\ 50\ \ 35'' + 4''\ 5$	$= 26°\ 50\ 39''\ 5$
Refraction correction $= -\ 57''\ \cot 26°50\ 39''\ 5$	$= -\ \ 1\ 52''\ 635$
Correction for parallax $= +\ 8''\ 77 \cos 26°50\ 39''\ 5$	$= +\ \ 7''\ 825$
Net observational correction	$= -\ \ 1\ 44''\ 81$
Apparent altitude	$= 26°\ 50\ 39''\ 5$
Deduct net observational correction	$= -\ \ 1\ 44\ \ 81$
True altitude (α)	$= 26\ 48\ 54\ \ 69$
	h m s
Mean time of observation	$= 4\ \ 13\ \ 20$

Correction for watch $= + \left(5 - \dfrac{1\ 2 \times 4\ 222}{24}\right) = +\ \ 4\ 79$

Corrected mean L M T of observation	$= 4\ \ 13\ \ 24\ 79$
Deduct longitude E	$= 3\ \ 15\ \ 10$
Corresponding G M T	$= 0\ \ 58\ \ 14\ 79$

Sun's declination at G M N	$-1°\ 32\ \ 12''\ 10$
Variation for 58 m 14 79 s $= 58''\ 24 \times (0\ 9707\ \text{h})$	$= -\ \ \ 56\ 53$
Declination at the instant of observation	$= 1\ \ 31\ \ 15\ 57$

Now

$ZP = 90° - \theta = 90° - (55°30\ 30'') = 34°\ 29\ \ 30''$

$ZS = 90° - \alpha = 90° - (26\ 48\ 54\ 69) = 63\ \ 11\ \ \ 5\ 31$

$PS = 90° - \delta = 90° - (\ 1\ 31\ 15\ 57) = 88\ \ 28\ \ 44\ 43$

$2s = 186\ \ \ 9\ \ 19\ 74$

or $s = 93\ \ \ 4\ \ 39\ 87$

Whence,
$$s - ZP = 58^\circ 35' 9'' 87$$
$$s - ZS = 29\ 53\ 34\ 56$$
$$s - PS = 4\ 35\ 55\ 44$$

$$\tan \frac{A}{2} = \left\{ \frac{\sin 58^\circ 35'\ 9''\ 87 \sin 29^\circ 53'\ 34''\ 56}{\sin 93^\circ\ 4'\ 39''\ 87 \sin 4^\circ 35'\ 55''\ 44} \right\}^{\frac{1}{2}}$$

$$\log \tan \frac{A}{2} = 0\ 3626523$$

$$\frac{A}{2} = 66^\circ 32'\ 45''\ 59$$

or Azimuth of the sun $= 133^\circ\ 5'\ 31''\ 18$ W

Add mean horizontal angle between PQ and the sun $= 80\ 24\ 55$ $(+ve)$

Azimuth of the line PQ $= 213\ 30\ 26\ 18$ west from north

Whence, Azimuth of PQ $= 360^\circ - (213^\circ 30'\ 26''\ 18)$

$= 146^\circ 29'\ 33''\ 82$ clockwise from north

Latitude

Latitude of a place may be determined by measuring (1) the altitudes of a circumpolar star at upper and lower transits (2) the meridian altitude of a star or the sun, (3) the extra meridian altitudes of the sun or a star, (4) the extra meridian altitudes of Polaris, (5) circum meridian altitudes of the sun or a star, and (6) the meridian altitudes of two stars

(1) By the Altitudes of a Circumpolar Star at Upper and Lower Transits —In this method the altitudes of a circum polar star are measured at its upper and lower transits (culminations) The observed altitudes are then corrected for refraction and other errors As already explained, the latitude of a place is equal to the altitude of the Pole The altitude of the Pole being the mean of the altitudes of a star at its upper and lower culminations the latitude θ is given by the mean of the two altitudes

$$\text{Latitude } \theta = \tfrac{1}{2}(\alpha_1 + \alpha_2)$$

in which α_1 and α_2 are the corrected altitudes of the circumpolar star The circumpolar star most commonly observed is Polaris (α Ursa Minoris), since it can be easily identified and is very near the pole. In this method the declination of the star need not be known, but the times of culminations must be obtained by calculation This method is used when the declination of the star is not known If the declination of the star is known, the method 2 is preferred The disadvantage of this method is that since the interval between two observations is 12 sidereal hours, one of the observations has to be made in daylight, which cannot be done with a small instrument The method is not, therefore, commonly used

(2) (a) **By the Meridian Altitude of a Star** —In this method the altitude of a star is measured when crossing the meridian with a theodolite on both faces The necessary corrections are then applied to the observed altitude in order to obtain the true altitude Since the declination of the star is known

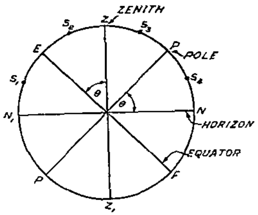


Fig 135

one meridian altitude of the star is sufficient Knowing the altitude (α) and the declination (δ) of the star, the latitude (θ) may be determined as follows Let the plane of the paper represent the plane of the meridian $ZPNZ_1E$ Four cases arise according to the position of the star (Fig. 135).

(2) From the known declination, obtain the polar distance or co declination $(90 - \delta)$ If the sun is observed, its declination at the instant of observation may be deduced as already explained

(3) Knowing the L M T of observation, determine the hour angle of the sun or the star

(a) If the sun is observed, the interval since L M N (i e L M T of observation) should be converted into the interval since L A N, using the equation of time The interval since L A N when converted to arc gives the value of the hour angle (H) of the sun

(b) If the star is observed, the L M T of observation should be converted into sidereal time as already explained Knowing the R A of the star, the hour angle of the star may be obtained from the relation

$$\text{L S T} = \text{R A} + \text{hour angle}$$

The hour angle in time thus obtained, may then be converted to arc

(4) Knowing the sides ZS and PS and the angle ZPS (H) of the triangle ZPS compute the angle PZS (i e the azimuth A) by the Sine rule

$$\sin A = \frac{\sin PS}{\sin ZS} \sin H$$

(5) Having obtained the angle A, calculate the side ZP from the formula

$$\tan \frac{ZP}{2} = \frac{\sin \frac{1}{2} (A + H)}{\sin \frac{1}{2} (A - H)} \tan \frac{1}{2} (PS - ZS)$$

(6) ZP being equal to the co latitude, determine the latitude θ by deducting ZP from 90°

It may here be noted that the local mean time of the observation need not be noted in the case of *an observation on a star*, provided the *direction of the meridian is accurately known*, in which case the azimuth may be obtained by direct measurement Knowing the azimuth (A), the hour angle (H) may be calculated by the sine formula used in step 4

However, in the case of an observation upon the sun it is necessary to note the local mean time in order to find the sun's declination at the instant of observation.

(4) **By Extra-Meridian Observation of Polaris**.—In this method the altitude of Polaris is measured when it is out of the meridian, Face left and Face right observations being taken, and the correct local mean time of each observation being noted. Since Polaris is very close to the pole, the latitude is calculated from the special formula

Latitude $\theta = \alpha - p \cos H + \frac{1}{2} \sin 1'' (p \sin H)^2 \tan \alpha$
in which α = the corrected altitude of Polaris
p = the polar distance, in *seconds* $(90° - \text{declination})$
H = the hour angle of Polaris in arc

From the L M T of observation, the hour angle of Polaris may be obtained as in method 3

Example 1.—The meridian altitude of a star was observed to be 74° 26′ 20″ on a certain day, the star lying between the zenith and the equator

The declination of the star was 45° 56′ 17″·56 N. Find the latitude of the place. (Fig. 135).

Refraction correction	Observed altitude	= 74° 26′ 20″
= 57″ cot 74° 26′ 20″ = 15″ 87.	Deduct refractin correction	= − 15 87
	True altitude	= 74 26 4 13

Now Zenith distance $z = 90° - (74° 26' 4''\cdot13) = 15°33'55''\ 87$

Since the star lies between the pole and the zenith, the latitude θ is given by

$\theta = \delta + z = 45° 56' 17''\ 56 + 15° 33' 55''\ 87 = \mathbf{61° 30' 13''\cdot 43\ N.}$

Example 2:—The meridian altitude of a star was observed to be 72° 30′ 10″ on a certain day, the star lying between the pole and the zenith. The declination of the star was 56° 40′ 38″ N. Find the latitude of the place. (See Fig 135).

	m s	
E T at 0 h	$=+6$ 19 82	Declination $=$ 23 8 20 08
Decrease at 1 177s		at the instant
per hour for 22h 1m	$=-$ 25 91	of observation
$=25$ 91 s		Co declina $=115$ 8 20 0″
E T at the time of observation	$=+5$ 53 91	tion $(90°-\delta)$

		h m s
Interval since L M N		$=$ 2 40 0
Add E T		$=+$ 5 53 91
Interval since L A N		$=$ 2 45 53 91
Hour angle (H) in arc		$=$ 41° 28 28″ 65
Now in the $\triangle$ZPS	ZS	$=$ 45 4 42 75
	PS	$=113$ 8 20 0″
	ZPS $=$ H	$=$ 41 28 28 65

Using the sine rule we get

$$\sin PZS = \frac{\sin PS}{\sin ZS} \sin ZPS = \frac{\sin 113° 8' 20'' 07}{\sin 45° 4' 42'' 75} \sin 41° 28' 28'' 65$$

$\log \sin PZS \quad \bar{1}\ 9345453$

$PZS = 120° 40' 22'' 8 =$ Azimuth (A)

Knowing the angles A and H and the sides ZS and PS, the side ZP may be calculated from the formula

$$\tan \frac{ZP}{2} = \frac{\sin \frac{1}{2}(A+H)}{\sin \frac{1}{2}(A-H)} \tan \frac{1}{2}(PS - ZS)$$

$$A = 120° 40' 22'' 80$$

$$H = 41\ 28\ 28\ 65$$

$$\frac{A+H}{2} = 81\ 4\ 25\ 73$$

$$\frac{A-H}{2} = 39° 35' 57'' 08$$

$$\tan \frac{ZP}{2} = \frac{\sin 81° 4' 25'' 73}{\sin 39° 35' 57'' 08} \tan 34° 1' 48'' 66$$

$$\log \tan \frac{ZP}{2} = 0\ 0197681$$

whence $\quad \dfrac{ZP}{2} = 46° 18' 12'' 74$

$$ZP = 92\ 36\ 25\ 48$$

The latitude of the place $= 2° 36' 25'' 48$ S

Example 5 —In longitude 6° 12 W , an observation for latitude was made on Polaris on a certain day The mean of the observed altitudes was 51° 31 24" and the average of the local mean times 22 h 48 m 6 s The readings of the barometer and thermometer were 30 3o inches ([illegible] cm) and 60° F (15 67° c) respectively Find the latitude, given the following

R A of Polaris = 1 h 41 m 59 15 s , S T of G M M

Declination of = 88° 58 26" 34 , = 17 h 9 m 48 15 s

From Chambers Mathematical Tables,	Observed altitude = 51° 31′ 24"
Mean refraction for the observed altitude = 47"	Deduct refraction = − 47 correction
Correction for barometer = +1	True altitude = 51 30 37
„ , temperature = −1	Declination of Polaris = 88 58 26 34
Refraction correction = 47"(−ve)	Polar distance = 1 1 33 66
Longitude = 6° 12 W	$(p) = (90° - \delta)$ = 3693" 66
= 24 m 48 s W	h m s
Correction for longitude at 9 86 s per hour	G S T of G M M (0 h) = 17 9 48 15
for 24 m = $\frac{24}{60}$ × 9 86 = 3 944s	Acceleration = + 4 08
„ 48 s = $\frac{48}{3600}$ × 9 86 = 0 132s	
Total = 4 076 = 4 08 s	L S T of L M M = 17 9 52 23
	Add S I = 22 51 50 75
h m s	L S T of obs = 40 1 42 98
L M T of obs — 22 48 6	Deduct 24 h = 24
Acceleration at 9 856os per hour	
for 22 h —216 843 s	L S T of obs = 16 1 42 98
48 m = 7 886 s	Deduct R A of Polaris = 1 41 59 15
6 s = 0 016 s	
224 745 s	
Acceleration = 3 m 44 75 s	Hour angle(H) = 14 19 43 83
	H in arc = 214° 55 57" 5
S I since L M M = 22 51 50 75	(L S T = R. A + H)

Now latitude $(\theta) = \alpha - p \cos H + \frac{1}{2} \sin 1'' p^2 \sin^2 H \tan \alpha$

First correction $p \cos H = 3693'' 66 \cos 214° 5ɔ 57'' ɔ$

$= -3028'' 15 = -50 \quad 28'' 15$

Second correction $\frac{1}{2} \sin 1'' p^2 \sin^2 H \tan \alpha$

$= \frac{1}{2} \sin 1'' \ (3693 \ 66) \ \sin^2 214° 5ɔ \ 57'' 5 \times \tan 51° 30 \ 3$

$= 13'' 64$

By formula $\theta = \alpha - p \cos H + \frac{1}{2} \sin 1'' (p \sin H)^2 \tan \alpha$ we have

Latitude $(\theta) = 51° 30 \ 37'' + 50 \ 28'' 15 + 13'' 64$

$= 52° 21 \ 18'' 79$ N

Determination of Longitude

As already explained the difference of longitude between any two places on the earth s surface is equal to the difference of their local times at the same instant whether the times are apparent, mean or sidereal Usually the local time at some instant is obtained by astronomical observation at a place whose longitude is required and the corresponding mean time at some standard meridian such as Greenwich meridian is ascertained The longitude of the place is then found by noting the difference between the two times If the local time is greater than the corresponding Greenwich time the place is east of Greenwich and the long tude of the place is east while if it is less than the corresponding Greenwich time the place is west of Greenwich and the longitude of the place is west The chief methods of determining long tude are (1) by triangulat on (2) by chronometer (3) by time signals

1 By Triangulation —This method is the most accurate but very expensive It involves knowledge of the earth s figure and the calculations are complicated It is described in chapter VIII

2 By Chronometer —The chronometer is a very accurately constructed time piece and is much larger and heavier than a watch It is a very delicate instrument and must be handled with great care It should be wound at the same time each day It may be regulated for either mean time or sidereal time the beat being half a second The temperature of the chronometer must be kept as uniform as possible

In this method the longitude of a place is determined by first finding the local mean time at some instant by any of the methods already described, and then obtaining the corresponding Greenwich time by noting the chronometer reading and correcting it for error and rate, the chronometer being previously compared with Greenwich time and its error and rate being ascertained. By the rate of a chronometer is meant the amount that it gains or loses in 24 hours. The difference between these two times gives the longitude of the place east or west of Greenwich. If the difference of longitude between two places A and B is required, the errors and rates of a number of chronometers (2 or 3) keeping local time at A are first ascertained.

They are then transported to the place B and compared there with the chronometers whose errors and rates on the local time at B have been ascertained. The chief difficulty in this method is to find the travelling rate of a chronometer and to ascertain that it is uniform. It may be noted that the travelling rate, i e. the rate whilst being transported is seldom the same as when stationary This method is now rarely used by the surveyors, but it is still used at sea

3. **By Electric Telegraph**.—If two places are connected by an electric telegraph, the difference of longitude may be determined with great accuracy. In this method a number of telegraphic signals are sent in opposite directions, and by averaging the results, the difference of longitude may be very accurately determined, the error due to the time required for transmission of the signal being entirely eliminated. Suppose, for instance, A and B are two places connected by an electric telegraph, and A is to the east of B. A signal is sent out at A at the time t_1 of A, and it is received at B at the time t_2 of B, where t_1 and t_2 are the times (both solar or sidereal) obtained from the clocks at A and B after correcting them for the errors of the clocks Assuming the transmission of the signal is instantaneous, the difference of longitude $= t_1 - t_2$. However, if s is the time required for the transmission of the signal, $t_1 + s =$ the time at A corresponding to the tim t_2 at B. Then the difference of longitude $(\phi_1) = (t_1+s) - t_2 = (t_1-t_2) + s$. Now suppose that

a signal is sent out in the reverse direction from B to A at the time t'_2 and it is received at A at the time t'_1 Then the difference of longitude $= t_1 - t_2$, if the transmission of the signal is instantaneous But if the time (s) required for the transmission of the signal is taken into account, the difference of longitude (ϕ_2) $= t'_1 - (t'_2 + s) = t'_1 - t_2 - s$ By averaging the results, we have the difference of longitude (ϕ) $= \frac{1}{2}(\phi_1 + \phi_2)$

$$= \tfrac{1}{2}\left\{(t_1 - t_2) + (t'_1 - t'_2)\right\}$$

4 **By Wireless Time Signals** —This method is very simple and the most accurate except triangulation due to the universal development of wireless telegraphy Greenwich time signals are sent out at stated intervals on the standard Rhythmic system from several wireless stations In this system the time signal is sent out at the rate of 61 dots per minute of mean time for five minutes At the beginning of each minute a dash is sent instead of a dot Full particulars regarding wireless stations, wave lengths, times of transmission, etc are given in the Admiralty List of Radio Signals published annually The local mean time at a place may be found by astronomical observations By comparing it with Greenwich mean time as obtained from the time signals, the longitude of the place may be easily determined

The Solar Attachment —The Solar Attachment is a special apparatus fitted to the telescope of an ordinary theodolite for determining the direction of the meridian, the latitude, and local time It is a device which enables the Surveyor to solve the astronomical triangle mechanically However, the results obtained by the use of this device are approximate only within one minute of truth

Fig 137 shows the Burt Solar Attachment which is in common use It essentially consists of (1) the "polar axis" which is fixed to the centre of the trunnion axis of the telescope and perpendicular to the line of collimation (2) The declination arc which revolves about the polar axis It is read by means of a vernier fitted at the end of a rotating (or radial) arm which also carries a lens and a small silver plate at each end Two horizontal and two vertical lines are ruled, upon each silver plate for centering the image of the sun formed by the lens The horizontal lines are known as "equitorial

lines,", while the vertical lines are called hour lines' (4) The hour circle which is attached to declination arc It revolves about the polar axis with the declination arc and is read by a fixed index mark

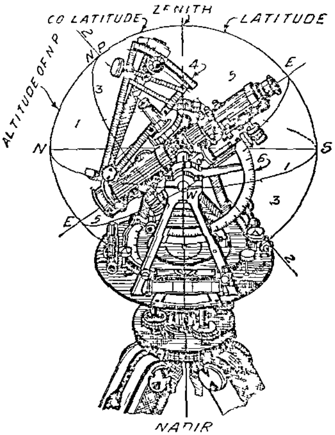


Fig 137

1—1	Azimuth Circle or Horizon	4	Declination Arc
2—2	Polar Axis	5—5	Equator or Hour Circle
3—3	Declination	6	Latitude Arc

(a) Procedure for determining the direction of the meridian and local time —

(1) Set up the instrument over a convenient station and level it accurately by means of the altitude bubble

(2) Set off the co latitude of the place on the vertical circle of the theodolite and the declination of the sun at the time of observation corrected for refraction on the declination arc

(3) Clamp the horizontal plates at zero and turn the instrument about the outer axis until one of the lenses on the arm of the declination arc is directed towards the sun

(4) Rotate the declination arc slowly and also the horizontal limb of the theodolite in azimuth and observe the path of the sun s image on the silver plate

(5) Tighten the lower clamp when the sun's image remains exactly between the equatorial lines The telescope will now lie in the meridian and the polar axis is parallel to the earth s axis

If the azimuth of the line joining the instrument station to any object is to be determined release the vernier plate and the vertical circle and bisect the object in the usual way The mean of the two vernier readings will give the required azimuth

The above procedure may be adopted for determining the local apparent time When the sun s image is brought exactly in the square formed by the horizontal (equatorial) and vertical (hour) lines read the hour circle This reading gives the local apparent time which is then converted into the mean time

(b) To determine the latitude of the observer (i) Set up the instrument and level it accurately

(ii) Correct the declination of the sun at apparent noon (12 o clock) for refraction and set it on the declination arc

(iii) About 15 or 20 minutes before noon direct the telescope towards north and move the telescope and the declination arc from side to side so as to bring the image of the sun between the equatorial lines Clamp the instrument

(iv) Turn the declination arc until it is exactly in line with the telescope (parallel to the telescope) by means of the tangent screw of the hour circle

(v) Bring the image of the sun exactly between the equatorial and hour lines and keep it within the small square of the

solar screen using the vertical circle tangent screw (for vertical adjustment) and the tangent screw of the hour circle (for horizontal adjustment)

(vi) When the image ceases to fall below the lower equatorial line, it is apparent noon when the index of the hour circle should indicate XII Read the vernier of the vertical circle of the theodolite, which gives the co-latitude The required latitude (complement of the angle read on the vertical arc) is then obtained by subtracting it from 90°

PROBLEMS

1 (a) Explain briefly the following —

(i) Various systems of Co-ordinates adopted in Astronomical Surveying
(ii) Correction for refraction (iii) Elongation (iv) Correction for Semi diameter

(b) A star was observed from station A (lat. 5° N) when it was at its western elongation The horizontal angle subtended at A by a reference object B and the star measured clockwise from B was noticed to be 190° 56′ 30″ Find the true bearing of AB if the declination and R A of the star were respectively 74° 27′ 30″ N and 14 h 50 m 54 s.

(c) Find the L. M. T of elongation if longitude of A is 64° 30′ E and Sidereal time of mean noon at Greenwich is 5 h 16 m 54 s (U B)
(Ans. (b) 143° 15′ 30″ 9 (c) 14 h 8 m 58 3s.)

2 The following notes were recorded at 4 p.m on June 14 1916, while determining the azimuth of a reference point P from a station A of a triangulation survey the opposite faces of the theodolite being used in observing the upper and lower limbs of the sun

Latitude of station A 41° 40′ 40″N
True altitude of sun 34° 32′ 50″
Declination at 4 p. m + 23° 17′ 18″

Mean observed horizontal angle of sun, right of reference point 202° 26′ 45″
Find the azimuth of the reference point. (U B)
(Ans. 69° 15′ 25″)

3 A star was observed at western elongation at a station A in latitude 54° 30′ N and longitude 52° 30′ W The declination of the star was 62° 12′ 21″ N and its right ascension 10 h 58 m 36 s., the G S T of G M N being 4 h 38 m. 32 s. The mean observed horizontal angle between

the referring object P and the star was 65° 18 42" Find (a) the altitude of the star at elongation, (b) the azimuth of the line AP, and (c) the local mean time of elongation. (U B)

(Ans. (a) 66° 58 6" 7 (b) 241° 16 19" 7, (c) 9 h 7 m 24 82 s)

4 A star was observed at western elongation at a place in latitude 60° 4' N and longitude 127° 30 W when its whole circle bearing from a reference line OP was 207° 4" Determine the local mean time, and also the azimuth of OP, given that the star s declination was 80° 17 N and its right ascension 9h 49 m 11 s the G S T of G M M. being 16 h 54 m. 13 s. (U P)

(Ans 21 h 40 m 48 31 s , 132° 26 50" 5°)

5 What is meant by (1) a Sidereal day, (2) Apparent Solar day, (3) Mean Solar day? Why is sidereal time of such great use in connection with astronomical observations? State the relation between Sidereal time, Right Ascension and Hour angle

Find the local mean time of transit of a star in longitude 7° 18 E on December 26 Given that the Sidereal time at Greenwich Mean Noon = 18 h. 18 m 48 s and P A of the star 10h 2 m. 34 s. (U P)

(Ans 15 h 41 m. 16 18 s)

6 The meridian altitude of a star was observed to be 75° 18 25" on Octo. 15 1916 the observation being made with face left, the star lying between the zenith and the pole The declination of the star on the given date was 58° 41 43" N and the index correction —5" Find the latitude of the place of observation (U P)

(Ans 43° 59 48" 04)

7 State the various instrumental and other corrections which must be applied to the observed altitude of a heavenly body and explain the reasons for each correction Find the local mean time from the following data

Longitude of the place 45° 30 15"E R A of star 6 h. 32 m 5° 44 s. Date March 3rd 1916 Hour angle of star at given instant 4 h. 3 m. 18s. S T at G M N 2° h 43 m 42 69 s (U P)

(Ans 11 h 51 m 0 85 s. P M March 2.)

8 At a place in latitude 52° 45'N and longitude 80° 30 W., a star was observed at eastern elongation when its clockwise horizontal angle from a survey line was 84 24 12" The declination of the star was 60° 55 2" N and its right ascension 8 h 25 m 12 8 s the G S T of G M M being 4 h 6 m 24 s Find the azimuth of the survey line and the local mean time of elongation (U P)

(Ans. 329° 0' 3" 38, 1 h 25 m 39 33 s)

9 From the following data determine (i) the true altitude of the sun and (ii) the declination of the sun at the instant of observation

Observed altitude of the upper limb	= 46° 26 46"
Index correction	= + 10"
Readings of altitude bubble	= 6 8 O, 5 8 E

Value of 1 bubble division	= 15″
Semi diameter	= 15 49″
Declination of the sun at G M N	= 14° 24 42″ 7 N decreasing at 46″ 5 per h
G M T of observation	= 2 h 30 m p m

(Ans (i) 46° 10 25″ 38 (ii) 14° 22 45″ 75)

10 An observation was made on a star lying west of the meridian at a place in latitude 40° 20 36″N to determine the azimuth of the survey line AB The mean observed altitude was 42° 10 24″ and the clockwise horizontal angle from AB to the star was 100° 18 48″ The declination of the star was 24° 54 35″N Find the azimuth of the survey line AB

(Ans 168° 20 19 74)

11 Determine the azimuth of the survey line AB from the following data

Latitude of the place	= 48° 34 40″ N
Mean observed altitude of the sun	= 40° 50 20″
Mean of G M T of two observations	= 3 h 30 m p m
Mean clockwise horizontal angle from the survey line to the sun	= 98° 17 24″
Declination of the sun at G M N	= − 15° 45 24″ decreasing at 43″ 8 per hour

(Ans 145° 43 32″ 96)

12 A star was observed for time by equal altitudes when on the prime vertical at a place in latitude 34° 20 N given that the declination of the star was + 20° 30 38″ 4″ and its R A 16 h 51 m 15 89 s Determine the altitude when on the prime vertical and local sidereal times of prime vertical transits

(Ans 38° 25 26″ 97 5 h 38 m 25 28 s 22h 4 m 6 50 s)

13 Determine the error of chronometer from the following data

Observed altitude of the star east of meridian	= 38° 12 42″
Latitude of the place	= 42° 15 30″ N
Longitude of the place	= 72° 48 E
Declination of the star	= 23° 16 6″ 4 N
R A of the star	= 6 h 0 m 24 63 s
G S T of G M M	= 1 h 2m 54 63 s
Chronometer time	= 1 h 3 m 10 s. A M

(Ans 5 02 s fast)

14 From the following observations determine the error of the chronometer:

True altitude of the sun	= 46° 20 21″ 9
Latitude of the place	= + 52° 38 36″
Longitude of the place	= 62° 24 E
Declination of the sun at G M N	= + 18°35 50″ 6 increasing at 35″ 92 per hour
E T at G A N	= + 3 m 45 01s.
Approximate time of observation	= 14h 30 m

(Ans 2 m 52 92 s fast)

15. Determine the latitude of the place from the following observations

Mean altitude of Polaris	= 50° 36′ 28″.
L. M. T of observation	= 21 h. 35 m 15 s.
G S. T. of G. M M.	= 17h. 57 m 6·85 s.
Longitude of the place	= 60° 45′ W.
Declination of Polaris	= 88° 58′ 26″·24
R A of Polaris	= 1 h 41 m 59·15 s.
Correction for barometer reading 30·3 inches	= + 1″.
temperature 58° F.	= − 1″.
Mean refraction correction	= −47″

(Ans. 51° 29′ 51″·84 N).

+ + +

GEODETIC SURVEYING

Geodetic or Trigonometrical surveying differs from plane surveying in that it takes into account the curvature of the earth, since very extensive areas and very large distances are involved In work of this nature highly refined instruments and methods are used Geodetic work is usually undertaken by the State agency In India it is done by the Survey of India department It involves two operations (i) triangulation, and (ii) precise levelling

The object of geodetic surveying is to accurately determine the relative positions of a system of widely separated points on the surface of the earth, and also their absolute positions The relative positions are determined in terms of the azimuths and lengths of the lines joining them and the absolute positions in terms of latitudes and longitudes, and elevations above mean sea level In geodetic work distances are usually expressed in metres The geodetic points so determined furnish the most precise control for a more detailed survey of the intervening country The methods employed in geodetic surveying are (1) Triangulation and (2) Precise Traversing The former is the most accurate method and is invariably used while the latter is inferior and is mainly used in cases where triangulation is physically impossible or very expensive, e g densely wooded country or very flat country

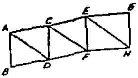


Fig 138

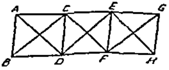


Fig 139

Triangulation —Triangulation is based on the trigonometrical proposition that if one side and the three angles of a triangle be known the remaining sides can be computed by the application of the sine rule In this method suitable points called triangulation stations are selected and established through-

out the area to be surveyed The stations may be connected by a chain of triangles (Fig 138) or a chain of quadrilaterals as shown in Fig 139 These stations form the vertices of a series of mutually connected triangles, the complete figure being called a *triangulation system* In this system of triangles one line, say AB, and all the angles are measured with the greatest care and the lengths of all the remaining lines in the system are then computed For checking both the field work and computations another line such as GH is very accurately measured at the end of the system The line whose length is actually measured is known as the *base line* or the *base* and that measured for checking purposes is called the *check base* When the work is of a large extent, intermediate check bases are introduced The triangulation stations at which azimuth, latitude, or longitude is directly determined by astronomical observations are called azimuth latitude or longitude stations respectively

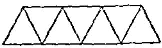


Fig 140

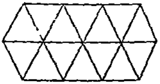


Fig 141

Triangulation Figures —The geometrical figures used in a triangulation system are (1) triangles (2) quadrilaterals and (3) quadrilaterals pentagons, or hexagons with central stations If it is desired to connect two distant points by a triangulation system a chain of single triangles as shown in Fig 140 may be used This arrangement, although simple and economical, is the least accurate, since the number of rigid geometrical conditions to be fulfilled in the figure adjustment is comparatively small If the greatest area is to be covered a double row of single triangles (Fig 141) or a chain of hexagons (Fig 142) may be used This arrangement covers greater area and gives more accurate results than the first system For very accurate work, a chain of quadrilaterals as in Fig 143 may be used. A

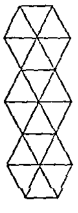


Fig 142

geodetic quadrilateral is the ordinary quadrilateral with both its diagonals included There is *no station* at the *intersection* of the

Fig 143

diagonals This system is the most accurate, since the number of conditions involved in its adjustment is much greater than in the first two systems

On very extensive surveys (e g Indian Survey) primary triangulation is usually laid out in two series of chains of triangles, *one series of chains of triangles is* run roughly along the meridian (north and south) while the other approximately at right angles to the meridian (east and west) as in Fig 144 The areas enclosed are then covered with a network of smaller triangles of secondary and tertiary accuracy This system is known as the *gridiron* system Another system called the *central* system is used for the survey of an area of moderate extent e g Great Britain In this system the whole area may be covered with a network of primary triangles extending outwards in all directions from the initial base line Every triangulation system is essentially made up of triangles In order to minimise the effect of small errors in measurement of angles, the triangles should be well shaped or well proportioned, i e they should have no angles less than 30° nor greater than 120°, since a given error in a small angle produces a much larger effect in the computations than the same error in an angle nearing 90° The best shaped triangle is equilateral and the best shaped quadrilateral is square Wherever possible, the triangles should be equilateral or nearly so, and the quadrilaterals the squares

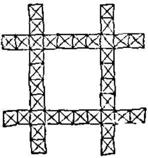


Fig. 144

Classification of Triangulation Systems —Triangulation systems may be classified according to the degree of accuracy

desired and the magnitude of the work as (1) primary or first order (2) secondary or second order and (3) tertiary or third-order

Primary or First Order Triangulation —In primary triangulation very large areas (the whole country) are covered and the highest possible degree of precision is secured In work of this character large and well proportioned triangles and most refined instruments and methods of observation and computation are used It furnishes the most precise horizontal control for small scale mapping surveys The average triangle closure is one second and the maximum three seconds The length of the base line varies from 5 to 20 or more km and that of the sides of the triangles from 30 to 160 or more km The degree of accuracy is 1 in 500 000 and the check on the base 1 in 25 000

Secondary or Second-Order Triangulation —Within the primary triangles other points are fixed at closer intervals so as to form a secondary series of triangles which are tied to the primary system at intervals In work of this nature comparatively smaller triangles are used The instruments and methods used are not of the same utmost refinement The average triangle closure is 3 seconds and the maximum one 8 seconds The length of the base line varies from 2 to 5 km and the length of the sides of the triangles from 8 to 70 km The degree of accuracy is 1 in 50 000 and the check on base 1 in 10 000

Tertiary or Third Order Triangulation — Within the secondary triangles points are established at short intervals to furnish horizontal control for detail surveys In this case the triangles are still smaller and 12 cm to 20 cm instruments are used The average triangle closure is 6 seconds and the maximum one 12 seconds The base line varies from 1 to 3 km in length and the triangle sides from 1 5 to 10 km in length The degree of accuracy is more than 1 in 5000 while the check on base 1 in 5000

The triangulation work is carried out in the following steps — (1) Reconnaissance (2) Erection of signals and towers (3) Measurement of horizontal angles (4) Astronomical observations necessary to determine the true meridian and the absolute positions of the stations (5) Measurement of the base lines

(6) Computations including (a) adjustment of the observed angles, (b) computation of the lengths of the sides of each triangle, and (c) computation of the latitudes and longitudes of the stations It may be observed that the azimuths of all the sides, and the latitudes and longitudes of all the stations can be calculated, if the azimuth of one side of the triangulation system, and the latitude and longitude of one station are determined However, astronomical observations for azimuth, latitude, and longitude are made at intervals for checking purposes

Reconnaissance —The reconnaissance is of the greatest importance, since the economy and accuracy of triangulation work depend to a great extent upon an exhaustive reconnaissance It requires skill, experience, and judgment on the part of the chief of the reconnaissance party It consists of (a) examination of the country to be surveyed, (b) selection of the most favourable sites for base lines, (c) selection of suitable positions for triangulation stations (d) determination of (i) intervisibility of stations and computation of the heights of towers and signals, and (ii) the amount and direction of cutting and clearing necessary to make the line of sight clear of the obstructions, and (e) collection of information regarding (i) access to the stations, (ii) transport facilities, (iii) supplies of food, water, and other materials required, and (iv) camping ground or suitable accommodation

The instruments required for the reconnaissance are (1) a small theodolite and a sextant for measuring angles, (2) a prismatic compass for measuring directions, (3) an aneroid barometer for determining the elevations, (4) guyed ladders, ropes, creepers for climbing trees, (5) a steel tape, (6) a powerful field glass, and (7) drawing instruments and materials

The reconnaissance party is often called upon to establish the direction of the line joining two stations which are not intervisible on account of forest growth, but which can be seen from each other after the forest growth is cleared out along the line In such a case, the direction of the connecting line may be computed by selecting two points from each of which the two stations and the other point are visible.

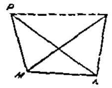


Fig 145

Thus in Fig 145, let PQ be the line whose direction is to be determined M and N the points from each of which both points P and Q are visible. The angles PMQ and QMN are then observed at M Similarly the angles MNP and PNQ are measured at N Assuming the value of MN as unity, find the lengths of PN and QN from the triangles PMN and QMN respectively Now in the triangle PNQ the two sides PN and QN and the included angle PNQ are known and therefore the other angles QPN and PQN may easily be calculated The line may then be aligned from P or Q

Selection of Stations —In selecting stations a careful study of all existing maps of the region should be made since the information regarding the height and the relative location of the stations and the possible arrangement of triangles can be had from such maps If they are not available small scale maps should be prepared by conducting a rough triangulation The selection of stations is based upon the following considerations —

(1) The stations should be clearly visible from each other. For this purpose the highest available ground (commanding positions) such as tops of hills or mountains is selected

(2) They should form well shaped triangles.

(3) They should be easily accessible

(4) They should be useful for detail surveys

(5) They should be so fixed that the length of sight is not too large nor too small

When the length of sight is too large the signal may be too indistinct for accurate bisection while if it is too small errors of centering and bisection become appreciable

(6) They should be so located that the cost of clearing and cutting and of building towers is minimum

Station Marks —The triangulation station should be permanently marked with copper or bronze tablet on which its name and the year in which it is set are stamped It should be referenced with at least two reference marks and the reference sketches giving a complete description of the station and its location should be drawn for identification and future use The station mark is securely set in rock or in a concrete monument In earth, two marks are set, one about 75 cm below the surface of

ground and the other extending a few cm above the surface The underground mark may consist of a stone with a copper bolt in the centre or a concrete monument with a tablet mark set in it

Intervisibility and Height of Stations —In order that two stations may be intervisible the line of sight must clear all the intervening obstructions Stations are therefore fixed on the highest available ground such as summits mountain peaks ridges, or tops of hills Whether the proposed stations are actually intervisible or not can usually be ascertained by direct observation at the ground level or from tops of trees or ladders But when the distance between two stations is great and the difference in elevation between them is small it is necessary to raise both the instrument and the signal to overcome the curvature of the earth and to clear all the intervening obstructions In such a case, the height of the station must be determined by calculation It is well to note the distinction between the terms the *elevation* (or *altitude*) of a station and the *height* of a station By the elevation of a station is meant the elevation of the observing instrument above mean sea level while the height of a station is the elevation of the instrument above the natural ground The height of both the instrument and the signal above the ground depends upon (1) the distance between the stations (2) their relative elevations and (3) the profile of the intervening country

(1) **Distance Between the Stations** —If the intervening ground is free from any obstruction the distance of the visible horizon from a station of known elevation above datum as well as the elevation of the signal which may be just visible at a given distance may be determined from the formula

$$h = (1 - 2m) \frac{D^2}{2R} \qquad (1)$$

in which h = the height of the station above a datum

D = the distance from the station to the point of tangency,

R = the mean radius of the earth

m = the coefficient of refraction (0 07 for sights over land and 0 08 for sights over water)

h D and R being expressed in the same units
Alternatively from the formula $h = 0.0673\ D^2$ (1a)
in which h is in m and D in km

(2) **Relative Elevations of Stations** —In Fig 146 let

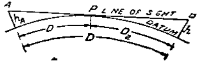


Fig 146

A and B = the two stations
D = the distance in km between A and B
h_A = the known elevation of A above datum.
h = the required elevation at B above datum.
D_1 = the distance in km from A to the point (P) of tangency
D_2 = the distance in km from B to the point (P) of tangency

Then the distance D_1 may be calculated from the formula
$h = 0.0673\ D^2$

$$h_A = 0.0673\ D_1^2 \text{ or } D_1 = \sqrt{\frac{h_A}{0.0673}}$$

Now $D_2 = D - D_1$

Whence the required elevation $= h = 0.0673 D_2^2$
Knowing the ground level at B and the elevation h it may be ascertained if the station B requires to be elevated, and if so, the height of scaffold at the station B may be determined from
Height of Scaffold at B = elevation of datum + h − R L of station B

The actual height of the signals and instruments will be greater than the calculated one by a few metres

It is not advisable to have the line of sight close to the surface of the ground at the point of tangency on account of the strata of disturbed air and the risk of lateral refraction. It should therefore be kept at least 2 m above the ground preferably 3 m and this allowance should be made in determining the heights of stations

(3) **Profile of Intervening Ground** —If the peaks in the intervening ground are likely to obstruct the line of sight, their

elevations and locations must be ascertained The elevations of the line of sight at the respective points may then be computed and the results compared with the ground elevations at those points to ascertain whether the line of sight clears all the intervening obstructions The method of procedure is shown in the following examples

Towers —(Fig 147) When the station is to be elevated a rigid support must be provided for the instrument and the signal A tower is a structure erected over a station for the support of the instrument and the observing party It consists of two independent structures, the inner tripod supports the instrument and the outer scaffold entirely surround

Fig 147

ng the instrument tripod carries a platform for the observing party and a light awning at its summit to protect the instrument from the sun and wind The two scaffolds are built entirely independent so that any movement of the outer scaffold due to movements of the observing party or to the wind may not be transmitted to the inner one They must be properly braced and guyed so as to make them absolutely immovable (or rigid) Towers may be of timber, steel or masonry For small heights, masonry structures are most suitable Steel towers made of light sections and rods (Bilby towers) are very portable and can be easily erected and dismantled

Example 1 —The triangulation stations A and B, 50 km apart, have elevations 243 m and 258 m respectively The intervening ground may be assumed to have a uniform elevation of 216 m Find the minimum height of the signal required at B, so that the line of sight may not pass nearer the ground than 2.4 m

$$\text{Minimum elevation of the line of sight} = 216 + 2.4$$
$$= 218.4 \text{ m}$$

Thus elevation being taken as the datum level, the elevation of $A = h_1 = 243 - 218.4 = 24.6$ m

The tangent distance D_1 corresponding to h_1 may be calculated from the formula $h = 0.0673 D^2$

$$24.6 = 0.0673\ D_1^2 \text{ or } D_1 = \sqrt{\frac{24.6}{0.0673}} = 19.12 \text{ km}$$

$$\therefore\ D_2 = D - D_1 = 50 - 19.12 = 30.88 \text{ km}.$$

The elevation h_2 corresponding to the distance D_2 is

$$h_2 = 0.0673 \times (30.88)^2 = 64.18 \text{ m}$$

$\therefore$ The elevation of the line of sight at B $= 218.4 + 64.18$
$= 282.58$ m

Ground level at B = 258 m.

$\therefore$ Minimum height of signal above ground at B
$= 282.58 - 258 = 24.58$ m say 25 m.

Example 2.—The elevations of two triangulation stations A and B, 120 km apart, are respectively 210 m and 1050 m above mean sea level. The elevations of two peaks C and D on the profile between them are respectively 360 m and 542 m the distances being AC = 50 km and AD = 80 km. Ascertain if A and B are intervisible, and, if necessary, find the minimum height of a scaffolding at B, assuming A as the ground station.

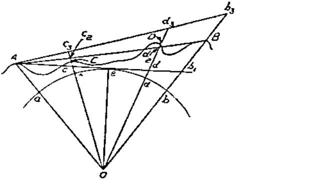


Fig. 148

In Fig. 148, let a horizontal sight through A cut the horizon in e.

Then the distance Ae to the visible horizon from station A of an altitude 210 m is given by

$$D = \sqrt{\frac{h}{0.0673}}. \quad \therefore\ Ae = \sqrt{\frac{210}{0.0673}} = 55.86 \text{ km}$$

Now AC = 50 km; AD = 80 km; and AB = 120 km

Whence, $ec = 55\ 86 - 50 \quad = 5\ 86$ km.

$ed = 80 \quad - 55\ 86 = 24\ 14$ km.

$eb = 120 \quad - 55\ 86 = 64\ 14$ km.

The corresponding heights cc_1, dd_1, and bb_1 can be obtained from the above formula Thus we get

$cc_1 = 0\ 0673 \times (5 \cdot 86)^2 = 2\ 311$ m

$dd_1 = 0\ 0673 \times (24\ 14)^2 = 39\ 21$ m

$bb_1 = 0\ 0673 \times (64\ 14)^2 = 227\ 0$ m

To ascertain if the line of sight AB will clear the peaks C and D, we have

$$\frac{c_1c_2}{b_1\text{B}} = \frac{\text{A}c_1}{\text{A}b_1} = \frac{50}{120}; \quad \frac{d_1d_2}{b_1\text{B}} = \frac{\text{A}d_1}{\text{A}b_1} = \frac{80}{120}.$$

But $b_1\text{B} = 1050 - 277\ 0 = 773\ 0$ m

$$c_1c_2 = \frac{50}{120} \times 773\ 0 = 322\ 08 \text{ m}.$$

$$d_1d_2 = \frac{80}{120} \times 773\ 0 = 515\ 33 \text{ m}.$$

The elevation of the line of sight at C $= cc_1 + c_1c_2$
$= 2\ 31 + 322\ 08 = 334\ 39$ m.

The elevation of the line of sight at D $= dd_1 + d_1d_2$
$= 24\ 14 + 515\ 33 = 539\ 47$ m

Now the elevation of C $= 360$ m and that of D $= 542\ 0$ m

Thus the line of sight clears the peak at C, but fails to clear that at D by $542 - 539\ 47 = 2\ 53$ m

To clear by $3\ 0$ m at D, $d_2d_3 = 3\ 0 + 2\ 53 = 5\ 53$ m. The line of sight should therefore, be raised at B by the amount

$$\text{B}b_2 = \frac{\text{AB}}{\text{AD}}\, d_2d_3 = \frac{120}{80} \times 5\ 53 = 8\ 30 \text{ m}$$

Hence the minimum height of the scaffold at B $= 8\ 30$ m

The above procedure may be adopted when there is only one intervening peak

Example 3 —The elevations of two triangulation stations A and B 100 km apart, are 180 m and 450 m respectively The intervening obstruction situated at C, 75 km from A has

an elevation of 259 m Ascertain if A and B are intervisible If not, by how much B should be raised so that the line of sight must nowhere be less than 3 m above the surface of the ground, assuming A as the ground station ?

(i) Drawing a figure similar to Fig 148 and using the same notation, we have.

The distance Ae to the visible horizon from station A is

$$= \sqrt{\frac{h}{0\ 0673}}$$
$$= \sqrt{\frac{180}{0\ 0673}} = 51\ 72\text{ km.}$$

Now $Ac = 75$ km.

$$ec = Ac - Ae = 75 - 51\ 72 = 23\ 28\text{ km.}$$
$$eb = AB - Ae = 100 - 51\ 72 = 48\ 28\text{ km}$$

Whence $cc_1 = 0\ 0673\ (23\ 28)^2 = 36\ 48$ m.
and $bb_1 = 0\ 0673\ (48\ 28)^2 = 156\ 8$ m

(ii) To ascertain if the line of sight AB will clear the obstruction at C, we have

$$\frac{c_1c_2}{b_1B} = \frac{Ac_1}{Ab_1} = \frac{75}{100} \quad \text{But } b_1B = bB - bb_1 = 450 - 156\ 8 = 293.2\text{ m}$$

$$\therefore\ c_1c_2 = \frac{75}{100} \times 293\ 2 = 219\ 9\text{ m.}$$

Now the elevation of the line of sight at $C = cc_1 + c_1c_2$
$= 36\ 48 + 219\ 9 = 256\ 38$ m.

But the elevation of the obstruction at C = 259 00

Thus the line of sight AB fails to clear the obstruction at C by $259\ 00 - 256\ 38 = 2\ 62$ m.

(iii) To clear by 3 m, the line of sight should be raised at C by an amount $= 2\ 62 + 3 = 5\ 62$ m

$\therefore\ c_2c_3 = 5\ 62$ m.

Whence, the line of sight must be raised at B by an amount

$$Bb_2 = \frac{AB}{AC} \cdot c_2c_3 = \frac{100}{75} \times 5\ 62 = 7\ 49\text{ m}$$

$\therefore$ Minimum station height above the ground at B
$= 7{\cdot}49$ m say $7{\cdot}5$ m.

Alternative method —By Captain G T. McCaw's solution.

In this method the height of the line of sight at the intervening obstruction may be obtained by the formula

$$h = \tfrac{1}{2}(h_2+h_1) + \tfrac{1}{2}(h_2 - h_1)\ \frac{x}{s} - (s^2 - x^2)\ \text{cosec}^2\zeta \left(\frac{1-2K}{2R}\right)$$

In which $2s$ = the distance between the two stations (A and B).
$s + x$ = the distance of the obstruction from station A
$s - x$ = „ „ „ „ „ station B
h_1 = the height of station A
h_2 = the height of station B
h = the height of the line of sight at the obstruction C

$\text{cosec}^2\zeta$ may be taken equal to unity; $\left(\dfrac{1-2K}{2R}\right) = 0{\cdot}0673.$

Hence $2s = 100$ km, $s = 50$ km, $s + x = 75$ km, $s - x = 25$ km;
$x = 25$ km
$h_1 = 180$ m ; $h_2 = 450$ m

$$\therefore\ \tfrac{1}{2}(h_2 + h_1) = \frac{180+450}{2} = \frac{630}{2} = 315 \text{ m}$$

$$\tfrac{1}{2}(h_2 - h_1) = \frac{270}{2} = 135 \text{ m}$$

Now $s^2 - x^2 = 75 \times 25 = 1875$

$$(s^2 - x^2)\left(\frac{1-2K}{2R}\right) = 0\ 0673 \times 1875 = 126\ 2 \text{ m}$$

Hence $h = 315 + 135 \times \dfrac{25}{50} - 126\ 2 = 315 + 67{\cdot}5 - 126{\cdot}2$
$= 256\ 3.$

The height of the obstruction at C $= 259$ m

$\therefore$ The line of sight AB fails to clear the obstruction at C by $259 - 256\ 3 = 2\ 7$ m

To clear by 3 m the line of sight should be raised at C by an amount $= 2\ 7 + 3 = 5\ 7$ m

Minimum station height above the ground at B

$$= \frac{100}{75} \times 5{\cdot}7 = 7{\cdot}6 \text{ m}$$

Signals —A signal is any object such as a pole, target erected at a station upon which a sight is taken by the observer at another station The signals may be classified as (i) non luminous (opaque), (ii) sun signals, and (iii) night signals The signals should be conspicuous (clearly visible against any background) free from *phase*, capable of being accurately centred over the station and readily bisected When the cylindrical signal is partly illuminated and partly in the shadow, the observer sees only the illuminated portion and bisects it The error of bisection thus *introduced is called phase It is the apparent displacement of* the centre of the signal It is therefore, necessary to apply the correction for phase to the observed direction in order to determine that to the centre of the signal

There are two cases according as (1) an observation is made on the bright portion and (2) an observation is made on the bright line

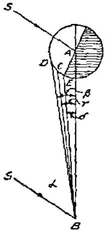


Fig 149 (a)

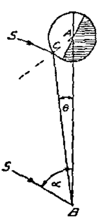


Fig 149 (b)

(1) When the bright (illuminated) portion is bisected

In Fig 149 a, B is the position of the observer, A the centre of the signal, the visible portion of the illuminated surface extends from D to E, BC the line of sight. Then the phase correction (γ) is

$$CBA = \gamma = \tfrac{1}{2}(\beta + \delta) \text{ radians} = \frac{r \cos^2 \frac{1}{2}\alpha}{D \sin 1''} \text{ seconds}$$

in which r = the radius of the signal, α = the angle which the direction of the sun makes with BA, D = the length of sight

(2) When the observation is made on the bright line formed by the reflected rays, SCB representing their path. (Fig 149 b)

The phase correction θ is given by

$$=\theta \frac{r \cos \frac{1}{2} \alpha}{D \sin 1''} \text{ seconds}$$

The observed angle is then corrected by applying the correction algebraically according to the relative position of the sun and the signal

Opaque Signals —The opaque signals include the pole or target signals For sights under 7 km, pole signals consisting of round poles painted black and white in alternate sections and supported by a tripod or quadripod (Fig 150) may be used For distances upto 25 or 30 km the target signals are generally used The target signal consists of a pole carrying two square or rectangular targets made of cloth stretched on wooden frames and placed at right angles to each other

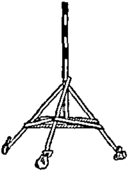


Fig 150

Sun Signals —The sun signals include the heliotrope and heliograph They are invariably used when the distances between stations exceed about 30 km The heliotrope essentially consists of (i) a plane mirror which reflects the rays of the sun and (ii) a line of sight to transmit the reflected light in the direction of the observer s station

Night Signals —The night signals are used for night observations They include (i) large kerosene oil lamps with Argand burners furnished with parabolic reflectors used for sights under about 80 km (ii) the acetylene gas lamps (McCaw lamp) (iii) Drummond s lights consisting of a small ball of lime placed in the focus of a parabolic reflector and raised to a very high tem-

perature by impinging upon it a stream of oxygen gas, (iv) electric lamps, and (v) the magnesium lamps with parabolic reflectors, used for long lines

Atmospheric Conditions —Horizontal angles should be measured when the air is the clearest and the lateral refraction minimum, the best time in clear weather being from 6 a m to 9 a m and from 4 p m till sunset In densely clouded weather satisfactory work can be done all day. First order work is generally done at night, since night observations are more accurate than day observations and the number of hours a day available for good work is doubled Night operations are confined to the period from sunset to midnight The best time for measuring vertical angles is from 10 a m to 2 p m when the vertical refraction is the least variable

Measurement of Angles

Instruments for Measuring Angles —The geodetic instruments (theodolites) used in triangulation work are specially designed for very accurate measurement of horizontal angles They differ from those used in plane surveying in the following respects —

(1) They are made of the best material (2) They are of a larger size and of a higher grade of workmanship (3) The telescope must be of the best quality and of high magnifying power (usually ranging from 30 to 80) (4) The size of the aperture of the objective is large (6 5 to 7 5cm) (5) Greatest care is taken in marking the graduations and in making and fitting the centres (6) All the levels are more sensitive, the sensitiveness of the plate bubbles being 10 to 20 seconds per division and that of the striding level 1 to 5 seconds per division (7) The cross hairs used in the telescope are of different pattern For sighting poles or targets, cross hairs placed in the form of × are used (the angle at which they are set varying from 45° to 90°), while for observing light signals two parallel vertical hairs are used, the distance between them being such as to subtend an angle of 25 to 35 seconds (8) Lifting rings are provided

It was formerly supposed that greater precision could be secured by increasing the size of the circles (the larger the circle

the greater the accuracy) and, therefore the 45 cm (18 in) to 90 cm (36 in) instruments were used for triangulation work of different grades But it has now been found that there is no advantage in having the circles more than 30 cm in diameter The 25 cm to 30 cm instruments are now used for first order work, 18 cm to 20 cm instruments for second order work, and 15 cm instruments for third order work

There are two types of instruments used in triangulation of high precision, viz (1) the *repeating* instrument and (2) the *direction* instrument The repeating instrument has a double vertical axis (two centres and two clamps) and is provided with two or more verniers reading to 10 to 5 seconds It is used when the angles are to be measured by the method of repetition. The direction instrument has only one vertical axis so that it cannot be used for measuring angles by repetition and is provided with two or three micrometer microscopes instead of verniers to read fractional parts of the angle smaller than the smallest division of graduated circle The horizontal circle of 25 cm or 30 cm instrument is usually graduated to 5 minutes and by means of micrometer microscopes an angle can be read directly to the nearest second and 0 1 second by estimation The direction instruments are generally used for primary triangulation, while the repeating instruments are used for secondary and tertiary triangulation

In recent times the geodetic theodolites of the micrometer type are being replaced by those of the double reading type" such as the Zeiss Wild, or Tavistock theodolite The distinguishing features of the double reading theodolite with optical micrometer are

(1) It is very small and light Its horizontal circle is made of glass and is only 7 to 8 cm in diameter

(2) The graduations of the circle are very much finer

(3) All readings can be taken from the eye piece end of the telescope The observer need not, therefore, move round the instrument to read the different levels and micrometers

(4) By means of a system of prisms the graduat ons of the diametrically opposite parts of the circle are brought together

or slightly separated into the same field of view and can be read in a single microscope

(5) The micrometers are so designed that the observed single reading gives the mean of the readings at the diametrically opposite parts of the circle, thus eliminating the error due to eccentricity of the circle

(6) It is completely water proof and dust proof

(7) It is electrically illuminated

Adjustments of the Instrument —The adjustments of the geodetic instrument are the same as those of the ordinary engineer's transit Prior to the measurement of horizontal angles, the following adjustments must be carefully made and preserved

(1) Plate bubble adjustment, (2) Striding level adjustment, (3) Collimation adjustment, (4) Horizontal axis adjustment, (5) Micrometer adjustment (in the case of the direction instrument)

Methods of Observation —There are two general methods of observing angles in triangulation work (1) the method of *repetition* and (2) the *direction* method also called the method of *series*, or the *reiteration* method The former is used in secondary or tertiary triangulation, while the latter is employed in primary triangulation

I The Method of Repetition —In this method the angle is measured a number of times by successive additions on the limb The true value of the angle is then obtained by reading this multiplied angle and dividing it by the number of repetitions Suppose it is required to measure the angle AOB (Fig 151) It is measured three times with the telescope *direct* or *normal* (Face Left) and the equal number of times with the telescope reversed or inverted (Face Right) Its

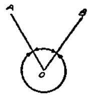


Fig 151

explement, i e the exterior angle BOA is also measured the same number of times and in exactly the same manner, measuring the exterior angle being called *closing the horizon* The observed values of the two angles are then added, and if their sum differs from 360°, the discrepancy is equally divided among

the two angles The six repetitions of the angle and the same number for its explement make one set Six such sets are usually taken for first order work, while two to four such sets for second order and third order work The results are then averaged to determine the true value of the angle

Programme of Measurement —(i) Centre the instrument over O and level it accurately Set vernier A to zero and read vernier B (or all verniers if there are more than two verniers)

(ii) Set the telescope direct, swing clockwise, and bisect the left station A

(iii) Loosen the upper clamp, turn clockwise and bisect the right station B

(iv) Read vernier A to find the approximate value of the angle

(v) Unclamp the lower plate, turn clockwise, and again bisect A

(vi) Release the upper clamp, swing clockwise and set on B

(vii) Loosen the lower clamp, turn clockwise, and set on A.

(viii) Slacken the upper clamp, swing clockwise, and set on B

(ix) Reverse the telescope, and leaving verniers *unchanged*, turn clockwise and set on A Make three repetitions exactly as above

(x) Read both verniers (or all verniers if there are more than two verniers)

(xi) Leaving the verniers *unchanged* and the telescope *reversed*, set on B Measure the exterior angle BOA three times in exactly the same manner

(xii) Set the telescope direct and again measure it three times

(xiii) Finally read both verniers (or all verniers if there are more than two verniers)

(xiv) Relevel the instrument, if necessary, and proceed as above for the second set, the verniers remaining unchanged.

If it is required to measure more than one angle at a station, the procedure is as follows

In Fig 152, suppose the angles AOB, BOC, and COD are to be measured at the station O Each of the angles AOB, BOC, and COD, and also the exterior angle DOA are measured six times according to the above programme. The difference between the sum of the observed values of these four angles and 360° is *equally* divided among the four angles irrespective of their magnitude.

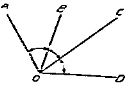


Fig 152

Alternatively, the angle AOB and its exterior angle BOA, the angle BOC and its exterior angle COA, and finally, the angle COD and its exterior angle DOA are measured as above.

In using the method of repetition, the following precautions should be taken

(1) Do not relevel the instrument while a measurement is being made.

(2) Handle the instrument very carefully, turn the clamps very slowly and not too tightly

(3) Do not revolve the instrument on its vertical axis by taking hold of the telescope.

(4) Turn the instrument upon its vertical axis always clockwise.

(5) Do not walk around the instrument to read verniers but release the lower clamp and revolve the instrument so as to bring vernier A before you and then vernier B

(6) Read each vernier independently

(7) Great care must be taken to see that the instrument and the signal are accurately centred over the stations.

The following errors are eliminated in the above procedure

(1) Instrumental Errors :—(i) Errors due to eccentricity of the verniers, and also due to eccentricity of centres i.e.

eccentricity between the centre of the alidade (vernier plate) and the centre of the limb (lower plate) are eliminated by reading both verniers and taking the mean

(ii) Errors of collimation and horizontal axis (i e errors due to the line of collimation not being at right angles to the horizontal axis, and the horizontal axis not being perpendicular to the vertical axis) are eliminated by reversing the telescope (using both faces)

(iii) Errors due to inaccurate graduation are eliminated by taking readings on different parts of the circle (by repetitions)

(iv) Errors due to pointing and to repeated clamping are eliminated by closing the horizon

(2) **Observational Errors** —Errors in the pointings tend to compensate each other and the remaining error minimised by the division

(3) Errors due to atmospheric influences are eliminated by taking different sets of measurements on different days

It may be noted that error due to the vertical axis not being truly vertical cannot be eliminated Care must, therefore, be taken to keep the plate bubbles in true adjustment Otherwise the graduated limb will not be horizontal and the measured angles will always be too large Similarly, errors due to inaccurate centering of the instrument or the signal cannot be eliminated

II **The Direction Method** —In this method the angles at a station are determined by measuring the direction to each station from an initial or reference station and by taking the differences of successive readings One of the triangulation stations which is likely to be always clearly visible may be selected as the initial or reference station Suppose the angles AOB, BOC, and COD are to be measured at the station O (Fig 146) A may be taken as the initial station With the telescope direct (or normal), A is bisected and all the micrometers read Each of the stations B, C, and D is then bisected successively, reading all the micrometers after each bisection The stations are again bisected successively but in the opposite direction (from right to left) as C, B, and A, reading the micrometers at each bisection The telescope is then inverted and the observations repeated as

before Thus we get four measures of each angle, which make one set The observations for the second set or series should commence from a different " zero " so that the readings will be observed on different parts of the circle The limb is then shifted through a number of degrees equal to $\frac{360°}{mn}$, where m is the number of micrometers and n the number of sets The micrometers should not be set to an exact number of degrees The second set is then taken in exactly the same manner Six or eight such sets or series are taken for first order work four for second order work and 2 for third order work. If any set differs by about 4", it is discarded The results are then averaged to obtain the true value of the angle

In addition to measurement of individual angles, summation angles (angles in various combinations) such as AOC, AOD and BOD are sometimes measured

Programme of Measurement —

(i) Set up the instrument at O and level it accurately Set one of the micrometers to zero

(ii) With the telescope direct (or normal), bisect A Read all micrometers

(iii) Bisect successively each of the stations B, C, and D and read all micrometers after each bisection

(iv) Bisect, C, B, and A successively and read all micrometers at each bisection

(v) Reverse the telescope, set on A and read all micrometers

(vi) Set on B, C, and D successively and read all micrometers after each bisection

(vii) Set on C, B, and A successively and read all micrometers at each bisection The entire operation completes the first series

(viii) Shift the limb Relevel, if necessary Repeat the observations for the second series in exactly the same manner

The routine of observation is specially arranged to eliminate the following instrumental and observational errors

(1) To eliminate errors of eccentricity of the vertical axis and of the microscopes, all the micrometers are read at each bisection of the stations

(2) To eliminate errors caused by imperfect adjustment of the line of collimation and horizontal axis, observations are taken on both faces (half of the measures with the telescope direct and half with the telescope reversed)

(3) To eliminate errors of graduation, each angle is read on different parts of the circle by changing or shifting "zero". To do this, the limb is shifted after each set of readings through an angle equal to $\frac{360°}{mn}$

(4) To eliminate errors of manipulation, and those due to twist of the instrument and station caused by the effect of the sun and wind, and those due to slip due to the defective clamping apparatus, one half of the measures are taken from left to right and the other half from right to left and bringing the cross-hairs into coincidence from left to right alternately It may be noted that when the station is elevated, its top is, in clear weather, usually twisted in the direction of the sun's movement. The twist has been observed to be as much as 1 second per minute of time on a station 23 m in height

(5) To eliminate errors of pointing and reading, a large number of observations are taken

(6) To eliminate errors of atmospheric influences, different sets of observations are taken on different days

Reduction to Centre

It sometimes happens that it is impossible to set up an instrument exactly over the stations as when objects such as church spires, steeples, flagpoles, towers, etc, are selected as triangulation stations in order to secure well shaped triangles or because of their visibility In such a case, a subsidiary station is established as near the true or principal station as

possible, the station so established being called a *satellite station*, or an *eccentric* or a *false station* The true station is referred to the satellite station by a distance and an angle The distance between the true station and the satellite station known as the *eccentric* distance may be determined by (a) trigonometrical levelling (Vide method 2, page 589, Part I) or (b) triangulation. The instrument is then centered over the satellite station and all the angles at this station are measured with the same precision as would have been used in the measurement of angles at the true station

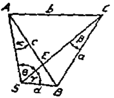


Fig 153

These angles will not be the same as those when measured at the true station They may, however, be readily reduced to what they would have been if the true station were occupied by computing corrections and applying them algebraically to their observed values This operation is known as '*reduction to centre*"

In Fig 153, let A, B, and C = the triangulation stations

B = the true station to which sights are taken from the stations A and C.
S = the satellite station
BS = the eccentric distance (d)
α = the angle BAS
β = the angle BCS
θ = the angle ASC measured at S
γ = the angle CSB measured at S
B = the required angle ABC
a, b, and c = the lengths of BC, CA and AB respectively

In the triangle ABC the angles BAC and BCA are known by actual measurement, and the side AC is known by computation from its connection with adjacent triangulation. The sides AB and BC may, therefore, be calculated by the application of the sine rule

Now $\angle ABC = 180° - \angle BAC - \angle BCA$

$$AB = c = \frac{CA \sin BCA}{\sin ABC} \quad \text{and} \quad BC = a = \frac{CA \sin BAC}{\sin ABC}$$

Knowing the sides AB and BC, and the angles ASC and CSB, the angles BAS ($\propto$) and BCS (β) may be calculated by the sine rule.

Therefore, $\sin \propto = \dfrac{d \sin (\theta + \gamma)}{c}$ and $\sin \beta = \dfrac{d \sin \gamma}{a}$

Since $\propto$ and β are very small angles, we may write

$$\propto'' = \frac{\sin \propto}{\sin 1''} = \frac{d \sin(\theta + \gamma)}{c \sin 1''}$$

and $$\beta'' = \frac{\sin \beta}{\sin 1''} = \frac{d \sin \gamma}{a \sin 1''}.$$

Having determined the values of $\propto$ and β, the true angle ABC (B) may be determined thus :

$$\text{AEC} = \text{ASC} + \text{BAS} = \theta + \propto.$$

Similarly, $\text{AEC} = \text{ABC} + \text{BCS} = \text{B} + \beta.$

$$\therefore\ \theta + \propto = \text{B} + \beta \text{ or } \text{B} = \theta + \propto - \beta.$$

Whence, the true angle $\text{ABC} = \text{B}$

$$= \theta + \left\{\frac{d \sin (\theta + \gamma)}{c \sin 1''} - \frac{d \sin \gamma}{a \sin 1''}\right\}.$$

There are four cases corresponding to the four positions of the satellite station S as shown in Fig. 154.

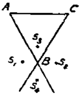


Fig. 154

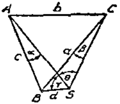


Fig. 155

The corresponding equations are

Case I (Fig. 153) : $\text{B} = \theta + \propto - \beta$
Case II (Fig 155) . $\text{B} = \theta - \propto + \beta$
Case III (Fig. 156) : $\text{B} = \theta - \propto - \beta$
Case IV (Fig 157) : $\text{B} = \theta + \propto + \beta.$

When a round of angles or directions is taken from an eccentric or a satellite station, the necessary corrections may

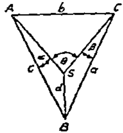


Fig. 156

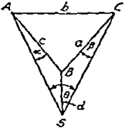


Fig. 157

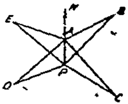


Fig. 158

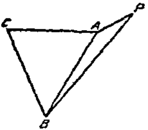


Fig. 159

be calculated as follows (1) The line joining the eccentric station to the true station is assumed to be a meridian. (2) The measured angles are then reduced to this meridian (3) The corrections required to refer these directions to the true station are then computed by the formula

$$\text{Correction } (\alpha) \text{ in seconds} = \frac{d \sin \theta}{D \sin 1''}$$

in which d = the eccentric distance, θ = the observed angle reduced to the assumed meridian; and D = the distance from the true station to the observed station.

The signs of the corrections are the same as those of sin θ. Thus in Fig 158, P is the eccentric station; A, the true station; PA, the assumed meridian; APB (θ_1), APC (θ_2), etc. the reduced directions; PA, the eccentric distance (d); AB, AC, etc. the distances D_1, D_2, etc. of the observed stations

B, C, etc. Then the corrections ABP (α_1), ACP (α_2), etc are obtained by solving the triangles APB, APC, etc. Thus we have

$$\alpha_1 = \frac{d \sin \theta_1}{D_1 \sin 1''}; \qquad \alpha_2 = \frac{d \sin \theta_2}{D_2 \sin 1''}; \text{ etc.}$$

Satellite stations should be avoided as far as possible in primary triangulation, but they are of frequent occurrence in secondary and tertiary triangulation

Eccentricity of Signal —When observations are made upon a signal which is found to be out of centre (eccentric signal), it is necessary to apply the corrections to the observed angles The corrections will be the same as those computed above. Thus if in Fig 158, the signal for station B is at S, the observed angles CAS and ACS must be corrected by α and β respectively. In Fig 159, the signal for the station A is situated at P instead of at A The observed angle CBP must, therefore, be corrected by the angle ABP in order to obtain the true angle CBA.

Now in the triangle ABC, the angles BAC and BCA are known by actual measurement, and the side AC is known by computation from its connection with adjacent triangulation. The side BA can, therefore, be calculated The angle APB and the distance AP being measured, the required correction may be obtained from the relation

$$\text{ABP (in seconds)} = \frac{\text{AP} \ \sin \text{APB}}{\text{BA} \ \sin 1''} .$$

Example 1 ·—Form an eccentric station E, 13 8 m from station A, the angles measured to three trigonometrical stations A, B, and C are as follows, the stations C and E being on opposite sides of the line AB :

$$\angle \text{BEC} = 68^\circ\ 26'\ 36'', \ \angle \text{CEA} = 32^\circ\ 45'\ 48''.$$

The lengths of AC and AB are 5588 4 m and 4371·0 m respectively. Calculate the angle BAC. (Fig 153)

Let $\angle$CBA = α and $\angle$ECA = β ; AE=13 8 m, AB=4371·0 m AC= 5588 4 m $\angle$BEA = $\angle$BEC + $\angle$CEA = 68° 26 36″ + 32° 45′48″ = 101° 12′ 24″. The required angle BAC = $\angle$BEC + $\alpha - \beta$.

Now $\sin \alpha = \dfrac{AE \sin BEA}{AB}$ and $\sin \beta = \dfrac{AE \sin CEA}{AC}$

Since α and β are very small angles, we may write as

$$\alpha \text{ (in seconds)} = \frac{13 \cdot 8 \sin 101^\circ 12' 24''}{4371 \cdot 0 \sin 1''} = 638 \cdot 788$$

$$\beta \text{ (in seconds)} = \frac{13 \cdot 8 \sin 32^\circ 45' 48''}{5588 \cdot 4 \sin 1''} = 275 \cdot 645.$$

Hence $\angle BAC = 68^\circ 26' 36'' + 638'' \cdot 788 - 275'' \cdot 645$
$= 68^\circ 32' 39'' \cdot 143$

Example 2 —In measuring angles at a triangulation station C, it was found necessary to set the transit over another station P south west of C and 3 m from C so that the angle APB is approximately bisected by the line PC. The angles APC and *CPB were found to be 28° 20′ 35″ and 31° 26′ 45″ respectively* The side AB was computed to be 975 m in the adjacent triangle, and when the station C was observed, the mean values of the angles CAB and CBA were recorded as 61° 30′ 25″ and 58° 34′ 20″ respectively. Determine the angle ACB (See Fig 157)

(i) Let $\angle PAC = \alpha$, $\angle PBC = \beta$; $PC = 3$ m; $AB = 975$ m.
In the $\triangle ABC$, $\angle CAB = 61^\circ 30' 25''$, $\angle CBA = 58^\circ 34' 20''$.

$$\therefore \angle ACB = 180^\circ - \angle CAB - \angle CBA = 180^\circ - 61^\circ 30' 25'' - 58^\circ 34' 20''$$
$$= 59^\circ 55' 15''.$$

By the sine rule, $CA = \dfrac{975 \sin 58^\circ 34' 20''}{\sin 59^\circ 55' 15''} = 761 \cdot 44$ m.

$$CB = \frac{975 \sin 61^\circ 30' 25''}{\sin 59^\circ 55' 15''} = 990 \cdot 258 \text{ m}$$

$$\alpha \text{ (in seconds)} = \frac{3 \sin 28^\circ 20' 35''}{961 \cdot 44 \sin 1''} = 305'' \cdot 55 \text{ or } \alpha = 5' 5'' \cdot 55$$

$$\beta \text{ (in seconds)} = \frac{3 \sin 31^\circ 26' 45''}{990 \cdot 258 \sin 1''} = 331'' \cdot 1 \text{ or } \beta = 5' 31'' \cdot 1$$

Now $\angle ACB = \angle APB + \alpha + \beta = (\angle APC + \angle CPB) + \alpha + \beta$
$= 28^\circ 20' 35'' + 31^\circ 26' 45'' + 5' 5'' \cdot 55 + 5' 31'' \cdot 1$
$= 59^\circ 57' 56'' \cdot 65.$

Example 3 —From a satellite station E at a distance of 4 2 m from the main triangulation station D, the following directions were observed

D, 0° 0 0″, A, 140° 23 40″, B, 208° 47 20″, C, 282° 34′ 10″

The lengths of DA, DB, and DC were 2870 1, 3791 4, and 2677 5 m respectively Determine the directions of DA, DB, and DC

Let α_1, α_2, and α_3 be the corrections to the observed bearings of EA, EB, and EC respectively

Now in the $\triangle$ BED, $\angle$ BED $= 360° - 208°47'20'' = 151°12'40''$.

In the $\triangle$ CED, $\angle$ CED $= 360° - 282°34'10'' = 77°25'50''$.

Then $$\alpha_1 = \frac{\text{ED} \sin \text{DEA}}{\text{DA} \sin 1''} = \frac{4\ 2 \sin 140°23\ 40''}{2870\ 1 \sin 1''}$$

$$= 192\ 42 \text{ seconds or } 3'\ 12''\ 42 \quad (+ve)$$

Similarly, $$\alpha_2 = \frac{4\ 2 \sin 151°12\ 40''}{3791\ 4 \sin 1''} = 110\ 039 \text{ seconds} \quad \text{or}$$

$$1\ 50''\ 04 \quad (-ve)$$

$$\alpha_3 = \frac{4\ 2 \sin 77°\ 25\ 50''}{2677\ 5 \sin 1''} = 315{\cdot}798 \quad ,, \quad \text{or} \quad 5'\ 15''\ 8.$$

$$(-ve)$$

Hence

Direction of DA $=$ direction of EA $+ \alpha_1$

$= 140°\ 23\ 40'' + 3\ 12''\ 42 = 140°\ 26'\ 52''\ 42$

,, of DB $= 208°\ 47\ 20'' - 1\ 50''\ 04 = 208°\ 45'\ 29''\ 96.$

,, of DC $= 282°\ 34'\ 10'' - 5\ 15''\ 80 = 282°\ 28'\ 54''\ 20$

Example 4 —The following notes refer to observations made on P, Q, and R from a satellite station S near the main triangulation station P on a church spire

Q, 0° 0 0″, R, 63° 32 20″, P, 309° 17′ 30″

To determine the distance of the satellite station S from P, the station A was fixed towards station P at a distance of 21 m from S so that S, A, and P were in the same vertical plane The vertical angles observed at A and S to the church spire were

21° 53′ and 14° 24′ respectively. The staff readings on a reference point taken with the horizontal sight from S and A were 0·996 m and 0 852 m The lengths of PQ and PR were 6415 8 m and 7129·2 m respectively. Calculate the angle QPR. (Fig. 155)

(i) To find SP :—The difference of level between the inst. axes at S and A $= h_d = 0{\cdot}996 - 0{\cdot}852 = 0{\cdot}144$ m.

$\therefore$ The correction to be applied to distance AS $= h_d \cot \beta$ $= 0{\cdot}144 \cot 14°24' = 0{\cdot}561$ m Since the inst. axis at station S is higher than that at station A, the correction is additive

Now the distance (D) from A to the church spire (P) may be obtained from

$$D = \frac{d \tan \alpha_2}{(\tan \alpha_1 - \tan \alpha_2)}. \quad \text{Here } d = 21 + 0{\cdot}561 = 21{\cdot}561 \text{ m}$$

$$\alpha_1 = 21°53'; \ \alpha_2 = 14°\,24'.$$

$$\therefore \quad D = \frac{21\ 561 \tan 14°\,24'}{(\tan 21°\,58' - \tan 14°\,24')} = 38{\cdot}204 \text{ m.}$$

Hence the distance SP $= 21 + 38{\cdot}204 = 59{\cdot}204$ m.

(ii) Now $\angle$QSR $= 63°\,32'\,20''$; $\angle$PSQ $= 360° - 309°\,17'\,30''$
$= 50°\,42'\,30''$.

and $\angle PSR = \angle PSQ + \angle QSR = 50°\,42'\,30'' + 63°\,32'\,20''$
$= 114°\,14'\,50''$.

Let $\angle$ PQS $= \alpha$ and $\angle$PRS $= \beta$.

$$\text{Then } \alpha \text{ (in seconds)} = \frac{SP \sin PSQ}{PQ \sin 1''} = \frac{59{\cdot}204 \sin 50°\,42'\,30''}{6415{\cdot}8 \sin 1''}$$

$$= 1473{\cdot}08.$$

$$\beta \text{ (in seconds)} = \frac{SP \sin PSR}{PR \sin 1''} = \frac{59.204 \sin 114°\,14'\,50''}{7129{\cdot}2 \sin 1''}$$

$$= 1561{\cdot}81.$$

$$\text{Now} \quad \angle QPR = \angle QSR - \alpha + \beta$$

$$= 63°\,32'\,20'' - 1473''{\cdot}08 + 1561''{\cdot}81$$
$$= 63°\,32'\,20'' + 1'\,28''{\cdot}73 = 63°\,33'\,48''{\cdot}73.$$

Base Line Measurement

In triangulation the base line is of prime importance Since the accuracy of the computed sides of the triangulation system depends upon the accuracy of measurement of the base line, utmost care should be taken in its measurement The length of the base varies from a fraction of a 1 5 km to 15 km according to grades of triangulation It generally lies between one third and two-thirds of the length of the average side of the triangulation system In India ten bases were used The lengths of the nine bases varied from 6 4 miles (10 7 km) to 7 8 miles (13 km) and that of the tenth base was 1 7 miles (2 83 km)

Base Line Site —In selecting the site for a base line, the following requirements should be taken into consideration —

(1) The site should be fairly level or uniformly sloping or gently undulating

(2) It should be free from obstructions throughout the whole of its length

(3) The ground should be firm and smooth

(4) The site should be such that the whole length can be laid out, the extremities of the base line being intervisible at ground level

(5) The site should be so selected that well shaped triangles can be obtained in connecting the end stations of the base to the main triangulation stations In very flat open country there is a considerable choice of sites and the base may be selected to suit the location of the triangulation stations, while in rough country the choice is limited and some of the triangulation stations must be selected to suit the location of the base line

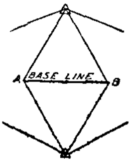


Fig 160

Base Net —A series of triangles connecting a base line to the main triangulation is called a *base net* The base should be expanded gradually by triangu-

lation Figs 160, 161, and 162 show the various ways in which the base is connected to the main triangulation

Base Measuring Apparatus —The instruments used for base line measurement are (1) Rigid bars, (2) Flexible apparatus consisting of (a) Steel or Invar tapes, and (b) steel and brass wires

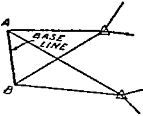


Fig 161

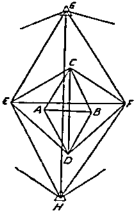


Fig 162

Rigid Bars —The rigid base bars were formerly used for work of highest precision They were made of Deal, glass or metal They include (1) Contact apparatus in which the base-bars were placed successively end to end, (2) Compensating base bars, which were designed to maintain a constant length under varying temperature by a combination of two metals e g Colby's apparatus, (3) Bimetallic and non compensating base-bars e g Duplex apparatus, (4) Monometallic base-bars in which the temperature is either kept constant at the melting point of ice, e g Ice Bar apparatus, or is otherwise ascertained

Tapes —The tapes may be made of steel or 'invar' Invar is an alloy of steel and about 36% nickel The chief advantage of this alloy is that it possesses a very low coefficient of expansion ($\frac{1}{10}$th that of steel) The invar tape is very soft and easily kinked It should, therefore, be handled very carefully It should be wound upon a large reel or drum It is 6 mm × $\frac{1}{2}$mm in cross section The end scales of the 30 m invar tape are usually divided to 1 mm The tapes used in

base line measurement are usually from 30 m to 100 m long and the value of the coefficient of expansion rarely exceeds 1×10^{-6} per degree C For measurements of ordinary precision, the steel tape or the invar tape may be used but for those of high precision, the invar tape 100 m in length is invariably used. The tape must be standardized, i e its actual length under specified conditions must be determined very accurately by comparing it with a standard of known length Standardization is done by the survey and standards department The certificate issued states the actual length (absolute length) of the tape (or the error of its length) for a certain temperature and pull, and whether the tape was standardized flat or in catenary At least two tapes are usually standardized, one for use in the field (field tape) and the other kept for comparison It is well to note here the distinction between the *nominal* length and *absolute* length of a tape By the former is meant the designated length of a tape (e g 30 m tape or 100 m tape), while by the latter is meant its actual length under specified conditions The expression such as " the tape is standard at 15° C," means that its actual length is exactly equal to its designated length at 15° C The tape is a very convenient instrument, and measurement can be done easily and rapidly, and also economically Steel and brass wires may also be employed for base measurement (Jaderin's method) The method is however no longer used

Equipment for Base Line Measurement —The equipment consists of (1) three standardized tapes (one for field measurements and the other two used only for standardizing the field tape at frequent intervals, (2) straining device, (3) spring balance or weight and pulley, (4) six thermometers, and a finely divided pocket scale, (5) marking tripods or stakes, (6) supporting tripods or stakes, and (7) a spacing steel tape for setting out tripods or stakes Two of the six thermometers should be standardized and kept for use as standards The spring balance should be sufficiently sensitive It should be tested at the beginning and at the end of each day's work

Field Work —The survey personnel consists of (1) a setting-out party and (2) a measuring party. The former places the

tripods or stakes in advance of the measurement at correct intervals, while actual measurement of the line is done by the latter

To begin with, the line is cleared of obstructions such as trees, bushes, etc and is first approximately set out and levelled The line is then divided into sections of about 0 8 to 1 2 km in length and accurately aligned with a transit, stout posts 10 cm × 10 cm being driven firmly into the ground at each end of the section A series of such posts (marking or measuring posts) are accurately driven on the line with their tops about 0 6 m above the ground surface at intervals slightly less than one tape length A strip of zinc or copper is nailed on the top of each post to provide a flat surface for marking the end of the tape Supporting stakes 2 5 cm ×5 cm are driven with their faces in line at the proper intervals (15 to 30 m), the nails being

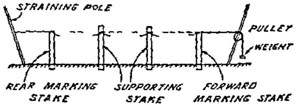


Fig 163 a

driven in their sides to carry hooks to support the tape. The points of support are set either on a uniform grade between the marking posts or at the same level For very accurate work, tripods are used as tape supports instead of posts The differences of level between the stakes are then very accurately determined by spirit levelling

To measure a base line, the tape is stretched between the marking posts (or tripods) and allowed to hang freely The rear end of the tape is connected to the straining stake (or pole) driven behind the rear marking post and the forward end to the spring balance or other stretching apparatus (Fig 163 a) The rear end of the tape is adjusted to coincide with the mark on the zinc strip of the rear marking post The proper tension is then applied by means of a spring balance and the position of the forward end of the tape is marked on the zinc strip with a steel scriber. The temperature of the tape is determined by three

thermometers one placed near each end, and one near the middle of the tape. The tape is then carried forward and the process repeated until the end of the section is reached. The section is again measured in the reverse direction as a check. If the two measurements agree within the permissible limit, their mean is adopted for the length of the section Otherwise, additional measurements must be made. It is advisable to make several measurements of each section at different times and with different pulls and temperatures. The most serious source of error in precise base-line measurements is due to the difficulty of measurement of the actual temperature of the tape When steel tapes are used, it is essential to determine the temperature of the tape very accurately, but when the invar tapes are used, errors in determining the temperature of the tape are less important, since they have an exceedingly small coefficient of expansion. Very precise measurement by steel tape can be done only on densely cloudy days or at night (when the air and the ground are at the same temperature). But by invar tape, best work can be done at all hours of the day

Another method of measuring the base is to measure the distance between the fine marks on two successive tripods (Fig. 163 b). By means of a small graduated scale at each end of the tape, the exact distance between the marks on tripod

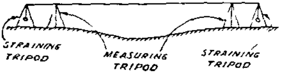


Fig. 163 b

heads is determined, the reading on the scale corresponding to the mark being read with a microscope

Corrections to Base Line Measurements

It is necessary to apply the following corrections to the field measurements of a base line in order to obtain its true length :—

(i) Correction for absolute length, (ii) Correction for temperature, (iii) Correction for tension or pull, (iv) Correction for

sag, (v) Correction for slope or vertical alignment, (vi) Correction for horizontal alignment, and (vii) Reduction to sea level

A correction is said to be *plus* or *positive* when the uncorrected, length is to be increased, and *minus* or *negative* when it is to be decreased in order to obtain the true length

It may be noted that each section of the base line is separately corrected

Correction for Absolute Length —It is the usual practice to express the *absolute* length of a base measuring unit as its *nominal* or *designated* length plus or minus a correction The correction is given by the formula

$$C_A = \frac{Lc}{l} \tag{1}$$

where C_A = the correction for absolute length

L = the measured length of base

l = the nominal length of measuring unit

c = the correction to measuring unit

The sign of the correction (C_A) will be the same as that of c It may be noted that L and l must be expressed in the same units, and the unit of C_A is the same as that of c

Correction for Temperature —It is necessary to apply this correction since the length of a tape is increased as its temperature is raised and consequently, the measured distance is too small It is given by the formula

$$C_t = a\,(T_m - T_0)\,L \tag{2}$$

in which C_t = the correction for temperature

a = the coefficient of thermal expansion

T_m = the mean temperature during measurement

T_0 = the temperature at which the measuring unit is standardized

L = the measured length

The sign of the correction is plus or minus according as T_m is greater or less than T_0 The coefficient of expansion for steel varies from 0 0000099 to 0 000012 per degree C and that

for invar is 0 0000010 per degree C or less If the coefficient of expansion of a steel tape is not known an average value of 0 000011 may be assumed For very precise work the coefficient of expansion for the tape in question must be carefully determined

Correction for Pull (or Tension) —The correction is necessary when the pull used during measurement is different from that at which the tape or wire is standardized It is not required in the case of rigid apparatus i e base bars It is given by the formula

$$C_p = \frac{(P - P_0) L}{AE} \quad (+ve) \tag{3}$$

where C_p = the correction for pull in metre
P = the pull applied during measurement in kilogram
P_0 = the pull for which the tape is standardized in kilogram
L = the measured length in metre
A = the cross sectional area of the tape or wire in square centimeter
E = the modulus of elasticity of the tape or wire

The value of E for steel may be taken as $2\cdot1 \times 10^5$ kg per sq cm and that for invar $15\cdot4 \times 10^5$ kg per sq cm For very precise work its value must be ascertained The sign of the correction is always plus as the effect of the pull is to increase the length of the tape and consequently to decrease the measured length of the base

Correction for Sag —(Fig 164) When a tape is stretched over points of support it takes the form of a catenary In practice however the curve of the tape is assumed to be a parabola The correction for sag is the difference in length between the arc and its chord i e the difference between the curved length of the tape and the distance between the supports It is required only when the tape is suspended during measurement Since the effect of the sag on the tape is to make the measured length too large this correction is always *subtractive* It is given by the formula

$$C_s = \frac{l_1 (wl_1)^2}{24P^2} \quad (-ve) \tag{4}$$

in which C_s = the sag correction for a single span, in metre
l_1 = the distance between supports, in metre.
w = the weight of the tape in kg per metre.
P = the applied pull, in kg

If there are n equal spans per tape length, the sag correction per tape length is given by

$$C'_s = \frac{nl_1(wl_1)^2}{24P^2} = \frac{l(wl_1)^2}{24P^2} = \frac{l(wl)^2}{24n^2P^2} \quad \ldots \quad (4a)$$

in which l = the length of the tape $= nl_1$ and $l_1 = \frac{l}{n}$

The total sag correction to the measured length (L) is:

Total sag correction $= N \times$ sag correction per tape length + sag correction for any fractional tape length . (4b)

in which N = the number of whole tape lengths.

The formula for the sag correction for a parabola with level supports may be derived as follows

Referring to Fig 164, let x be the deflection or dip at the

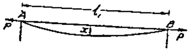


Fig 164

middle of the tape Passing a section through the tape midway between supports and taking moments of the external forces on one side of this section about one support, we get

$$Px = \frac{wl_1}{2} \cdot \frac{l_1}{4} = \frac{wl_1^2}{8} \quad \text{or} \quad x = \frac{wl_1^2}{8P}$$

Now the difference in length between the arc and chord of a very flat parabola (i. e. when $\frac{x}{l_1}$ is small) is very nearly equal to $\frac{8x^2}{3l_1}$ $\therefore$ Sag correction $= \frac{8x^2}{3l_1} = \frac{8}{3l_1}\left(\frac{wl_1^2}{8P}\right)^2 = \frac{l_1(wl_1)^2}{24\ P^2}$.

Normal Tension —The *normal* tension of a tape is a tension which will cause the effects of pull and sag to neutralize each other The corrections for pull and sag being of opposite sign, the elongation due to increase in tension is exactly counterbalanced by the shortening due to sag It may be obtained by equating the corrections for pull and sag Thus, we have

$$AE \quad = \frac{l_1(wl_1)^2}{24P_n^2} \quad \text{or} \quad (P_n - P_0)\,P_n^2 = \frac{W^2AE}{24}$$

$$P_n = \frac{0\ 204W\sqrt{AE}}{\sqrt{(P_n - P_0)}} \tag{4c}$$

in which P_n = the normal tension in kg

W = the weight of tape between supports, in kg.

The value of P_n may be determined by trial or by the use of the slide rule To use the slide rule set the cursor to N on the D scale, where N is the numerator of the right hand member of the equation (4c) Move the slide until by inspection $P_n - P_0$ on the right B scale is at the cursor, when the index of the C scale is at P_n on the D scale

Correction for Slope or Vertical Alignment —This correction is required when the points of support are not exactly at the same level

Let l_1, l_2 etc = the successive lengths of uniform grades

h_1 h_2 etc = the differences of elevation between the extremities of each of these grades

C_g = the total correction for slope

If l is the length of any one grade, and h the difference of elevation between the ends of the grade (Fig 165)

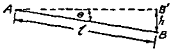


Fig 165

The slope correction $= l - \sqrt{l^2 - h^2}$

$$l - l\left(1 - \frac{h^2}{2l^2} - \frac{h^4}{\;} - \text{etc.}\right)$$

$$= \left(\frac{h^2}{2l} + \frac{h^4}{8l^3} + \text{etc.}\right) = \frac{h^2}{2l} \quad (-ve) \quad . \tag{5}$$

Whence

$$C_g = \left(\frac{h_1^2}{2l_1} + \frac{h_2^2}{2l_2} + \qquad + \frac{h_n^2}{2l_n}\right) \quad (-ve) \tag{5a}$$

When the grades are of uniform length l, we have

$$C_g = \frac{1}{2l}\,(h_1^2 + h_2^2 + \qquad + h_n^2) = \frac{\Sigma h^2}{2l} \;(-ve) \tag{5b}$$

This correction is always subtractive from the measured length. If the grades are given in terms of vertical angles (plus or minus angles), the following formula may be used

The correction for slope $= l - l\cos\theta = l \text{ versin } \theta$

$$= 2l \sin^2 \frac{\theta}{2} \quad (-ve) \tag{6}$$

in which l = the length of the section

θ = the grade (or the angle of slope) of the section.

Correction for Horizontal Alignment.—(Fig 166)

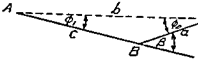


Fig 166

Ordinarily a base line is set out in one continuous straight line but it sometimes becomes necessary to deviate it due to some intervening obstructions. It is then called a *broken base*. The two sections AB and BC, and the exterior angle β being measured, the length of AC may be computed by the cosine rule. Thus we have

$$b^2 = a^2 + c^2 + 2ac\cos\beta \tag{7}$$

where b = the length of the broken base AC
c = " of the section AB
a = " " " BC
β = the exterior angle at B

Whence, the correction (C_H) for horizontal alignment is given by

$$C_H = (a + c) - b \qquad (-ve)$$

Adding $2ac$ to both sides of equation (7), we get

the $2ac + c^2 - b^2 = 2ac - 2ac \cos \beta$

or $(a + c)^2 - b^2 = 2ac\,(1 - \cos \beta)$

$$\therefore \quad a + c - b = \frac{2ac\,(1 - \cos \beta)}{(a + c) + b} = \frac{4ac \sin^2 \frac{1}{2}\beta}{(a + c) + b}$$

$$\therefore \quad C_H = \frac{ac\beta^2 \sin^2 1'}{2\,(a + c)} \quad (-ve) \quad . \tag{8}$$

in which β is expressed in minutes

Hence $$AC = b = a + c - \frac{ac\beta^2 \sin^2 1'}{2\,(a + c)} \quad .. \quad . \tag{9}$$

If A and C are mutually visible, the angles CAB (ϕ_1) and BCA (ϕ_2) should be measured The length AC (b) may then be determined from

$$b = AB \cos \phi_1 + BC \cos \phi_2 = c \cos \phi_1 + a \cos \phi_2 \tag{10}$$

and $$C^H = \{c\,(1 - \cos \phi_1) + a\,(1 - \cos \phi_2)\} \quad (-ve) \tag{11}$$

Reduction to Mean Sea Level —In geodetic work all horizontal distances are reduced to their equivalent distances at mean sea-level called the geodetic distances If the length of the base be reduced to its equivalent length at mean sea-level, the computed lengths of all other lines of the triangulation system will correspond to this level The mean elevation of the base must, therefore, be ascertained This correction is required for comparison of the various bases

The correction is given by the formula

$$C_{msl} = \frac{Lh}{R} \quad (-ve) \tag{12}$$

where C_{msl} = the correction to the length L

L = the measured length of the base.

h = the average height of the base above mean sea-level

R = the mean radius of the earth

It may be derived as follows . Let l be the length of the base reduced to mean sea level (Fig 167). Then

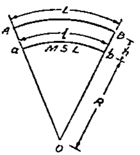


Fig 167

$$\frac{l}{L}=\frac{R}{R+h}=\left(1+\frac{h}{R}\right)^{-1}=\left(1-\frac{h}{R}\right).$$

since h is always very small as compared with R.

$$\therefore\quad l=L-\frac{Lh}{R}.\quad \text{Whence, } C_{msl}=\frac{Lh}{R}\ (-ve).$$

It may be remembered that the angles that are measured at the triangulation stations are the *horizontal* angles and are not affected by the difference of elevation of those stations.

The above corrections, being very small, may be calculated individually from the measured length of the base, and then added algebraically to obtain the total correction which, when applied to the measured length, gives the true length of the base.

To Compute a Portion of a Straight Base which cannot be Directly Measured :—It sometimes happens that a portion of a straight base cannot be directly measured due to an intervening

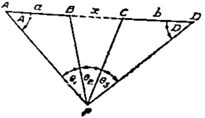


Fig. 168

obstruction such as a stream. In such a case, the following procedure may be adopted. In Fig. 168, let AB (a) and CD (b)

be the portions of the base which are measured directly, and BC (x) the portion which cannot be measured directly. Select a suitable station P and measure the angles APB, BPC, and CPD, and denote them by θ_1, θ_2 and θ_3 respectively.

Then from the $\triangle$ APC, $CP = AC \dfrac{\sin A}{\sin(\theta_1 + \theta_2)}$

" , APB $BP = AB \dfrac{\sin A}{\sin \theta_1}$

$$\frac{CP}{BP} = \frac{AC \sin \theta_1}{AB \sin(\theta_1 + \theta_2)} = \frac{(a + x) \sin \theta}{a \sin(\theta_1 + \theta_2)}$$

Similarly, from the triangles CPD and BPD,

$$\frac{CP}{BP} = \frac{b \sin(\theta_2 + \theta_3)}{(b + x) \sin \theta_3}$$

Whence, $\dfrac{(a + x) \sin \theta_1}{a \sin(\theta_1 + \theta_2)} = \dfrac{b \sin(\theta_2 + \theta_3)}{(b + x) \sin \theta_3}$

or $\qquad (a + x)(b + x) = \dfrac{ab \sin(\theta_1 + \theta_2) \sin(\theta_2 + \theta_3)}{\sin \theta_1 \sin \theta_3}$

$$x = + \sqrt{\left\{\frac{ab \sin(\theta_1 + \theta_2) \sin(\theta_2 + \theta_3)}{\sin \theta_1 \sin \theta_3} + \left(\frac{a - b}{2}\right)^2\right\}} - \tfrac{1}{2}(a + b)$$

Examples on Base Line Measurement

Example 1 —A tape 30 m long of standard length at 20° C was used to measure a line the mean temperature during measurement being 15°C. The measured distance was 221.65 m, the following being the slopes

1° 10 for 60 m	1° 30 for 30 m
2° 20 for 30 m	3° 48 for 30 m
5° 18 for 30 m	7° 20 for 18 m
4° 40 for 12 m	1° 20 for 11.65 m

Find the true length of the line, if the coefficient of expansion is 116×10^{-7} per 1° C.

Correction for temperature = L × diff. in temp × coefficient of expansion

Here L = 221 65, diff in temp $= 15° - 29° = -14°$ and coeff of expansion $= 116 \times 10^{-7}$

Correction for temperature
$= 221\ 65 \times (-14) \times 116 \times 10^{-7}$
$= 0\ 036$ m ($-$ ve)

This is subtractive, since the mean temperature at the time of measurement is below that at which the tape was standard

Correction for slope $= l(1 - \cos\theta)$ ($-$ ve)

C_1	for	60 m =	60 (1 $-$ cos 1° 10) =	012 m
C_2	for	30 m =	30 (1 $-$ cos 1° 30) =	009 m
C_3	for	30 m =	30 (1 $-$ cos 2° 20) =	024 m
C_4	for	30 m =	30 (1 $-$ cos 3° 48) =	060 m
C_5	for	30 m =	30 (1 $-$ cos 5° 18) =	129 m
C_6	for	18 m =	18 (1 $-$ cos 7° 20) =	033 m
C_7	for	12 m =	12 (1 $-$ cos 4° 40) =	039 m
C_8	for	11 65 =	11 65 (1 $-$ cos 1° 20) =	003 m
			Total =	0 309 m ($-$ve)

. True length of the line $-$ 221 65 $-$ 036 $-$ 0 309
= 221 305 m

Example 2 —Calculate the sag correction for a 100 m tape weighing 1 kg under a pull of 10 kg in three equal spans of $\frac{100}{3}$ m each

$$\text{Sag correction } C_s = \frac{w^2 l_1^3}{24 P^3} \ (-\text{ve}).$$

Here $w = \frac{1}{100}$ kg per m length

$$l_1 = \frac{100}{3}\text{m}; \qquad P = 10 \text{ kg}$$

$$\therefore \text{ Correction for } \frac{100}{3} \text{ m span} = \frac{(\ 01)^2 \times \left(\frac{100}{3}\right)^3}{24 \times 10^2}$$

$$= \frac{0001 \times 10}{27 \times 24 \times 600} = 0\ 00154 \text{ m}$$

Correction for a 100 m tape $= 3 \times 0\ 00154 = 0\ 00462$ m

Example 3 —A steel tape 30 m long, standardized at 10° C with a pull of 10 kg was used for measuring a base line Find the correction per tape length, if the temperature at the time of measurement was 20° C and the pull applied was 15kg Wt. of 1 cubic cm of steel = 8 grammes Wt of tape = 600 grammes

$E = 21 \times 10^5$ kg per sq cm Coeff of expansion of tape per 1° C $= 12 \times 10^{-6}$

(1) Correction for pull $(C_p) = \dfrac{(P - P_0) L}{AE}$

$P = 15$ kg, $P_0 = 10$ kg, $L = 30$ m

$E = 21 \times 10^5$ kg / cm², Wt of 30 m tape = 600 grammes.

If A is the area of the cross section of the tape in sq cm.

$A \times 30 \times 100 \times 8 = 600 \qquad A = \dfrac{600}{30 \times 100 \times 8} = \dfrac{1}{40}$ sq cm

$$\therefore \quad C_p = \frac{5 \times 30}{\frac{1}{40} \times 21 \times 10^5} = 0\ 0029 \text{ m } (+\text{ve})$$

(2) Correction for temperature $(C_t) = \alpha (T_m - T_0) L$

Difference in temperature $= 20° - 10° = 10°$ C

$\alpha = 12 \times 10^{-6}$, $L = 30$m

$C_t = 12 \times 10^{-6} \times 10 \times 30 = 0\ 0036$ m (+ve)

(3) Correction for sag $(C_s) = \dfrac{LW^2}{24P} = \dfrac{30 \times (0\ 6)^2}{24 \times 15^2}$

$= 0\ 002$ m (−ve)

Total correction to be applied $= 0\ 0029 + 0\ 0036 - 0\ 002$

$= 0\ 0045$ m (+ve)

Example 4 —A steel tape is 30 m long at a temperature of 15° C when lying horizontally on the ground Its sectional area is 0 08 sq cm it weight 1 8 kg and the coeff of expansion $11^{-} \times 10^{-7}$ per 1° C The tape is stretched over three supports which are at the same level and at equal intervals Calculate the actual length between the end graduations under the following conditins Temperature = 25° C, Pull = 18 kg

Here $L = 30$ m; $T_m = 25°$ C; $T_0 = 15°$ C; $P = 18$ kg; $P_0 = 0$ kg; $\alpha = 117 \times 10^{-7}$.

(a) Correction for temp $= \alpha (T_m - T_0) L$

$$= 117 \times 10^{-7} \times 10 \times 30$$

$$= 0\ 00351 \text{ m } (+\text{ ve})$$

(b) Correction for pull $= \dfrac{(P - P_0) L}{AE}$

$$= \frac{18 \times 30}{0\ 008 \times 21 \times 10^5}$$

$$= 0\ 0032 \text{ m. } (+\text{ ve}).$$

(c) Correction for sag $= \dfrac{nl_1 (wl_1)^2}{24P^2}$; $n = 2$; $l_1 = 15$ m

$$= \frac{2 \times 15 (0\ 9)^2}{24 \times 18^2} = 0.0031 \text{ m } (-\text{ ve})$$

∴ Actual length of tape $= 30 + 0\ 00351 + 0\ 0032 - 0\ 0031$

$$= 30\ 00361 \text{ m}$$

Extension of a Base —The length of a base line is usually not greater than 10 to 20 km, as it is not often possible to secure a favourable site for a longer base The usual practice is, therefore, to measure a short base and extend it by means of well-conditioned triangles

In Fig 169, suppose it is required to prolong a base line AB. Let C be the extremity of the base when prolonged

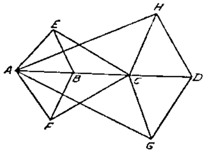


Fig 169

(i) Select stations E and F on either side of AB so that they are clearly visible from A and B and form well shaped triangles

(ii) With the theodolite centred over the station A or B, fix the station C accurately in the line AB prolonged such that E and F are both clearly visible from it and the triangles ACE and ACF formed by it are well-conditioned

(iii) Set up the instrument at each of the stations A, B, C, E, and F, and measure the angles of each of the triangles ABE, ABF, BCE, BCF, ACE, and ACF.

From the data thus obtained, compute the length of BC. In the triangle ABE, the three angles, and the side AB are known. The sides AE and BE may, therefore, be computed by the application of the sine rule Similarly, from the triangle ABF, the sides AF and BF can be calculated Knowing BE and the three angles of the triangle BCE, BC may be computed Similarly, BC may be computed from the triangle BCF. Thus we get two values for BC. Two more values for BC may be obtained by solving the triangles ACE and ACF, BC being equal to AC − AB. The mean of these four values gives the required value of BC. By repeating the above process, the base may be further prolonged to D.

This method may also be used to check the accuracy of the measurements of the sections of a base line Suppose the base line is divided into sections AB, BC, and CD Assuming the measured length of AB to be correct, four values for BC may be determined by the repeated application of the sine rule from the triangles ABE, ABF, BCE, BCF, ACE, and ACF, their mean being adopted as the computed value of BC This value when compared with the measured length of BC checks the accuracy of the measurements of both AB and BC. Similar procedure may be followed to check the measurement of CD.

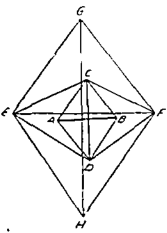


Fig 170

Another and more common method of extension of a base is shown in Fig. 170 In this method the base is gradually enlarged through the medium of well proportioned triangles. The base AB is expanded to CD by selecting

suitable stations C and D on opposite sides of AB From the measured value of AB and the angles of the triangles ABC, ABD, ACD, and BCD, two values for CD can be computed, their mean being adopted as the length of CD The new base CD is then enlarged to EF by means of the triangles ECD and FCD by selecting suitable stations E and F on either side of CD EF now becomes a new base which is expanded to GH in a similar way The process may be repeated as many times as required

PROBLEMS

1 The elevations of two stations A and B 75 km apart are respectively 64 m and 96 m above mean sea level Calculate the approximate height of the scaffold at A if the height of the signal above station B is 14 m Assume intervening ground at mean sea level

(Ans 17 m)

2. The elevations of two stations A and B are 97 5 m and 147 m above M S L. respectively, and the distance between them is 50 km The intervening ground may be assumed to have a uniform elevation of 75 m. Determine the minimum height of the signal required at B in order that the line of sight may nowhere be less than 2 m above the surface of ground

(Ans. 16 5 m.)

3 Two stations A and B are 80 km apart The elevation of an instrument at A is 40 m above M S L The line of sight crosses a portion of the sea. Compute the minimum elevation required for the signal at B given that the coefficient of refraction is 0 08 and the mean radius of the earth is 6370 km

(Ans. 202 m)

4. The elevations of two proposed triangulation stations A and B, 100 km apart are respectively 140 m and 416 m above mean sea level The elevation of the intervening peak at C, 60 km from A, which is likely to obstruct the line of sight is 150 m Ascertain if A and B are intervisible, and if not, find the height required for the scaffold at B so that the line of sight may clear C by 3 m

(Ans. The line of sight fails to clear C by 5 92 m Height of scaffold at B = 14 9 m)

5 Two proposed triangulation stations A and D are 120 km apart and their respective elevations above mean sea level are 282 m and 1105 m. The

altitudes of two peaks B and C on the profile between them are respectively 375 m and 640 m and the distances AB and AC are 47 km and 83 km respectively Find whether the stations A and D are intervisible If not, compute the height of the scaffold at D in order that the line of sight may clear the obstacle by 3 m taking A as a ground station

(Ans The line of sight clears C, but fails to clear B by 3 m Height of scaffold at B = 8 7 m)

6 Two stations, A and B, are 110 km apart, the top of the scaffold at A is 24 m above mean sea level and the height of the ground at B is 750 m above the same datum The highest intervening point is at C, 50 km from B, at a height of 155 m above mean sea level Ascertain if A and B are intervisible, and if necessary, determine a suitable height for the scaffold at B in order that the line of sight may clear the point C by 3 m

(Ans 11 m).

7 What is meant by "a Satellite station"? Explain the reasons for using it during a trigonometrical survey Directions were observed from a satellite station, 68 m from station C, with the following results

A, 0° 0 0", B, 71° 54 32" 25, C, 296° 12 00"

The approximate lengths of AC and BC are respectively 18024 m and 23761 m Compute the angle subtended at station C

(Ans 71° 49 46" 22.)

8 What is meant by a satellite station and reduction to centre? State the reasons for using a satellite station in a triangulation survey

Derive the formulæ used for reducing the angles measured at satellite stations to centre

9 What is meant by eccentricity of signal? How would you correct the observations when made upon an eccentric signal?

From an eccentric station E, 24 24 m from C, the following angles were measured to three triangulation stations A, B, and C, the stations B and E being on opposite sides of AC

AEB = 62° 32 40", AEC = 78° 24' 30"

The approximate lengths of AC and BC were 4705 5 m and 5695 8 m respectively Find the angle ACB

(Ans. 62° 46 0" 94)

10 Directions were observed from a satellite station D, 58 5 m from station B and the following results were obtained

A, 0° 0 0", C, 69° 14' 24", B, 108° 26 49".

The approximate lengths of AB and BC were respectively 5771 4 m and 11017 8 m Determine the angle ABC.

(Ans 69° 24 36"·69)

11. From a satellite station P, 12 m from A, the following angles were measured APB, 60° 24' 18", BPC, 80° 12 24", CPD, 74° 32 48", DPE, 93° 16' 36" The approximate distances from A to stations B, C, D, and E

were as follows AB = 1754 m AC = 2416 m, AD = 2238 m, AE = 1992 m Reduce the measured angles to centre (Fig 158)

[Taking PA as the meridian, find the bearings of PB, PC, PD, and PE Work out the corrections in the usual manner]

(Ans BAC = 89° 0′ 53″ 973, CAD = 74° 27′ 17″ 027, DAE = 93° 15′ 4″ 91)

12. A church spire P was sighted from three triangulation stations A, B, and C, and the following angles were recorded, BAP 60° 32′ 48″ PBA = 72° 48′ 24″, CBP = 63° 22′ 36″, PCB = 44° 43′ 50″.

The lengths of AB and BC were 7573 5 m and 12322 5 m respectively From a satellite station D, 65 m from P and inside the angle APB, the angles observed to A, B, and C were as follows.

PDA = 148° 10′ 6″, ADB = 46° 56′ 38″, BDC = 67° 5′ 21″.

Compute the angles APB and BPC

(Ans APB = 46° 28′ 47″ 46, BPC = 66° 53′ 34″ 48)

13 Directions were observed from a satellite station P, 8 75 m from station A and the following results were obtained

Station	Observed direction	Distance in m from A
A	0° 0′	
B	38° 48′	2190
C	102° 36′	1895
D	256 12	2277
E	324 6	2522

Correct the observed directions to those which would have been measured if the transit had been set up at station A

(Ans 38° 50′ 41″ 61, 102° 40′ 52″ 12, 256° 7′ 53″ 04, 324° 3′ 48″ 11)

14. State, in detail, what precautions you would take in measuring a long base line with extreme accuracy by means of a steel tape or wire and describe how you would conduct the field operations.

A line, 3 km long, is measured with a tape of length 100 m which is standard under no pull at 12° C The tape in section is 4 mm wide by $\frac{1}{2}$ mm thick If the line is measured at a temperature of 15° C and the tape is stretched with a pull of 9 kg and is supported at every $\frac{100}{3}$ m of its length, find the length of the line corrected for (a) pull, (b) temperature, and (c) sag Coefficient of expansion = 115×10^{-7} weight of 1 cubic cm of steel = 8 grammes, E = 21×10^{5} kg per square cm

(Ans. 2 km 0 411 m)

15 What are the corrections that must be applied to the measurement of the length of a base line ?

A tape 100 m long was of standard length under a pull of 4 kg at 12° C It was then used in catenary, in three equal spans of $\frac{100}{3}$ m each to measure a level line which was found to measure 3400 m. Calculate the true length of the line from the following data :—

Pull on tape = 10 k~, Section o' tap~ – 5 mm × ¼ mm
Weight of tape per cubic cm of steel = ~ ~ gms
Mean temperature during the field measurements = 20° C
Coefficient of expansion = 0 0000113, E = ~1 × 10^5 ~ per square cm
(Ans 3400 m 0 03o7 m)

16 What is meant by base net ? Explain how you would extend a base line ? A line is measured with a tape 100 ~ in length which is standard at 15°C, when supported thr ughout under a pull of 1 k~ The area of the tape is 0 022 sq cm and its weight is 1 6 kg The temperature at the time of measurement i s C and the pull on the tape is 9 kg the tape being supported ver w intermediate supports making three equal spans The recorded e gth of the line is 8~1 m Find the correct length of the line C ef cient of expansion = 110 10 E = 20 2 × 10^9 kg per sq cm

(Ans 871 159 m)

17 What are the principal objects to be kept in view in selecting the ground for a base line in a large su e ? Enumerate in sequence the operations ne essary before the measuremen of he base line commences State the correct ons to be applied in base line mea urements
Explain how you would prol ng a ven base line

18 Explain wi h sketches how y u w l extend a given base line and check the measurement of its segments by triangulation Give a list of corrections to be applied in base line mea urements

A base line 2 4 km long was measu ed with a tape of length 100 m This tape was suspended in three equal spans to measure the line The tape was stretched wi h a pull of 11 kg The tape was standard under a pull of ~ ~ k at 14 C The cross-sectional area of the tape was 0 038 sq cm and the mean temperature during the field measurements was 26° C Find the correct length of the ba e line given that the coefficient of expansion = 0 000011~ E = 1 × $10^?$ kg per sq cm and weight of the tape ~ 1 kg

(Ans. ~400 127 m)

19 A line was measured on a slope with a 30 m steel tape and its length was found to be 217 4~ m The true length of the tape was 30 00~ m at 2~°C The temperature at the time of measurement was 12° C and the following slopes were observed

2° 40' for 30 m 1° 30' for 60 m 3° 2~ for 60 m, 1° for ~ 47 m The coefficient of expansion was 11 × 10~ per 1°C Compute the true length of the line assuming the tape to be supported uniformly throughout its length

[Correction for standardization = + 0 0o0~ m, correction for temperature = – 0 0331 m correction for slope = – 0 2203 m]

(Ans. 217 2~~ m)

20. A 30 m steel tape was standardized on the flat and was found to be exactly 30 m under a pull of 7 kg at 17° C. It was used in catenary to measure a base of 5 bays The temperature during the measurement was 30° C and the pull exerted during the measurement was the same as that under which the tape was standardized The supports, of the tape were 0 45 0 695, ·850, 0 590, 0 445 m above the first support. The weight of the tape was 1·2 kg and the coefficient of expansion 110×10^{-7} per 1° C. Find the true length of the base

(Correction for temperature = + 0 0215 m, correction for sag = −0 1837 m; correction for slope = −0 0326 m)

(Ans. 149 8052 m).

21. A portion of a straight base line between B and C cannot be directly measured due to some intervening obstruction. To determine its length, two auxiliary points A and D are taken on the base on either side of BC and a theodolite is set up at a point P on the right side of the base line The angles APB, BPC, and CPD are measured and found to be 20°50′ 25″ 140° 15′ 20″, and 28° 32 10″ respectively Compute the length of BC, if the corrected lengths of AB and CD are 411 52 m and 549 55 m respectively

(Ans 563 86 m)

22 A base was deflected from the line proper at station A and the measured length of the deflected section AC was found to be 900 m It was again deflected at C at an angle of 3° 15′ so as to reach the original direction of the base at B. The measured length of CB was 1099 5 m

The tape 100 m long was standardized on the flat and was correct under a pull of 9 kg at 15° C The mean temperature during measurement was 27° C The tape was used in catenary, in three equal spans during measurement and was stretched with a pull of 12 kg The supports of the tape were at the same level The sectional area of the tape was 0 026 sq cm and the weight of one cubic cm of steel 7 5 gm Compute the length of the broken base AB E = 16×10^{5} kg per sq cm, coefficient of expansion = 117×10^{-7}.

[Corrected length of AC = 900 0196 m, corrected length of CB = 1099 5606 m]

(Ans 1998 7637 m)

✣ ✣ ✣

CHAPTER VIII

TRIANGULATION ADJUSTMENT

Definitions.—The observed quantities may be classified as (i) *independent* and (ii) *conditioned*.

(i) **Independent Quantity**.—A quantity is called independent when its value is independent of the values of any other quantities so that change in one does not affect the values of others. No necessary relation exists between the independent quantities, e.g. the reduced levels of several bench marks.

(ii) **Conditioned Quantity**.—A quantity is said to be conditioned when its value is dependent upon the values of one or more quantities on account of some necessary relation between them. If one is varied the values of other quantities are necessarily affected. It is also called a *dependent* quantity, e.g. the angles round a station, the relation between the various observed angles being that their sum is 360°, or the angles of a plane triangle, the condition being that the sum of the three angles is equal to 180°. In this case any two angles may be regarded as independent and the third as dependent or conditioned.

(iii) **Observation**.—An observation is the numerical value of a measured quantity.

(iv) **Direct Observation**.—An observation is said to be direct when it is made directly upon a quantity whose value is desired, e.g. a single measurement of an angle.

(v) **Indirect Observation**.—An observation is called indirect when made upon some function of quantities whose values are to be determined, e.g. measurement of a summation angle for total sum of the individual angles, measurement of an angle by repetition.

(vi) **Weight of an Observation**.—Weight of an observation is a number indicating (a measure of) its relative worth or trustworthiness. Thus if a certain observation is of weight 4, it means that it is four times as much as an observation of weight 1.

(vii) **Weighted Observations** —Observations are called weighted when different weights are assigned to them When observations are made with unequal care and under dissimilar conditions they are required to be weighted When made with the same care and under similar conditions they are called observations of equal weight (or equal precision) Weights are assigned to the observations in direct proportion to the number of observations in the case of an angle Sometimes they are arbitrarily assigned

(viii) **Observed Value of a Quantity** —The observed value of a quantity is the value obtained as a result of an observation after applying the corrections for all known errors

(ix) **True Value of a Quantity** —The true value of a quantity is the value which is absolutely free from all errors. It can never be ascertained

(x) **Most Probable Value of a Quantity** —The most probable value of a quantity is the value which is more likely to be the true value than any other value It is deduced from the several measurements on which it is based

(xi) **A True Error** —A true error is the difference between the true value of a quantity and its observed value

(xii) **A Residual Error** —A residual error (or residual) is the difference between the most probable value of a quantity and its observed value

(xiii) **Observation Equation** —An observation equation is an equation expressing the observed quantity and its numerical value

(xiv) **Reduced Observation Equation** —A reduced observation equation is an equation obtained by substitution of the assumed values of quantities in the original observation equation The assumed value of a quantity is usually taken as its observed value plus a correction

(xv) **Conditional Equation** —A conditional equation is an equation expressing the relation existing between the several dependent quantities

(xvi) **Normal Equation** —A normal equation is an equation of condition by means of which the most probable value of

any unknown quantity may be determined corresponding to a set of values assigned to the other unknown quantities Normal equations must, therefore be formed for each of the unknowns to determine their values

Laws of Weights

The following laws of weights are established by the method of least squares

(1) The weight of the arithmetic mean of observations of unit weight is equal to the number of observations

(2) The weight of the weighted arithmetic mean is equal to the sum of the individual weights

(3) If two or more quantities are added algebraically, the weight of the result is equal to the reciprocal of the sum of the reciprocals of the individual weights

e g $\alpha = 42° 8\ 10''$ weight 4
$\beta = 22° 4\ \ 6''$ weight 2

Weight of $\alpha + \beta\ (= 64°\ 12\ 16'') = \dfrac{1}{(\frac{1}{4}+\frac{1}{2})} = \dfrac{1}{\frac{3}{4}} = \dfrac{4}{3}$

$\alpha - \beta\ (= 20°\ \ 4\ \ 4'') =\ \ ,\ \ \ ,, = \dfrac{4}{3}.$

(4) If a quantity is multiplied by a factor the weight of the product is equal to the weight of that quantity divided by the square of that factor

e g $\alpha = 42° 10\ 5''$ weight 4

Weight of $3\alpha\ (= 126° 30\ 15'') = \dfrac{4}{3^2} = \dfrac{4}{9}$

(5) If a quantity is divided by a factor, the weight of the result equals the weight of that quantity multiplied by the square of that factor

e g $\alpha = 141° 24\ 39''$ weight 3

Weight of $\dfrac{\alpha}{3}\ (= 47° 8\ 13'') = 3 \times 3^2 = 27$

(6) If an equation is multiplied by its own weight, the weight of the resulting equation is equal to the reciprocal of the weight of the equation

e g $A + B = 120° 39' 42''$ weight $\frac{3}{4}$

Weight of $\frac{2}{3}(A + B)\{= 80° 26' 28''\} = \frac{3}{2}$

(7) The weight of an equation remains unchanged, if all the signs of the equation are changed or if the equation is added to or subtracted from a constant

e. g $A + B = 148° 20' 48''$ weight 3 2

Weight of $180° - (A + B)\ \{= 31° 39'\ 12''\} = 3{\cdot}2$

The following rules may be employed in the adjustment of the field observations

(1) Weights vary directly as the number of observations in the case of angles and inversely as the lengths of the various routes in the case of lines of levels

(2) Weights are inversely proportional to the squares of the corresponding probable errors if the angles are measured a large number of times

(3) Corrections to be applied are in inverse proportion to the weights

The Most Probable Values of Quantities

The Method of Least Squares —By the method of least squares we determine (1) the most probable values of the observed quantities and (2) the precision of the observations and of the adjusted results The fundamental principle of the method may be stated as follows

In observations of equal precision the most probable values of the observed quantities are those that render the sum of the squares of the residual errors a minimum '

On account of this principle the method is known as the *method of least squares*

1 Direct Observations of Equal Weight (or Precision) — Let Z be the most probable value of a quantity, M_1 M_2 etc the observed values of the quantity, and n the number of observations taken.

Then the residual errors v_1, v_2 etc are $Z - M_1$, $Z - M_2$ etc.

Now $v_1^2 + v_2^2 + \quad + v_n^2 \qquad = $ a minimum

or $(Z - M_1)^2 + (Z - M_2)^2 + \quad + (Z - M_n)^2 =$ a minimum.

Differentiating the equation we have

$$(Z - M_1) + (Z - M_2) + \quad + (Z - M_n) = 0$$

$$\text{Hence } Z = \frac{M_1 + M_2 + \quad + M_n}{n} \tag{1}$$

The rule may, therefore be stated as follows

The most probable value of the observed quantity is equal to the arithmetic mean of the observed values

The weight of the arithmetic mean is equal to the number of observations $= n$

2 **Direct Observations of Unequal weight (or Presision)** —When the observations are weighted (i e have different weights) the general principle may be stated as follows

In observations of unequal precision the most probable values of the observed quantities are those that render the sum of the weighted squares of the residual errors a minimum

Let the observed values M_1 M_2 etc have the weights w_1 w_2 etc

Then by the above principle we have

$$w_1 v_1^2 + w_2 v_2^2 + \quad + w_n v_n^2 - \text{a minimum}$$

$$\text{or } w_1 (Z - M_1)^2 + w_2 (Z - M_2)^2 + \quad + w_n (Z - M_n)^2 = \text{a minimum}$$

Equating the first derivative to zero we get

$$w_1 (Z - M_1) + w_2 (Z - M_2) + \quad + w_n (Z - M_n) = 0$$

$$Z = \frac{w_1 M_1 + w_2 M_2 + \quad + w_n M_n}{w_1 + w_2 + \quad + w_n} \tag{2}$$

The rule may therefore be stated as follows

The most probable value of the observed quantity is equal to the weighted arithmetic mean of the observed values

The weight of the weighted arithmetic mean is equal to the sum of the individual weights $= w_1 + w_2 + w_3 + \quad + w_n = \Sigma w$

Indirect Observations on Independent Quantities

In the case of indirectly observed quantities the most probable values of the unknowns may be found by the method of *normal equation*

A normal equation is an equation of condition by means of which we determine the most probable value of any one unknown quantity corresponding to any particular set of values given to the remaining unknown quantities

The normal equations must, therefore, be formed for each of the unknown quantities from their observation equations The solution of these normal equations will give the most probable values of the unknowns

Case I Indirect Observations of Equal Weight —

Rule for forming a normal equation —

"To form a normal equation for each of the unknown quantities multiply each observation equation by the algebraic coefficient of that unknown quantity in that equation, and add the results

Check on the formation of normal equations —The coefficients of the unknown quantities in the first column are the same as those in the first row in *value sign* and *order* The same is true for each of the successive columns and rows

Example Find the most probable value of the angle A from the following observation equations

$A = 40° 20' 12''$ $2A = 80° 40' 20''$ $6A = 242° 1' 6''$

To form the normal equation in A multiply the first equation by 1 the second by 2 and the third by 6

$$\begin{aligned} \text{Then}\quad A &= 40° 20' 12'' \\ 4A &= 161\ 20\ 40 \\ 36A &= 1452\ 6\ 36 \\ \hline 41A &= 1653° 47' 28'' \quad \text{normal equation in A} \\ A &= 40° 20' 10''\!\cdot 9 \end{aligned}$$

Case II Indirect Observations of Unequal Weight

Rule for forming the normal equation —

To form the normal equation for each of the unknown quantities multiply each observation equation by the product of the algebraic coefficient of that unknown quantity in that equation and the weight of that observation and add the results

The *check* on the formation of the normal equations is the same as in Case I

Example —Find the most probable value of the angle A from the following observation equations

$2A = 20° 12' 20''.4$ weight 2 $4A = 40° 24' 42''$ weight 3

Now to form the normal equation for A multiply the first equation by 4 ($= 2 \times 2$) and the second equation by 12 ($= 4 \times 3$)

Then $8A = 80° 49' 21''.6$
$48A = 484 \; 56 \; 24$
$56A = 565 \; 45 \; 45.6$ normal equation for A
$A = 10° \; 6' \; 10''.46$

The most probable value of the angle $A = 10° 6' 10''.46$

Conditioned Quantities

In the case of the conditioned quantities there are one or more conditional equations in addition to the observation equations The number of independent conditional equations is always less than the number of unknown quantities There are two methods of determining the most probable values of the unknowns viz (1) the method in which the conditional equations are avoided or eliminated and (2) the method in which the observation equations are eliminated in which case the solution is obtained by the method of Correlates The first method is suitable for the simple problems while the second one for the complicated ones

First Method —In this method all the observation equations are written in terms of the independent quantities thus eliminating conditional equations The most probable values of the unknowns may then be found by the rules for independent quantities

Example 1 —The following are the observed values of A B and C at a station the angles being subject to the condition that $A + B = C$

$A = 20° 10' 32''.?$ $B = 40° 32' 18''.8$ $C = 60° 42' 53''.6$

Find the most probable values of A B and C

To avoid the conditional equation we write the third observation equation as $A + B = 60° 42' 53''.6$ thus expressing all the

observation equations in terms of the independent quantities A and B

Therefore, the observation equations are

$$A = 20^\circ\ 10'\ 32''.2 \qquad (1)$$
$$B = 40^\circ\ 32'\ 18''.8 \qquad (2)$$
$$A+B = 60^\circ\ 42'\ 53''.6 \qquad (3)$$

Forming the normal equations for A and B by the rule stated in case I we get

$$\begin{aligned} A \quad &= 20^\circ\ 10'\ 32''.2 \\ A + B &= 60^\circ\ 42'\ 53''.6 \\ \hline 2A + B &= 80^\circ\ 53'\ 25''.8 = \text{normal equation for A} \end{aligned}$$

Similarly,

$$\begin{aligned} B &= 40^\circ\ 32'\ 18''.8 \\ A + B &= 60^\circ\ 42'\ 53''.6 \\ \hline A + 2B &= 101^\circ\ 15'\ 12''.4 = \text{normal equation for B} \end{aligned}$$

Solving these normal equations, we get

$A = 20^\circ\ 10'\ 33''.07$, $B = 40^\circ\ 32'\ 19''.67$, $C = 60^\circ\ 42'\ 52''.74$

Example 2 —Find the most probable values of the angles A, B and C of the triangle ABC from the following observation equations

$$A = 58^\circ\ 24'\ 36'',\ B = 52^\circ\ 12'\ 43'',\ C = 69^\circ\ 22'\ 45''$$

Here the condition is that $A + B + C = 180^\circ$. This conditional equation is avoided by writing the third observation equation involving C in terms of A and B which are selected as independent quantities. Therefore the observation equations are

$$A = 58^\circ\ 24'\ 36'' \qquad (1)$$
$$B = 52^\circ\ 12'\ 43'' \qquad (2)$$
$$180^\circ - (A + B) = 69^\circ\ 22'\ 45'' \qquad (3)$$

or

$$A + B = 110^\circ\ 37'\ 15'' \qquad (3)$$

Now we form the normal equations for A and B by the rule stated in case I

$$\begin{aligned} A \quad &= 58^\circ\ 24'\ 36'' \\ A + B &= 110\ 37\ 15 \\ \hline 2A + B &= 169^\circ\ 1'\ 51'' \\ &= \text{normal equation in A} \end{aligned} \qquad \begin{aligned} B &= 52^\circ\ 12'\ 43'' \\ A + B &= 110\ 37\ 15 \\ \hline A + 2B &= 162^\circ\ 49'\ 58'' \\ &= \text{normal equation in B} \end{aligned}$$

$\therefore$ The normal equations are

$$2A + B = 169° \; 1' \; 51'' \quad \ldots \quad \ldots \quad (4)$$

$$A + 2B = 162 \quad 49 \quad 58 \quad \ldots \quad \ldots \quad (5)$$

Solving these simultaneous equations, we have

$A = 58° \; 24' \; 34''\cdot67$; $B = 52° \; 12' \; 41'' \; 67$,

and $C = 180° - (A+B) = 180° - (110° \; 37' 16''\cdot34) = 69° 22' 43'' \; 66$.

Check :—$A + B + C = 58° \; 24' \; 34''\cdot67 + 52° \; 12' \; 41'' \; 67 + 69° \; 22' \; 43''\cdot66 = 180°$.

Example 3 —Given the following observations at a station O :—

AOB(A) = 87° 34′ 22″ weight 2, COD (C) = 102° 26′ 9″ weight 4; BOC(B) = 98° 42′ 18″ weight 3; DOA (D) = 71° 17′ 4″ weight 1.

Find the most probable values of A, B, C, and D.

Here the condition is that $A + B + C + D = 360°$, since the horizon is closed.

Referring to the observation and normal equations in the preceding example, we find that the right hand members contain large numbers. In order to make them as small as possible, we introduce corrections to the observed values and find the most probable values of these corrections, and then determine the most probable values of the quantities by applying the corrections algebraically By this artifice numerical work is abbreviated.

Let c_1, c_2, and c_3 be the corrections to A, B, and C respectively so that the most probable values of A, B, and C are
$A = 87° \; 34' \; 22'' + c_1$; $B = 98° \; 42' \; 18'' + c_2$; $C = 102° \; 26' \; 9'' + c_3$.

In the fourth observation equation we substitute $360° - (A + B + C)$ for D, thereby avoiding the conditional equation. Then the equation becomes $360° - (A + B + C) = 71° \; 17' \; 4''$.

Now on substituting the above values of A, B, and C in the given observation equations, the reduced observation equations are

$$c_1 = 0 \quad \text{weight } 2; \quad \ldots \quad (1)$$

$$c_2 = 0 \quad \text{weight } 3; \quad \ldots \quad (2)$$

$$c_3 = 0 \quad \text{weight } 4; \quad \ldots \quad (3)$$

$$c_1 + c_2 + c_3 = +7 \quad \text{weight } 1; \quad \ldots \quad \ldots \quad (4)$$

By the rule for normal equations stated under case II we get

Normal equation in c_1

$$\begin{array}{l} 2c_1 \qquad = \quad 0 \\ \underline{c_1 + c_2 + c_3 = +7} \\ 3c_1 + c_2 + c_3 = +7 \end{array}$$

Normal equation in c_2

$$\begin{array}{l} 3c_2 \qquad = \quad 0 \\ \underline{c_1 + c_2 + c_3 = +7} \\ c_1 + 4c_2 + c_3 = +7 \end{array}$$

Normal equation in c_3

$$\begin{array}{l} 4c_3 = \quad 0 \\ \underline{c_1 + c_2 + c_3 = +7} \\ c_1 + c_2 + 5c_3 = +7 \end{array}$$

The normal equations are

$$3c_1 + c_2 + c_3 = +7 \qquad (5)$$

$$c_1 + 4c_2 + c_3 = +7 \qquad (6)$$

$$c_1 + c_2 + 5c_3 = +7 \qquad (7)$$

The solution of these normal equations gives

$$c_1 = 1'' \cdot 68, \quad c_2 = 1'' \cdot 12, \quad c_3 = 0'' \cdot 84$$

Therefore, the most probable values of A, B, and C are

$$\begin{aligned} A &= 87^\circ\ 34'\ 22'' + 1''\ 68 = 87^\circ\ 34'\ 23''\ 68 \\ B &= 98^\circ\ 42'\ 18'' + 1''\ 12 = 98^\circ\ 42'\ 19'' \cdot 12 \\ C &= 102^\circ\ 26'\ 9'' + 0''\ 84 = 102^\circ\ 26'\ 9''\ 84 \\ D &= 71^\circ\ 17'\ 4'' + 3''\ 36 = 71^\circ\ 17'\ 7'' \cdot 36 \end{aligned}$$

$$\text{Correction to } D = \text{total correction} - (c_1 + c_2 + c_3)$$
$$= 7 - (1\ 68 + 1\ 12 + 0\ 84) = 3''\ 36$$

It will be noticed here that the correction to the observation having the largest weight is the least, which should be the case

teAlrnative method —Let c_1, c_2, c_3, c_4 be the corrections to A, B, C, and D

The sum of the observed values of A, B, C, and D is found to be 359° 59 53″

The total correction is, therefore, equal to + 7″

Now by the rule that the corrections are inversely proportional to the respective weights, we have

$$c_1 \quad c_2 \quad c_3 \cdot c_4 \text{ as } \tfrac{1}{2} \quad \tfrac{1}{3} \cdot \tfrac{1}{4} \cdot 1 \text{ or as } 6 \cdot 4 \quad 3 : 12$$

$$c_1 = \frac{6}{25}(7) - 1''\ 68; \quad c_2 = \frac{4}{25}(7) = 1'' \cdot 12$$

$$c_3 = \frac{3}{25}(7) = 0'' \cdot 84, \quad c_4 = \frac{12}{25}(7) = 3'' \cdot 36$$

Check :—$c_1+c_2+c_3+c_4=1\cdot68+1\cdot12+0\cdot84+3\cdot36=7''\cdot00$
= total correction.

The Probable Error

Definition :—In any large series of observations the probable error is an error of such a value that the number of errors numerically greater than it is the same as the number of errors numerically less than it

Thus, if the probable error of an angular observation is 3 seconds, the probability of the error lying between the limits of -3 seconds and $+3$ seconds equals the probability of its lying outside these limits

The probable error of an observation is a mathematical quantity and gives an absolute idea of the precision of the results. The precision of different observations can also be compared from the known values of their probable errors The probable error is always written after the observed quantity with the plus and minus signs, e g $78°42'13''\ 24\pm3''$ 24or 3252 295m$\pm$0 0039m. It may be remembered that the probable error is a measure of the accuracy of observations only with regard to *accidental* errors.

1. Direct Observations of Equal Weight (or Precision) —

Note —The probable error of any observation with weight w

$$=\frac{\text{the probable error of an observation of unit weight}}{\sqrt{w}}$$

The probable error of a single observation

$$=E_s=0\ 6745\sqrt{\frac{\Sigma v^2}{(n-1)}} \qquad (1$$

where n = the number of observations

Σv^2 = the sum of the squares of the residuals.

The probable error of the arithmetic mean

$$=E_m=\frac{E_s}{\sqrt{n}}=0\ 6745\sqrt{\frac{\Sigma v^2}{n\ (n-1)}} \qquad (2)$$

2. Direct Observations of Unequal Weight (or Precision) :

The probable error of a single observation of unit weight

$$=E_s'=0\cdot6745\sqrt{\frac{\Sigma\ (wv^2)}{(n-1)}} \qquad \ldots \quad \ldots \quad (3)$$

The probable error of any observation whose weight is w

$$= \frac{E_s}{\sqrt{w}} \tag{4}$$

The probable error of the weighted arithmetic mean

$$= E_m = \frac{E_s}{\sqrt{\Sigma w}} = 0\ 6745 \sqrt{\frac{\Sigma(wv^2)}{\Sigma w\,(n-1)}} \tag{5}$$

in which n = the number of observations

w = the weight of an observation

Σw = the sum of the weights

Σwv^2 = the sum of the weighted squares of the residuals

3 **Indirect Observations on Independent Quantities** —The probable error of an observation of unit weight

$$= E_s = 0\ 6745 \sqrt{\frac{\Sigma wv^2}{(n-q)}} \tag{6}$$

The probable error of an observation of weight $w = \dfrac{E_s}{\sqrt{w}}$ (7)

in which n = the number of observation equations

q = the number of unknown quantities

4 **Indirect Observations Involving Conditional Equations** —The probable error of an observation of unit weight

$$= E_s = 0\ 6745 \sqrt{\frac{\Sigma wv^2}{(n-q+p)}} \tag{8}$$

where n = the number of observation equations.

q = the number of unknown quantities

p = the number of conditional equations

The probable error of an observation of weight $w = \dfrac{E_s}{\sqrt{w}}$ (9)

5 **Computed Quantities** —

Case I —The computed quantity is the sum or difference of an observed quantity and a constant

Let x = the computed quantity
e_x = its probable error
a = the observed quantity
e_a = its probable error
k = a constant

Then $x = \pm a \pm k$

and $e_x = e_a$ (10)

Example —Given the most probable value of A = 60°20 30" ± 1 5"

Find the most probable value of its supplement A

By the condition A + A = 180° we have

A = 180° − (60° 20 30") = 119° 39 30"

The probable error of A = the probable error of A

The most probable value of A = 119° 39 30" ± 1 5"

Case II —The computed quantity is obtained by the product of an observed quantity and a constant factor

Then $x = ka$

and $e_x = ke_a$ (11)

Case III —The computed quantity is the algebraic sum of two or more independently observed quantities

a b c etc — the independently observed quantities

e_a e_b e_c etc — the probable errors of a b c etc

Then $x = \pm a \pm b \pm c$ etc

and $e_x = \sqrt{e_a^2 + e_b^2 + e_c^2 + \text{etc}}$ (12)

Example —Given A = 60° 25 40 ± 1" 2
B = 38° 12 30" ± 1" 3

Find the probable error of the value of the angle obtained by addition

x = A + B = 98° 38 10"

$e_x = \sqrt{(e_a)^2 + (e_b)^2} = \sqrt{(1\ 2)^2 + (1\ 3)^2} = \mp 1"\ 769^2$

Angle x = 98° 38 10" ± 1" 77

Case IV —The computed quantity is any function of a single observed quantity

$$x = \phi(a)$$

Then $$e_x = e_a \frac{dx}{da} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

Example :—Find the most probable value and the probable error of the area of a circle whose radius in m is $24\cdot25 \pm 0\cdot02$.

$$x = \pi a^2 = \pi (24\cdot25)^2 = 1848 \text{ sq. m.}$$

$$\frac{dx}{da} = 2\pi a, \quad e_a = \pm 0\cdot02$$

Now $$e_x = e_a \frac{dx}{da} = \pm \cdot02 \times 2\pi \times 24\cdot25 = \pm 3\cdot05.$$

$\therefore$ Area $= 1848 \pm 3\cdot05$ sq. m.

Case V —The computed quantity is any function of two or more independently observed quantities.

$$x = \phi(a, b, \text{etc})$$

Then $$e_x = \sqrt{\left(e_a \frac{dx}{da}\right)^2 + \left(e_b \frac{dx}{db}\right)^2 + \text{etc.}} \quad \ldots \quad \ldots \quad (14)$$

Example —Find the most probable value and the probable error of the area of a rectangle whose sides in m., are $150 \pm 0\cdot02$ and $200 \pm 0\cdot03$.

Then $a = 150 \pm 0\cdot02$, $b = 200 \pm 0\cdot03$.

$$x = ab = 150 \times 200 = 30000 \text{ sq. m.}$$

$$\frac{dx}{da} = b, \quad \frac{dx}{db} = a; \quad e_a = \pm 0\cdot02; \quad e_b = \pm 0\cdot03.$$

Now $$e_x = \sqrt{\left(e_a \frac{dx}{da}\right)^2 + \left(e_b \frac{dx}{db}\right)^2}$$

$$= \sqrt{(0\cdot02 \times 200)^2 + (0\cdot03 \times 150)^2} = \pm 6\cdot02.$$

$\therefore$ Area $= 30000 \pm 6\cdot02$ sq. m

Example 1 —The following are the direct measurements of a base line

3678·32 m., 3678·38 m.; 3678·09 m.
3678·29 m., 3678·26 m.; 3677·98 m.

Find the most probable value of the length of the base line and its probable error.

Observed value (M)	Arithmetic mean (Z)	Residual v	(Residual)^2 v^2
3678 32 m	3678 22 m	-0 10	0 0100
3678 29 m		-0 07	0 0049
3678 38 m		-0 16	0 0256
3678 26 m		-0 04	0 0016
3678 09 m		$+0$ 13	0 0169
3677 98 m		-0 24	0 0576
Sum = 22069 32 m $n = 6$		$\Sigma v = 0$ 00	$\Sigma v^2 = 0$ 1166

$Z = 3678$ 22 m

The probable error of a single observation

$$= 0\ 6745 \sqrt{\frac{0\ 1166}{(6-1)}} = \pm\ 0\ 103$$

The probable error of the arithmetic mean

$$= \frac{Es}{\sqrt{n}} = \frac{0\ 103}{\sqrt{6}} = \pm\ 0\ 042 \text{ m}$$

Hence the most probable value of the length of the base line = 3678 22 $\pm$ 0 042 m

Example 2 —The following are the direct observations of the angle B

45° 17	34′ 26	weight 2			45° 17	33′ 96	weight	2
45° 17	35 82	,,	3 ,		45 17	36 28	,,	1
45 17	35 04	,	4		45 17	33 44	,,	3

Find the probable error of the angle B

M	Z	v	w	wv^2
45° 17 34 26	45° 17 34′ 71	$+$ 45	2	0 4050
35 82		-1 11	3	3 6963
35 04		-0 33	4	0 4356
33 96		$+0$ 75	2	1 1250
36 28		-1 57	1	2 4649
33 44		$+1$ 27	3	4 5387

Weighted mean

$Z = 45°\ 17'\ 34''{\cdot}71 \quad n = 6, \ \Sigma w = 15, \ \Sigma wv^2 = 12.6655$

The probable error of the weighted arithmetic mean

$$= \pm 0.6745 \sqrt{\frac{12.6696}{15(6-1)}}$$

$$= \pm 0''.28$$

Station Adjustment

After the completion of field work of measurement of angles, it is necessary to adjust the angles so as to satisfy the geometrical conditions involved e g the sum of the angles around a station should equal 360° or the sum of the three angles of a triangle should be equal to 180° In every important work the entire triangulation system is adjusted in one operation by the method of least squares The process being very laborious it is usual to divide the adjustment of the triangulation system into two parts which are separately adjusted More generally the angles of a triangle or a chain of triangles etc are adjusted under two heads (1) *Station adjustment and* (2) *Figure adjustment* the former being made prior to the latter

(1) **Station Adjustment** —Station adjustment is the determination of the most probable values of two or more angles measured at a station so as to satisfy the condition of being geometrically consistent We shall now consider the various cases of station adjustment which necessarily involve one or more conditional equations

Case I When the Horizon is Closed with Angles of Equal Weight —In Fig 171 the angles A, B and C are measured at a station O with equal care Since the horizon is closed the conditional equation is $A + B + C = 360°$ The most probable value of each angle may be obtained by equal distribution of the error of closure (i e the difference between the actual sum of the angles and the theoretic sum)

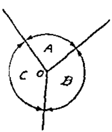


Fig 171

Case II When the Horizon is Closed with Angles of Unequal Weight When the angles have been assigned different weights the discrepancy is distributed among the angles inversely as the respective weights

Case III : Summation Adjustment :—(Fig 172). When several angles are measured at a station individually, and in combination (summation angles), their most probable values are determined by the method of normal equations. However, if two or more angles, and also their sum are measured at a station, the following rules may be employed to determine their most probable values.

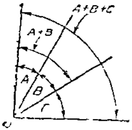


Fig 172

Rule 1 :—When several angles, and also their sum have equal weights, distribute the discrepancy equally among all the measured angles, the sign of the correction for the summation angle being opposite to that of the corrections for the individual angles

(If the measured sum is less than the sum of individual measurements, the correction for the summation angle is positive and that for the individual measurements negative, and vice versa).

Rule 2 :—When the measured angles have different weights, distribute the discrepancy among all the measured angles inversely as their respective weights, the sign of the correction for the summation angle being opposite to that of the corrections for the individual angles

Example 1 :—Find the most probable values of the angles A, B, and the summation angle A + B from the following observations .

$A = 42°$	$20'$	$30''.4$	weight	1
$B = 36°$	$18'$	$25''.2$	,,	2
$A + B = 78°$	$38'$	$50''.3$	,,	3

The sum of the measured values of A and $B = 78° 38' 55''.6$.
The measured value of the summation angle $A + B = 78° 38' 50''.3$.

$\therefore$ Discrepancy $= 5''.3$.

Now this discrepancy is to be distributed in the proportion of $1 : \frac{1}{2} : \frac{1}{3}$ i. e. $6 : 3 : 2$

Since the summation angle is less than the sum of the angle

A and B, the correction to the summation angle is positive and that to the angles A and B negative

Therefore, the correction to $A = \frac{6}{11}(5.3) = 2''.90 \qquad (-ve)$

" to $B = \frac{3}{11}(5.3) = 1''.44 \qquad (-ve)$

" to $A + B = \frac{2}{11}(5.3) = 0''.96 \qquad (+ve)$

Hence the most probable values are

$$\begin{aligned} A &= 42^\circ\ 20'\ 27''.50 \\ B &= 36\ \ 18\ \ 23''.76 \\ A + B &= 78^\circ\ 38'\ 51''.26 \end{aligned}$$

Example 2.—Given the following observations

$$\begin{aligned} A &= 45^\circ\ 26'\ 48''.34 \\ B &= 52\ \ 43\ \ 24.62 \\ C &= 48\ \ 34\ \ 22.78 \\ A + B &= 98\ \ 10\ \ 12.46 \\ B + C &= 101\ \ 17\ \ 47.65 \end{aligned}$$

Find the most probable values of A, B, and C

Let e_1, e_2 and e_3 be the corrections to A, B, and C

Then the most probable value of $A = 45^\circ\ 26'\ 48''.34 + e_1$

" of $B = 52\ \ 43\ \ 24.62 + e_2$

" of $C = 48\ \ 34\ \ 22.78 + e_3$

On substituting these values in the above observation equations we get the following reduced observation equations

$$\begin{aligned} e_1 \qquad\qquad &= 0 \\ e_2 \qquad &= 0 \\ e_3 &= 0 \\ e_1 + e_2 \qquad &= -0.50 \\ e_2 + e_3 &= +0.25 \end{aligned}$$

By the rule for normal equations, the normal equations or c_1, c_2, and c_3 are

$$\begin{aligned} 2c_1 + c_2 \qquad &= -0{\cdot}50 \\ c_1 + 3c_2 + c_3 &= -0{\cdot}25 \\ c_2 + 2c_3 &= -0{\cdot}25 \end{aligned}$$

The solution of these equations gives

$$c_1 = -0''{\cdot}22,\ c_2 = -0''{\cdot}06,\ c_3 = +0''{\cdot}16$$

Whence, the most probable values are

$$\begin{aligned} A &= 45^\circ\ 26'\ 48''{\cdot}34 - 0''{\cdot}22 = 45^\circ\ 26\ \ 48''{\cdot}12 \\ B &= 52\ \ 43\ \ 24{\cdot}62 - 0{\cdot}06 = 52\ \ 43\ \ 24{\cdot}56 \\ C &= 48\ \ 34\ \ 22{\cdot}78 + 0{\cdot}16 = 48\ \ 34\ \ 22{\cdot}94 \end{aligned}$$

Example 3 —Find the most probable values of A, B, and C from the following observations

$$\begin{aligned} A &= 32^\circ 15'\ 36''{\cdot}2, \text{ weight } 2, & A + B &= 72^\circ\ 31\ \ 50''{\cdot}2, \text{ weight } 1, \\ B &= 40\ \ 16\ \ 18{\cdot}4 \quad ,, \quad 1, & A + B + C &= 107\ 44\ \ 25{\cdot}5 \quad ,, \quad 2 \\ C &= 35\ \ 12\ \ 26{\cdot}6 \quad ,, \quad 1, \end{aligned}$$

Let c_1, c_2, and c_3 be the most probable corrections to A, B, and C Then the most probable values of A, B, and C are $A = 32^\circ 15'\ 36''{\cdot}2 + c_1$, $B = 40^\circ 16'\ 18''{\cdot}4 + c_2$, $C = 35^\circ 12'\ 26''{\cdot}6 + c_3$

Substituting these values in the observation equations, the reduced observation equations may be written thus

$$\begin{aligned} c_1 \qquad\qquad &= 0 \quad \text{weight } 2 \\ c_2 \qquad &= 0 \quad ,, \quad 1 \\ c_3 &= 0 \quad ,, \quad 1 \\ c_1 + c_2 \qquad &= -4{\cdot}4 \quad ,, \quad 1 \\ c_1 + c_2 + c_3 &= +4{\cdot}3 \quad ,, \quad 2 \end{aligned}$$

From which we get the following normal equations for c_1, c_2, and c_3

$2c_1 = 0$	$c_2 = 0$	$c_3 = 0$
$c_1 + c_2 = -4{\cdot}4$	$c_1 + c_2 = -4{\cdot}4$	$2c_1 + 2c_2 + 2c_3 = +8{\cdot}6$
$2c_1 + 2c_2 + 2c_3 = +8{\cdot}6$	$2c_1 + 2c_2 + 2c_3 = +8{\cdot}6$	
$5c_1 + 3c_2 + 2c_3 = +4{\cdot}2$ = normal equation in c_1	$3c_1 + 4c_2 + 2c_3 = +4{\cdot}2$ = normal equation in c_2	$2c_1 + 2c_2 + 3c_3 = +8{\cdot}6$ = normal equation in c_3

Solving these normal equations we have

$$c_1 = -0''\cdot 24;\ c_2 = -0''\cdot 44;\ c_3 = +3''\cdot 305.$$

Therefore, the most probable value	of A = 32°	15′	35″·98
,, ,, ,,	of B = 40	16	17 ·96
,, ,, ,,	of C = 35	12	29 91

Example 4.—The following angles were measured at a station O so as to close the horizon.—

AOB (θ_1) = 83°	42′	28″ 75,	weight	3
BOC (θ_2) = 102	15	43 ·26,	,,	2
COD (θ_3) = 94	38	27 ·22,	,,	4
DOA (θ_4) = 79	23	23 ·77,	,,	2

Adjust the angles.

We shall work out the problem by the several methods

First Method.—By the application of the rule, viz. corrections to the angles are inversely proportional to the respective weights

Since the horizon is closed, $\theta_1 + \theta_2 + \theta_3 + \theta_4 = 360°$.

Now the sum of the observed values of the angles $= 360°\,0'3''$.

The discrepancy $= +3''$.

If, c_1, c_2, c_3, and c_4 be the corrections to the angles θ_1, θ_2, θ_3, and θ_4, respectively, then

$c_1 + c_2 + c_3 + c_4 = -3''$, and by the rule, $c_1 : c_2 : c_3 : c_4$ as $\frac{1}{3}$ $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{2}$ or 4 6 3 : 6, we have

$$c_1 = \frac{4}{19}(3) = -0''\ 63, \quad c_2 = \frac{6}{19}(3) = -0''\cdot 95.$$

$$c_3 = \frac{3}{19}(3) = -0''\ 47; \quad c_4 = \frac{6}{19}(3) = -0''\ 95$$

Here it will be noticed that the angle having the largest weight has the least correction, which should be the case

Whence, the most probable value	of θ_1 = 83°	42′	28″·12
,, ,,	of θ_2 = 102	15	42 ·31
,, ,,	of θ_3 = 94	38	26 ·75
,, ,,	of θ_4 = 79	23	22 ·82
Check:—	Sum = 360°	0′	0″·00

Second method :—By the method of normal equations :

(a) The most probable values are obtained directly from the observation equations.

Here the angles θ_1, θ_2, and θ_3 are considered as independent and the conditional equation is avoided by writing for θ_4 its value in terms of θ_1, θ_2, θ_3 and $\theta_4 = \{ 360° - (\theta_1 + \theta_2 + \theta_3) \}$ in the observation equation involving θ_4

Then the last observation equation becomes

$360° - (\theta_1 + \theta_2 + \theta_3) = 79° \; 23' \; 23''\, 77$

or $\quad \theta_1 + \theta_2 + \theta_3 = 280° \; 36' \; 36''{\cdot}23.$

Therefore, the observation equations are

$\theta_1 =$	83°	42′	28″·75	weight	3	(1)
$\theta_2 =$	102	15	43 ·26	,,	2	(2)
$\theta_3 =$	94	38	27 22	,,	4	(3)
$\theta_1 + \theta_2 + \theta_3 =$	280	36	36 23	,,	2	(4)

Applying the rule for normal equations, we have

$5\theta_1 + 2\theta_2 + 2\theta_3 = 812° \; 20' \; 38''\, 71$ = normal equation in θ_1

$2\theta_1 + 4\theta_2 + 2\theta_3 = 765 \; 44 \; 38\; 98$ = ,, ,, in θ_2

$2\theta_1 + 2\theta_2 + 6\theta_3 = 939 \; 47 \; 1\; 34$ = ,, ,, in θ_3

The solution of these equations gives

$\theta_1 = 83° \, 42' \, 28''\, 12$; $\theta_2 = 102° \, 15' \, 42''\, 31$, $\theta_3 = 94° \, 38' \, 26''{\cdot}75.$

Then $\theta_4 = 360° - (\theta_1 + \theta_2 + \theta_3) = 360° - 280° \; 36' \; 37''\, 18$

$\qquad\qquad = 79° \; 23' \; 22''\, 82$

(b) The numerical work may be simplified by introducing corrections to the observed values of the angles and then finding their most probable values, which, when applied algebraically, gives the most probable values of the angles.

Let c_1, c_2, and c_3 be the corrections to θ_1, θ_2, and θ_3 respectively. Then the most probable values of θ_1 θ_2, and θ_3 are

$\theta_1 = 83° \; 42' \; 28''{\cdot}75 + c_1$

$\theta_2 = 102 \; 15 \; 43 \cdot 26 + c_2$

$\theta_3 = 94 \; 38 \; 27 \cdot 22 + c_3$

The conditional equation being avoided as explained above, the reduced observation equations are

$$\begin{array}{llll} c_1 & = 0 & \text{weight} & 3 \\ c_2 & = 0 & ,, & 2 \\ c_3 & = 0 & ,, & 4 \\ c_1 + c_2 + c_3 & = -3 & ,, & 2 \end{array}$$

By the rule for normal equations, we have

$$\begin{array}{ll} 5c_1 + 2c_2 + 2c_3 = -6 = & \text{normal equation in } c_1 \\ 2c_1 + 4c_2 + 2c_3 = -6 = & \quad ,, \qquad ,, \quad \text{in } c_2 \\ 2c_1 + 2c_2 + 6c_3 = -6 = & \quad ,, \qquad ,, \quad \text{in } c_3 \end{array}$$

Solving these equations, we have

$$c_1 = -0''.632 \qquad c_2 = -0''.947 \qquad c_3 = -0''.474$$
$$= -0''.63, \qquad = -0''.95; \qquad = -0''.47.$$

$$\text{Correction to } \theta_4 = -3 - (-0.632 - 0.947 - 0''.474)$$
$$= -0.947 = -0''.95$$

which agree with the corrections previously obtained

Third method —By the method of Correlates:

Let c_1, c_2, c_3 and c_4 be the corrections to θ_1, θ_2, θ_3 and θ_4

By the conditional equation,

$$c_1 + c_2 + c_3 + c_4 = -3 \qquad (1)$$

By the principle of least squares,

$$w_1c_1^2 + w_2c_2^2 + w_3c_3^2 + w_4c_4^2 = \text{a minimum} \qquad (2)$$

Differentiating the equations 1 and 2, we have

$$\delta c_1 + \delta c_2 + \delta c_3 + \delta c_4 = 0 \qquad (3)$$

$$w_1c_1\delta c_1 + w_2c_2\delta c_2 + w_3c_3\delta c_3 + w_4c_4\delta c_4 = 0 \qquad \ldots \qquad (4)$$

Multiplying the equation (3) by $-\lambda$, adding the result to the equation (4) and then equating the coefficients of each δc to zero, we have

$$w_1c_1 - \lambda = 0,\ w_2c_2 - \lambda = 0;\ w_3c_3 - \lambda = 0;\ w_4c_4 - \lambda = 0$$

or $\quad c_1 = \dfrac{\lambda}{w_1}$; $c_2 = \dfrac{\lambda}{w_2}$; $c_3 = \dfrac{\lambda}{w_3}$; $c_4 = \dfrac{\lambda}{w_4}$

substituting these values in the equation (1), we get

$$\lambda\left(\frac{1}{w_1}+\frac{1}{w_2}+\frac{1}{w_3}+\frac{1}{w_4}\right)=-3\,;\ \text{Here } w_1=3\,;\ w_2=2\,;$$

$$w_3=4\,;\ w_4=2.$$

or $$\lambda\left(\frac{1}{3}+\frac{1}{2}+\frac{1}{4}+\frac{1}{2}\right)=-3\ \therefore\ \lambda=-\frac{36}{19}.$$

Whence, $$c_1=\frac{1}{3}\left(-\frac{36}{19}\right)=-0''\cdot63\,;\ c_2=\frac{1}{2}\left(-\frac{36}{19}\right)=-0''\cdot95.$$

$$c_3=\frac{1}{4}\left(-\frac{36}{19}\right)=-0''\cdot47\,,\ c_4=\frac{1}{2}\left(-\frac{36}{19}\right)=-0''\cdot95.$$

Example 5.—Find the most probable values of the following station observations closing the horizon :—

A =	28°	24′	28″·4	weight	2
B =	32	14	16 ·3	,,	1
A + B =	60	38	50 ·7	,,	1
C =	299	21	11 ·8	,,	2
B + C =	331	35	27 ·8	,,	3

Since the angles A, B, and C close the horizon, their sum must satisfy the conditional equation, viz. $A + B + C = 360°$.

Let A and B be regarded as independent quantities and C as a dependent quantity. To avoid the conditional equation, we write the observation equations in terms of the independent quantities A and B by subtracting all angles involving C from 360°. Therefore, we get

A =	28°	24′	28″·4	weight	2
B =	32	14	16 ·3	,,	1
A + B =	60	38	50 ·7	,,	1
A + B =	60	38	48 ·2	,,	2
A =	28	24	32 ·2	,,	3

Now let c_1 and c_2 be the most probable corrections for A and B. Then the most probable values of A and B are

$$A = 28°\ 24'\ 28''\cdot4 + c_1\,,\ B = 32°\ 14'\ 16''\cdot3 + c_2$$

Substituting these values in the above equations, the reduced observation equations are

$$
\begin{array}{lll}
c_1 = 0 & \text{weight} & 2 \\
c_2 = 0 & ,, & 1 \\
c_1 + c_2 = +6 & , & 1 \\
c_1 + c_2 = +3\ 5 & , & 2 \\
c_1 = +3\ 8 & ,, & 3
\end{array}
$$

Forming the normal equations in c_1 and c_2, we have

$$8c_1 + 3c_2 = 24\ 4 \text{ and } 3c_1 + 4c_2 = 13\ 0$$

Solving these two equations, we get

$$c_1 = +2''\ 55,\ c_2 = +1''\ 34$$

The most probable values of A, B and C are

$$A = 28^\circ\ 24\ 28''\ 4 + 2''\ 55 = 28^\circ\ 24'\ 30''\ 95$$
$$B = 32\ \ 14\ 16\ \ 3 + 1\ \ 34 = 32\ \ 14\ 17\ \ 64$$
$$C = 360^\circ - (A+B) = 360^\circ - (60^\circ\ 38\ 48''\ 59) = 299^\circ\ 21\ 11''\ 41$$

Example 6 —Find the most probable values of the angles A B, and C from the following observations at a station P

$$
\begin{array}{lll|lll}
A = 35^\circ\ 22\ 25''\ 6 & \text{weight} & 1 & A+B+C = 148^\circ\ 6\ 45''\ 4 & \text{weight} & 1 \\
B = 38^\circ\ 20\ \ 7''\ 7 & , & 1 & B+C = 112^\circ\ 44'\ 29''\ 1 & , & 2 \\
A+B = 73^\circ\ 42\ 32''\ 5 & , & 2 & & &
\end{array}
$$

Let c_1 c_2 and c_3 be the corrections to the angles A, B and C

Assume the value of C as (B + C) − B

$$= (112^\circ\ 44\ \ 29''\ 1) - (38^\circ\ 20\ \ 7''\ 7) = 74^\circ\ 24\ \ 21''\ 4$$

The most probable values of A, B, and C are

$$A = 35^\circ\ 22\ 25''\ 6 + c_1,\quad B = 38^\circ\ 20\ \ 7''\ 7 + c_2,$$
$$C = 74^\circ\ 24'\ 21''\ 4 + c_3$$

Substituting these values in the observation equations, we get the following reduced observation equations

$$
\begin{array}{lll}
c_1 = 0 & \text{weight} & 1 \\
c_2 = 0 & ,, & 1 \\
c_1 + c_2 = -0\ 8 & ,, & 2 \\
c_1 + c_2 + c_3 = -9\ 3 & , & 1 \\
c_2 + c_3 = 0 & ,, & 2
\end{array}
$$

From which the normal equations are

$$4c_1 + 3c_2 + c_3 = -10\ 9$$
$$3c_1 + 6c_2 + 3c_3 = -10\ 9$$
$$c_1 + 3c_2 + 3c_3 = -9\ 3$$

Solving these equations, we get

$$c_1 = -2''\ 88 \quad c = +1''\ 39, \quad c_3 = -3''\ 51$$

The most probable values of A B and C are

$$A = 35^\circ\ 22'\ 25''\ 60 - 2''\ 88 = 35^\circ\ 22\ \ 22''\ 72$$
$$B = 38^\circ\ 20\ \ 7''\ 70 + 1''\ 39 = 38^\circ\ 20\ \ 9''{\cdot}09$$
$$C = 74^\circ\ 24\ \ 21''\ 40 - 3''\ 51 - 74^\circ\ 24\ \ 17''\ 89$$

Figure Adjustment

The determination of the most probable values of the angles involved in any geometrical figure so as to fulfil the geometrical conditions is called the *figure adjustment* All cases of figure adjustment necessarily involve one or more conditional equations The geometrical figures used in a triangulation system are (i) triangles (ii) quadrilaterals and (iii) polygons with central stations Adjustment of the angles can conveniently be done by the *method of Correlates* (or *Correlatives*)

Triangle Adjustment —The following are the various rules for corrections to the observed angles of a triangle

Notation c = the correction to the observed angle
w = the weight of the angle
d = the discrepancy (error of closure)
n = the number of observations
E = the probable error of the angle

Rule 1 —When the angles are of equal weight, distribute the discrepancy equally among the three angles $\left(c = \frac{1}{3}d\right)$

Rule 2 —When the angles are of unequal weight, distribute the discrepancy among all the angles *inversely* as the respective weights $c_A\ \ c_B\ \ c_C = \frac{1}{w_A}\ \ \frac{1}{w_B}\cdot\ \frac{1}{w_C}$

$$c_A = \frac{\frac{1}{w_A}}{\left(\frac{1}{w_A}+\frac{1}{w_B}+\frac{1}{w_C}\right)}d, \quad c_B = \frac{\frac{1}{w_B}}{\left(\frac{1}{w_A}+\frac{1}{w_B}+\frac{1}{w_C}\right)}d,$$

$$c_C = \frac{\frac{1}{w_C}}{\left(\frac{1}{w_A}+\frac{1}{w_B}+\frac{1}{w_C}\right)}d$$

Rule 3 —The corrections are proportional to the reciprocals of the numbers of observations

$$c_A = \frac{\frac{1}{n_A}}{\left(\frac{1}{n_A}+\frac{1}{n_B}+\frac{1}{n_C}\right)}d \text{ etc}$$

Rule 4 —The corrections are inversely proportional to the squares of the numbers of observations

$$c_A = \frac{\left(\frac{1}{n_A}\right)^2}{\left\{\left(\frac{1}{n_A}\right)^2+\left(\frac{1}{n_B}\right)^2+\left(\frac{1}{n_C}\right)^2\right\}}d \text{ etc}$$

Rule 5 —The corrections are proportional to the squares of the probable errors

$$c_A = \frac{E_A^2}{(E_A^2 + E_B^2 + E_C^2)}d \text{ etc}$$

where E_A E_B and E_C are the probable errors of the angles A B and C

This follows from Rule 2 since the weight of a quantity is inversely proportional to the square of its probable error $\left(= \frac{1}{E^2}\right)$

Rule 6 —Gauss s Rule —(1) From the number of observations (n) of an angle find its weight by $w = \frac{\frac{1}{2}n^2}{\Sigma x^2}$ where v is

equal to the residual, i. e. the difference between the *mean* observed value of the angle and its observed value.

Let M = the mean value of the several observations of the angle A.

L_1, L_2, etc. = the several observations of the angle A.

Then $\Sigma v^2 = (M - L_1)^2 + (M - L_2)^2 + \quad + (M - L_n)^2$.

(ii) Knowing the weights of the angles, the corrections may be obtained by Rule 2

$$\therefore \; \frac{1}{w_A} = \frac{\Sigma v^2}{\frac{1}{2}n^2} = \text{say, } K_A.$$

Similarly, find $\frac{1}{w_B}$ ($= K_B$) and $\frac{1}{w_C}$ ($= K_C$)

Then the corrections are

$$c_A = \frac{K_A}{(K_A + K_B + K_C)}\, d\,; \quad c_B = \frac{K_B}{(K_A + K_B + K_C)}\, d\,;$$

$$c_C = \frac{K_C}{(K_A + K_B + K_C)}\, d.$$

The signs of the corrections are plus or minus according as the sum of the angles of the triangle is less or greater than the theoretic sum 180° (or 180° + spherical excess in the case of a spherical triangle)

In adjusting the angles of a triangle, Rules 1, 2, and 6 are commonly used

Example.—Adjust the following angles of the triangle ABC.

A = 52° 35′ 32″;	B = 70° 46′ 22″,	C = 56° 38′ 13″
30	24	10
31	23	12
28	25	11
26	26	
27		

Mean value of A = 52° 35′ 29″; number of observations = 6.

" " of B = 70 46 24 " " = 5.

" " of C = 56 38 11·5 " " = 4.

Now $A+B+C = 180° \, 0' \, 4''{\cdot}5$ $\quad\therefore$ Discrepancy $(d) = +4''{\cdot}5$

$$\text{Weight of A} \quad (w_A) = \frac{\frac{1}{2}n^2}{\Sigma v^2},$$

where $\Sigma v^2 = \Sigma(M-L)^2 = \{(-3)^2+(-1)^2+(-2)^2+(+1)^2+(+3)^2+(+2)^2\} = 28.$

$$\therefore \quad w_A = \frac{\frac{1}{2}(6)^2}{28} = \frac{9}{14} \quad \text{and} \quad K_A = \frac{1}{w_A} = \frac{14}{9} = 1{\cdot}556.$$

Similarly, Weight of B $(w_B) = \dfrac{\frac{1}{2}n^2}{\Sigma v^2}$

$$= \frac{\frac{1}{2}(5)^2}{\{(+2)^2+0+(+1)^2+(-1)^2+(-2)^2\}}$$

$$= \frac{\frac{1}{2}(5)^2}{10} = \frac{5}{4} \quad \text{and} \quad K_B = \frac{1}{w_B} = \frac{4}{5} = 0{\cdot}8$$

Weight of C $(w_C) = \dfrac{\frac{1}{2}n^2}{\Sigma v^2}$

$$= \frac{\frac{1}{2}(4)^2}{\{(-1{\cdot}5)^2+(+1{\cdot}5)^2+(-0{\cdot}5)^2+(0{\cdot}5)^2\}} = \frac{8}{5}$$

$$\text{and} \quad K_C = \frac{1}{w_c} = \frac{5}{8} = 0{\cdot}625.$$

Hence the correction to A $= \dfrac{K_A}{(K_A+K_B+K_C)}\, d$

$$= \frac{1{\cdot}556 \times 4{\cdot}5}{(1{\cdot}556+0{\cdot}8+0{\cdot}625)} = 2''{\cdot}35\,(-ve)$$

,, to B $= \dfrac{K_B}{(K_A+K_B+K_C)}\, d$

$$= \frac{0{\cdot}8 \times 4{\cdot}5}{2{\cdot}981} = 1''{\cdot}21\,(-ve)$$

,, to C $= \dfrac{K_C}{(K_A+K_B+K_C)}\, d$

$$= \frac{0{\cdot}625 \times 4{\cdot}5}{2{\cdot}981} = 0''{\cdot}94\,(-ve)$$

Check : Sum $= -4''{\cdot}5$

Figure Adjustment

Case I : Plane Triangle —It seldom happens that the sum of the measured angles of a triangle equals 180°. It is, therefore, necessary to adjust them so as to fulfil this condition. Rule 1 or 2 may be used for this purpose according as the angles are of equal weight, or of unequal weight If the number of observations of each angle be given, Gauss's rule should be used. After having corrected the angles, the sides of a triangle may be computed from a known side and the three angles, by the sine rule. The side may be known by direct measurement as a base line, or known from the preceding computations The co ordinates of the stations are computed as follows

In the $\triangle$ ABC, let the co-ordinates of A be given and AB, the known side, its azimuth being known from the previous computations

(i) From the known azimuth of AB, and the angles A and B, find the azimuths of BC and AC

(ii) Calculate the latitude and departure of AB

(iii) Find the co-ordinates of B by adding algebraically the latitude and departure of AB to the north co ordinate and east co-ordinate of A respectively

(iv) Calculate the latitudes and departures of BC and AC

(v) Find the co-ordinates of C from B, and also from A to check the results

If the computations be correctly made, the two values of the co ordinates of C must be exactly the same

Case II : Spherical Triangle .—A spherical triangle is a triangle bounded by three arcs of great circles The sum of the three angles of a spherical triangle always exceeds 180° by an amount known as *spherical excess*

Spherical Excess :—The spherical excess (E_s) depends upon the area of a triangle and is, therefore, ignored when the sides of the triangle are short (less than 3 5 km) But when they are great as in geodetic operations it must be taken into account.

It may be taken approximately as one second (1") for every 196·75 sq km Its exact value may be calculated from the following formula :

$$\text{Spherical excess in degrees } (E^\circ_s) = \frac{\Delta \times 180^\circ}{\pi R^2}$$

$$\text{,, ,, in seconds } (E''_s) = \frac{648000\,\Delta}{\pi R^2} \quad - \quad (1)$$

$$\text{,, ,, ,, } = \frac{\Delta}{R^2 \sin 1''} \quad - \quad (1a)$$

in which Δ = the area of the spherical triangle in sq m or sq km

R = the radius of the sphere of the earth

In using the formula care should be taken that Δ and R are expressed in the same units

For work of high precision, spherical excess should be calculated from the formula

$$E''_s \text{ in seconds} = \frac{\Delta(1 - e^2\sin^2\theta)^2}{a^2\,(1 - e^2)\,\sin 1''} \quad - \quad (2)$$

$$= \frac{\Delta}{RN \sin 1''} \quad - \quad (2a)$$

where θ = mean of the latitude of the bounding stations

a = the earth's equatorial semi-axis

e = the earths' eccentricity

R = the radius of curvature of a meridian section at latitude θ

N = the length of the normal or the redius of curvature of the arc cut out on the surface by a normal section, per pendicular to the meridian, at latitude θ

For the purpose of computing the area of the triangle, it is the usual practice to consider the triangle as plane, no serious error being involved in this assumption The area is then calculated from the formula

$\Delta = \frac{1}{2}a^2 \frac{\sin B \sin C}{\sin A}$, where a is the known side, and A, B, and C are the angles of the triangle

Most geodetic tables give values for the logarithms of $\frac{1}{2\,RN}$ or for $\frac{1}{2\,RN \sin 1''}$ for different latitudes and hence the value of $\frac{1}{2\,RN \sin 1''}$ can easily be obtained

To avoid confusion computation work must be done methodically in the following steps

Let A B and C = the mean observed values of the spherical angles of the $\triangle$ABC

A_o B_o and C_o = the approximate plane angles.
A_o, B_o, and C_o = the corrected plane angles
A_1 B_1 and C_1 = the corrected spherical angles

(1) *Spherical Excess* —(i) Add the three angles (A, B and C) and find the total discrepancy (*d*) between this sum and 180°

(ii) Distribute this error equally among the three angles ($\frac{1}{3}\,d$) thus obtaining the values of A_o, B_o and C_o

(iii) Using these values find the area ($\triangle$) of the triangle

(iv) Calculate the spherical excess

(v) Obtain the total error (*e*) in the observed angles by finding the difference between their sum (A+B+C) and (180°+E_s)

(2) (*a*) *When the angles are of equal weight* —

(i) Correct the angles by applying algebraically the correction equal to $\frac{1}{3}$ of the total error (*d*) to each of the observed angles (A B and C) thus obtaining the corrected plane angles A_o, B_o, and C_o

It may be noted that in this case the calculation for spherical excess is not required as the total discrepancy (*d*) includes both the spherical excess and the total error (*e*) in a plane triangle $A_o = A_1 - \frac{1}{3}E_s = (A - \frac{1}{3}\,e) - \frac{1}{3}E_s = A - \frac{1}{3}\,d$

(*b*) *When the angles are of unequal weight* —

(i) Having found the total error (*e*) find the corrections by Rule 2 (corrections inversely proportional to the weights of the angles)

(ii) Obtain the corrected spherical angles A_1, B_1, and C_1 by applying the corrections to each of the observed angles A, B, and C.

(iii) Find the corrected plane angles A_o', B'_o, and C'_o by subtracting $\frac{1}{3}$ of the spherical excess from each of the corrected spherical angles

Computation of the Sides of a Spherical Triangle — Three methods are available for computing the sides of a spherical triangle in which one side, and the three angles are known

First Method By Spherical Trigonometry —

Let BC (a) = the known side of the triangle ABC.
A_1, B_1, and C_1 = the adjusted spherical angles of the $\triangle$ ABC.
α, β and γ = the central angles subtended by BC, CA, and AB respectively

Then (i) Calculate the central angle α from the formula

arc = R $\times$ central angle in radians, or $\alpha^\circ = \dfrac{180^\circ \times a}{\pi R}$

R being the radius of the earth (6371000 m)

(ii) Using the sine rule, find β and γ

$$\sin\beta = \sin\alpha \, \frac{\sin B_1}{\sin A_1} \quad \text{and} \quad \sin\gamma = \sin\alpha \, \frac{\sin C_1}{\sin A_1}$$

(iii) Knowing β and γ, calculate the corresponding lengths of the arcs CA (b) and AB (c) by the relation

$$= \frac{\pi R \beta^\circ}{180^\circ} \quad \text{and} \quad c = \frac{\pi R \gamma^\circ}{180^\circ}.$$

Second Method By Delambre's Method —In this method the angular points A, B, and C are assumed to be joined by straight lines so that the triangle ABC formed by the corresponding chords of the arcs AB, BC, and CA is a plane triangle

(i) As before, calculate the central angle α.

(ii) Knowing the length of arc a and its central angle α, calculate the corresponding chord a by the relation

$$\text{ch } a = 2R \sin\frac{\alpha}{2}.$$

(iii) From the known length of chord a, and the corrected plane angles A_o', B_o', C_o', find the lengths of chord b and chord c by the sine rule

$$\text{Chord } b = \text{chord } a \frac{\sin B_o'}{\sin A_o'} \quad \text{and} \quad \text{chord } c = \text{chord } a \frac{\sin C_o'}{\sin A_o'}$$

(iv) Calculate the central angles β and γ by the relation

$$\sin \frac{\beta}{2} = \frac{\text{ch } b}{2R} \quad \text{and} \quad \sin \frac{\gamma}{2} = \frac{\text{ch } c}{2R}.$$

(v) Knowing β and γ find the lengths of arcs b and c by the relation $\text{arc } b = \dfrac{\pi R \beta^\circ}{180^\circ}$ and $\text{arc } c = \dfrac{\pi R \gamma^\circ}{180^\circ}$.

Third Method By Legendre's Method —The Legendre's theorem may be stated as "In any spherical triangle, the sides of which are small compared with the radius of the sphere, if each of the angles be diminished by one-third of the spherical excess, the sines of these angles will be proportional to the lengths of the opposite sides and the triangle may, therefore, be calculated as if it were plane" In this method each corrected spherical angle is diminished by one third of the spherical excess to determine the plane angles The sides are then computed by the sine rule, considering the triangle as if it were a plane triangle

(i) Find the corrected plane angles A_o, B_o' and C_o

(ii) From the known side a and the angles A'_o, B'_o, and C_o', calculate the lengths b and c by the sine rule

$$b = a \frac{\sin B_o'}{\sin A_o} \quad \text{and} \quad c = a \frac{\sin C_o}{\sin A_o}$$

Of the three methods, the first method is very laborious and is not, therefore, in common use The other two methods are commonly used, since they involve less labour and give equally accurate results.

Example —The mean observed angles in a spherical triangle ABC were recorded as follows:

A	58°	34′	27″ 52	weight	1
B	63°	46′	22″·65	,,	3
C	57°	39′	15″·95	,,	2

The length of the side BC = 56349·7 m. Calculate the lengths of the sides AB and AC.

(i) Summing up the observed angles, we get

A + B + C = 180° 0′ 6″·12; ∴ Summation error = +6″·12.

Let A_o, B_o, and C_o be the approximate plane angles. Then deducting $\frac{1}{3}$ (6″·12) from each angle, we have

$$A_o = 58°\,34'\,27''·52 - 2''·04 = 58°\,34'\,25''·48$$
$$B_o = 63°\,46'\,22''·65 - 2''·04 = 63°\,46'\,20''·61$$
$$C_o = 57°\,39'\,15''·95 - 2''·04 = 57°\,39'\,13''·91$$

(ii) Spherical excess :—

$$\text{Area of the triangle ABC} = \triangle = \frac{\frac{1}{2}a^2 \sin B_o \sin C_o}{\sin A_o}$$

$$= \frac{\frac{1}{2}(56349·7)^2 \sin 63°\,46'\,20''·61 \sin 57°\,39'\,13''·91}{\sin 58°\,34'\,25''·48}$$

$$\text{Spherical excess} = \frac{\triangle \times 648000}{\pi R^2} \text{ seconds}; \quad R = 6370291 \text{ m}.$$

$$= \frac{\frac{1}{2}(56349·7)^2 \sin 63°\,46'\,20''·61 \sin 57°\,39'\,13''·19 \times 648000}{\sin 58°\,34'\,25''·48 \times \pi\,(6370291)^2}$$

= 7·167 seconds

(iii) Correcting the observed angles :—

Theoretic sum of the angles of the △ ABC = 180° + 7″·167.
= 180° 0′ 7″·167.

Actual sum " " " = 180° 0′ 6″·12.

∴ Summation error = − 1·055 seconds.

Distributing the error among the angles inversely as the corresponding weights, i.e. as : 1 : $\frac{1}{3}$: $\frac{1}{2}$ or as 6 : 2 : 3.

$$\text{Correction to A} = \frac{6}{11}\,(1·055) = +\,0·575 \text{ seconds.}$$

$$\text{" to B} = \frac{2}{11}\,(1·055) = +\,0·192 \quad \text{"}$$

$$\text{" to C} = \frac{3}{11}\,(1·055) = +\,0·288 \quad \text{"}$$

$$\text{sum} = +\,1·055 \quad \text{"}$$

Hence the corrected spherical angles are

$$A_1 = 58°\,34'\;27''\cdot52 + 0''\cdot571 = 58°\,34'\;28''\cdot091$$
$$B_1 = 63°\,46'\;22''\cdot65 + 0\cdot190 = 63°\,46'\;22''\cdot840$$
$$C_1 = 57°\,39'\;15''\cdot95 + 0\cdot286 = 57°\,39'\;16''\cdot236$$
$$\text{sum} = 180°\,00'\;7''\cdot167$$

Subtracting $\frac{1}{3}$ spherical excess from each angle, the corrected plane angles are

$$A'_o = 58°\;34\;28''\cdot091 - 2''\cdot389 = 58°\,34'\,25''\cdot702$$
$$B'_o = 63°\;46'\,22''\cdot840 - 2\cdot389 = 63°\,46'\,20''\cdot451$$
$$C'_c = 57°\;39'\,16''\cdot236 - 2\cdot389 = 57°\,39'\,13''\cdot847$$
$$\text{sum} = 180°00\;00''\cdot000$$

(vi) Computation of sides:—

(a) By Legendre's Method:—Knowing the corrected plane angles A_o', B_o', and C_o' and the side BC (a), the remaining sides AB and AC are calculated by the sine rule.

$$\therefore \qquad AC = b = a\frac{\sin B_o}{\sin A_o'} = 56349\cdot7\;\frac{\sin 63°\,46'\,20''\cdot45}{\sin 58°\,34'\,25''\cdot7}$$

or $\log b = 4\;7725999$ $\qquad\therefore\; b = 59237\;90$ m.

$$AB = c = a\;\frac{\sin C'_o}{\sin A'_o} = 56349\;7\;\frac{\sin 57°\,39'\,13''\;85}{\sin 58°\,34'\,25''\cdot7}$$

or $\log c = 4\cdot7465506$ $\qquad\therefore\; c = 55789\cdot70$ m.

(b) By Delambre's Method:—Given the mean value of one minute of arc $= 1853\cdot79$ m. Let $\propto_a$, $\propto_b$, and $\propto_c$ = the central angles in minutes subtended by a, b, and c respectively.

Then $\quad \propto_a = \dfrac{184875\;5}{6076\cdot2} = 30'\;23''\cdot82.$

Now chord $a = 2R\sin\frac{1}{2}\propto_a = 2\times6370291\sin\frac{1}{2}(30'23''\cdot82) = K$
$\log$ ch. $a = 4\cdot7505688.$

Using the corrected plane angles and chord a, and applying the sine rule, we have

$$\text{ch. } b = \frac{\text{ch } a \sin B'_{\circ}}{\sin A'_{\circ}} = \frac{K \sin 63° 46' 20''\cdot 45}{\sin 58° 34' 25''\cdot 7};$$

log chord $b = 4\cdot7722773$.

$$\text{ch. } c = \frac{\text{ch } a \sin C'_{\circ}}{\sin A_{\circ}'} = \frac{K \sin 57° 39' 13''\cdot 85}{\sin 58° 34' 25''\cdot 7},$$

log chord $c = 4\cdot7462306$)

Now $\sin \frac{1}{2} \alpha_b = \frac{\text{ch } b}{2R} = \frac{\text{ch } b}{2 \times 20889000}$;

$$\log \sin \tfrac{1}{2} \alpha_b = \bar{3}\ 6670881.$$

$$\therefore \quad \frac{\alpha_b}{2} = 15' 58''\cdot 37 \quad \text{or} \quad \alpha_b = 31' 54''\cdot 06.$$

$\sin \frac{1}{2} \alpha_c = \frac{\text{ch } c}{2R} = \frac{\text{ch } c}{2 \times 20889000}$; $\log \sin \frac{1}{2} \alpha_c = \bar{3}\cdot6410414$

$$\therefore \quad \frac{\alpha_c}{2} = 15' 2'' 62 \text{ or } \alpha_c = 30' 5''\cdot 24.$$

Hence $b = (31' 54'' 06)\ 1853\ 79$, $\log b = 5\ 2885877$

$\therefore \quad b = 59221\ 18$ m

$c = (30' 5'' 24)\ 1853\ 79$,

$\therefore \quad c = 55775\ 54$ m

(*c*) By Spherical Trigonometry —Using the corrected spherical angles, and the angular value (α_a) of the side BC (*a*), and applying the sin rule we get

$$\sin \alpha_b = \frac{\sin \alpha \sin B_1}{\sin A_1} = \frac{\sin 30\ 25'' 36 \sin 63° 46' 22''\cdot 84}{\sin 58° 34' 28'' 1}$$

or $\log \sin \alpha_b = \bar{3}\ 9682008$

$$\alpha_b = 31' 57'' 10$$

$$\sin \alpha_c = \frac{\sin \alpha_a \sin C_1}{\sin A_1} = \frac{\sin 30' 25''\cdot 36 \sin 57° 39' 16''\cdot 24}{\sin 58° 34\ 28''\cdot 1}$$

or $\log \sin \alpha_c = \bar{3}\ 9421568$ $\qquad \alpha_c = 30' 5''\cdot 54$

Hence $\quad b = \alpha_b\ (6076\ 9) = (31' 57''\cdot 10)\ 1853\cdot 79$,

$\quad b = 59232\cdot30$ m

Similarly $\quad c = (30' 5'' 54)\ 1853\cdot79$

$\therefore \quad = 55784.25$ m

Adjustment of a Chain of Triangles

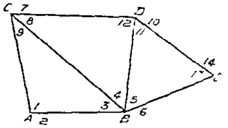


Fig 173

In Fig. 173, let ABC, BCD, BDE be the triangles. The angles indicated at the stations A, B, C, D, and E are measured with equal precision. The adjustment is made in two steps, viz. (1) *Station adjustment* and (2) *Figure adjustment*.

(1) **Station Adjustment** —Adjust the angles around each of the stations A, B, C, D, and E so as to fulfil the condition that their sum must be equal to 360°. Thus we have

$1 + 2 = 360°$; $3 + 4 + 5 + 6 = 360°$; $7 + 8 + 9 = 360°$;
$10 + 11 + 12 = 360°$, $13 + 14 = 360°$

The discrepancy should be equally distributed among all the angles at the station.

(2) **Figure Adjustment** —Using these adjusted values adjust the three angles in each triangle so that their sum equals 180°. Then we have

In the $\triangle$ ABC,		$1 + 3 + 9 = 180°$
„	BCD,	$4 + 8 + 12 = 180°$
„	BDE,	$5 + 11 + 13 = 180°$.

Since the angles are of equal weight, the correction is equal to one-third of the discrepancy and should be applied to each of the three angles of the triangle. If the angles are weighted, the corrections are applied inversely as the respective weights in both the adjustments.

Adjustment of Two Connected Triangles

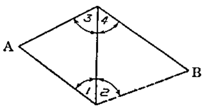


Fig. 174

Referring to Fig. 174, the triangles ACD and BCD are connected by the common side CD The eight angles A, C, C_3, C_4, B, D, D_1, and D_2 are measured. There are four independent conditional equations that the adjusted values of the angles must satisfy These conditional equations are called the *angle equations.*

Angle Equations —

(1) The sum of the angles in the triangle ACD must equal 180°.

(2) „ „ in the triangle BCD „ „ „

(3) The summation angle at C must be equal to the sum of its component parts

(4) The summation angle at D must be equal to the sum of its component parts

Therefore, the conditional equations are

$$A + C_3 + D_1 = 180° \quad (1); \quad B + C_4 + D_2 = 180° \; \ldots \; \ldots \; (2)$$

$$D = D_1 + D_2 \quad (3); \qquad C = C_3 + C_4 \; \ldots \; (4)$$

Out of the eight unknowns, D_1, D_2, C_3 and C_4 should be regarded as the independent unknowns, and the remaining four A, B, C, and D as the dependent ones, since they can be easily obtained from the conditional equations. From the conditional equations 1 and 2, the dependent unknowns A and B should be expressed in terms of the independent unknowns. Thus we have

$A = 180° - (C_3 + D_1)$;	$B = 180° - (C_4 + D_2)$.
$D = D_1 + D_2$,	$C = C_3 + C_4$.

Now substitute these values of A, B, C, and D in the given observation equations, and after reducing, obtain the new observation equations. Forming the normal equations from these new observation equations and solving them as simultaneous, we get the most probable values of D_1, D_2, C_3, and C_4. On substituting these values in the conditional equations, the most probable values of A, B, C, and D are obtained. The method is illustrated in the following example. The problem may also be solved by the method of correlatives.

Example :—The following are the measured values of equal weight :

$A = 70° \; 10' \; 24''.6$	$C_3 = 65° \; 40' \; 22''.4$
$B = 48 \; 20 \; 28 \; .2$	$C_4 = 74 \; 31 \; 43 \; .2$
$C = 140 \; 12 \; 4 \; .2$	$D_1 = 44 \; 9 \; 11 \; .5$
$D = 101 \; 17 \; 6 \; .4$	$D_2 = 57 \; 7 \; 51 \; .4$

Adjust the values of the angles.

The equations of condition are

$$C = C_3 + C_4 ; \qquad D = D_1 + D_2 .$$

$$A + C_3 + D_1 = 180° ; \qquad B + C_4 + D_2 = 180° .$$

Regarding A, B, C, and D as the dependent unknowns, expressing them in terms of the independent unknowns D_1, D_2, C_3, and C_4, and substituting their values in the observation equations, we have

$C_3 = 65° \; 40' \; 22''.4$	$C_3 + C_4 = 140° \; 12' \; 4''.2$
$C_4 = 74 \; 31 \; 43 \; .2$	$D_1 - D_2 = 101 \; 17 \; 6 \; .4$
$D_1 = 44 \; 9 \; 11 \; .5$	$C_3 + D_1 = 109 \; 49 \; 35 \; .4$
$D_2 = 57 \; 51 \; 51 \; .4$	$C_4 + D_2 = 131 \; 39 \; 31 \; .8$

Now let c_1, c_2, c_3, and c_4 be the corrections in seconds to the angles C_3, C_4, D_1, and D_2 respectively.

Then, the most probable value of $C_3 = 65° \; 40' \; 22''.4 + c_1$

" " of $C_4 = 74 \; 31 \; 43 \; .2 + c_2$

" " of $D_1 = 44 \; 9 \; 11 \; .5 + c_3$

" " of $D_2 = 57 \; 7 \; 51 \; 4 + c_4$

By substitution in the above equations, the reduced observation equations are

$$
\begin{array}{llll|l}
c_1 & & & = 0 & c_1+c_2 = -1{\cdot}4 \\
 & c_2 & & = 0 & c_3+c_4 = +3{\cdot}5 \\
 & & c_3 & = 0 & c_1 + c_3 = +1{\cdot}5 \\
 & & & c_4 = 0 & c_2 + c_4 = -2{\cdot}8
\end{array}
$$

Following the rule for the normal equations, we get

$$
\begin{array}{ll}
3c_1 + c_2 + c_3 = +0{\cdot}1 & \text{normal equation for } c_1 \\
c_1 + 3c_2 + c_4 = -4{\cdot}2 & \quad ,, \quad ,, \quad ,, \; c_2 \\
c_1 + 3c_3 + c_4 = +5{\cdot}0 & \quad ,, \quad ,, \quad ,, \; c_3 \\
c_2 + c_3 + 2c_4 = -0{\cdot}7 & \quad ,, \quad ,, \quad ,, \; c_4
\end{array}
$$

Solving these normal equations, we have

$$
c_1 = +0''{\cdot}0156 \qquad c_3 = +1''{\cdot}5900
$$
$$
c_2 = -1{\cdot}4770 \qquad c_4 = +0{\cdot}2156
$$

Whence, the adjusted values of the angle are

$$
\begin{array}{ll}
C_3 = 65\ 40\ 22''{\cdot}4 + 0''{\cdot}02 & = 65^\circ\ 40'\ 22''{\cdot}42 \\
C_4 = 74\ 31\ 43{\cdot}2 - 1{\cdot}48 & = 74\ 31\ 41{\cdot}72 \\
D_1 = 44\ 9\ 11{\cdot}5 + 1{\cdot}59 & = 44\ 9\ 13{\cdot}09 \\
D_2 = 57\ 7\ 51{\cdot}4 + 0{\cdot}22 & = 57\ 7\ 51{\cdot}62 \\
\hline
A = 180^\circ - (C_3+D_1) & = 70^\circ\ 10'\ 24''{\cdot}49 \\
B = 180^\circ - (C_4+D_2) & = 48\ 20\ 26{\cdot}66 \\
C = C_3 + C_4 & = 140\ 12\ 4{\cdot}14 \\
D = D_1 + D_2 & = 101\ 17\ 4{\cdot}71 \\
\textit{Check} - & \text{sum} = 360\ 00\ 00{\cdot}00
\end{array}
$$

Adjustment of a Triangle with a Central Station

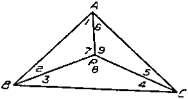


Fig 175

Referring to Fig 175, let ABC be the triangle and P the central Station Let the angles measured at the stations A, B, C, and P be designated by 1, 2, etc The angles 7, 8, and 9 are the central angles The angles 1, 3, and 5 to the left of an observer who traverses the boundary of the figure, always facing the central station P are called the left-hand angles and those (2, 4, 6) to his right, the right-hand angles

(Alternatively the angles are designated as left-hand (L. H.) and right-hand (R. H.) angles according as they appear to the left or right of an observer who faces the central station)

Let c_1, c_2, etc. = the corrections to the angles 1, 2, etc.
d_1, d_2, etc. = the tabular differences for 1" for log sin 1, log sin 2, etc.

(obtained from seven-figure logarithmic tables).

The equations of condition that must be fulfilled by the measured angles are

(i) The sum of all the angles around the common vertex P must be equal to 360°. This condition is called the *Apex condition*

(ii) The sum of the angles of each triangle must equal 180°. This condition is called the *Triangle condition*

(iii) The condition that the three lines AP, BP, and CP shall meet at a point P introduces another conditional equation that must be satisfied by the angles (1, 2, 3, 4, 5, and 6) measured at the stations A, B, and C

This conditional equation is derived as follows

In the $\triangle$ ABP, $AP = \dfrac{BP \sin 2}{\sin 1}$ Now calculating its length through the triangles ACP and BCP, we get

$AP = CP \dfrac{\sin 5}{\sin 6}$; $CP = \dfrac{BP \sin 3}{\sin 4}$ $\qquad AP = BP \dfrac{\sin 3 \sin 5}{\sin 4 \sin 6}$

Equating these two expressions, we have

$$BP \frac{\sin 2}{\sin 1} = BP \frac{\sin 3 \sin 5}{\sin 4 \sin 6}$$

i e $\sin 1 \sin 3 \sin 5 = \sin 2 \sin 4 \sin 6$

This equation is called a *side equation*, since it expresses the necessary relation between the three lines or sides meeting at P.

Therefore, the condition may be stated as follows

The product of the sines of the left-hand angles must be equal to the product of the sines of the right-hand angles

More usually, it is expressed as

The sum of the log sines of the left-hand angles must be equal to the sum of the log sines of the right hand angles.

This is called the *Log sine condition*

$$\Sigma \log \sin (\text{L H angle}) = \Sigma \log \sin (\text{R H angle})$$

Now by the Apex condition, $c_7 + c_8 + c_9 = \pm k_1$ (1)

By the Triangle condition $c_1 + c_2 + c_7 = \pm k_2$ (2)

$c_3 + c_4 + c_8 = \pm k_3$ (3)

$c_5 + c_6 + c_9 = \pm k_4$ (4)

By the Log sine condition,

$$d_1c_1 - d_2c_2 + d_3c_3 - d_4c_4 + d_5c_5 - d_6c_6 = \pm M \quad (5)$$

where M is in units of the seventh decimal place of logarithms

By the Least square condition,

$$c_1^2 + c_2^2 + c_3^2 + c_4^2 + c_5^2 + c_6^2 + c_7^2 + c_8^2 + c_9^2 = \text{a minimum} \quad (6)$$

The most probable values of the corrections c_1, c_2 etc may be found by the method of *Correlates* (or *Correlatives*) by the use of undetermined multipliers (λ_1 λ_2 etc), known as "correlates or correlatives which is illustrated in the following examples The most probable values of the angles are then determined by applying the corrections to their measured values

Example 1 —To locate a secondary station O in the triangle PQR, the following angles were measured (Fig 176)

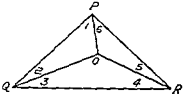


Fig 176

L H angle	R H angle
1 = 40° 30 36"	2 = 41° 21′ 30"
3 = 29° 48 24"	4 = 27° 31′ 48"
5 = 19° 34′ 4"	6 = 20° 53′ 48"

Find the most probable values of the angles

The conditional equations which must be satisfied in this case are

(1) Angle Equation :—$1+2+3+4+5+6=180^\circ$.

(2) Side Equation :—

Σ (log sin L H angle) $=\Sigma$ (log sin R H. angle)

Let c_1, c_2, c_3, etc be the corrections to the measured angles 1, 2, 3, etc.

Now the sum of the observed angles $=180^\circ\ 0'\ 10''$ The error is therefore, equal to $+10''$ and the total correction is $-10''$.

$\therefore$ By condition (1), $c_1+c_2+c_3+c_4+c_5+c_6=-10''$ (1)

Now the tabular differences for one second for log sin 1, log sin 2, etc. may be obtained from the seven figure log tables.

Then by condition (2),

$$24\cdot 4c_1-23\cdot 9c_2+36\cdot 8c_3-40\cdot 4c_4+59\cdot 2c_5-50\cdot 0c_6=+2350 \quad (2)$$

By the theory of least squares, we have

$$c_1^2+c_2^2+c_3^2+c_4^2+c_5^2+c_6^2=\text{a minimum} \quad (3)$$

Differentiating the equations 1 to 3, we get

$$\delta c_1+\delta c_2+ \qquad +\delta c_6=0 \quad (4)$$

$$24\cdot 4\delta c_1-23\cdot 9\delta c_2+ \qquad \text{etc.}=0 \quad (5)$$

$$c_1\delta c_1+c_2\delta c_2+ \qquad \text{etc}=0 \quad (6)$$

Multiplying the equations 4 and 5 by $-\lambda_1$ and $-\lambda_2$ respectively, and adding them to equation 6, we have

$$(c_1-\lambda_1-24\cdot 4\lambda_2)\,\delta c_1+(c_2-\lambda_1+23\cdot 9\lambda_2)\,\delta c_2$$
$$+(c_3-\lambda_1-36\cdot 8\lambda_2)\,\delta c_3\times(c_4-\lambda_1+40\cdot 4\lambda_2)\,\delta c_4$$
$$+(c_5-\lambda_1-59\cdot 2\lambda_2)\,\delta c_5+(c_6-\lambda_1+50\cdot 0\lambda_2)\,\delta c_6=0$$

Equating the coefficients of δc_1, δc_2, etc to zero, we get

$$c_1=\lambda_1+24\cdot 4\lambda_2;\qquad c_4=\lambda_1-40\cdot 4\lambda_2;$$
$$c_2=\lambda_1-23\cdot 9\lambda_2,\qquad c_5=\lambda_1+59\cdot 2\lambda_2,$$
$$c_3=\lambda_1+36\cdot 8\lambda_2;\qquad c_6=\lambda_1-50\cdot 0\lambda_2$$

By substituting these values in the original equations 1 and 2, we have

$$6\lambda_1+6\cdot 1\lambda_2 \qquad =-\ 10 \quad \ldots \quad \ldots \quad (7)$$

$$6\cdot 1\lambda_1+10157\cdot 61\lambda_2 \qquad =+2350 \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

$\therefore$ $\lambda_1=-1\cdot 903$ and $\lambda_2=+0\cdot 2325$.

By substitution of the values of λ_1 and λ_2 in c_1, c_2, etc, we get

$c_1 = -1 \cdot 903 + 24 \cdot 4\ (\cdot 2325) = +\ 3 \cdot 769 = +\ 3 \cdot 77$ secs
$c_2 = -1\ 903 - 23\ 9\ (\cdot 2325) = -\ 7\ 459 = -\ 7 \cdot 46$ „
$c_3 = -1\ 903 + 36\ 8\ (\cdot 2325) = +\ 6 \cdot 652 = +\ 6 \cdot 65$ „
$c_4 = -1 \cdot 903 - 40\ 4\ (\cdot 2325) = -11 \cdot 296 = -11 \cdot 30$ „
$c_5 = -1 \cdot 903 + 59\ 2\ (\cdot 2325) = +11 \cdot 857 = +11 \cdot 86$ „
$c_6 = -1 \cdot 903 - 50\ 0\ (\cdot 2325) = -13 \cdot 523 = -13 \cdot 52$ „

Check —sum $= -10$ secs $= -10$ secs.

Whence, the most probable values of the angles are

Angle	Observed value			Correction	Adjusted value		
	°	′	″	″	°	′	″
1	40	50	36	+ 3 77	40	50	39 77
2	41	21	30	− 7·46	41	21	22 54
3	29	48	24	+ 6 65	29	48	30 65
4	27	31	48	−11 30	27	31	36 70
5	19	34	4	+11 86	19	34	15 86
6	20	53	48	−13 52	20	53	34·48
Check —				sum =	180	00	00 00

Example 2 —To locate a secondary station O in the triangle PQR, the following angles were observed (Fig 177)

L H angle.	R. H angle	Central angle.
2 = 33° 2′ 9″	1 = 32° 15′ 30″	7 = 114° 42′ 15″
4 = 27° 0 13″	3 = 32° 27′ 30″	8 = 120° 32′ 20″
6 = 29° 48′ 13″	5 = 25° 26′ 24″	9 = 124° 45′ 27″

Determine the most probable values of the angles

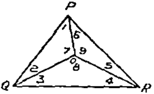


Fig 177

Here the angles are designated according as they appear to the left or right, if we face the central station

Let c_1, c_2, etc be the corrections to the observed values of the angles 1, 2, etc

In the △ OPQ, $1 + 2 + 7 = 179°\ 59'\ 54''$
„ „ OQR, $3 + 4 + 8 = 180°\ 0'\ 3''$
„ „ ORP $5 + 6 + 9 = 180°\ 0'\ 4''$

The sum of the central angles, $7 + 8 + 9 = 360°\ 0'\ 2''$

$\therefore$ Error $= -6''$ and total correction $= +6''$.
,, $= +3''$,, ,, ,, $= -3''$.
,, $= +4''$,, ,, ,, $= -4''$.
,, $= +2''$,, ,, ,, $= -2''$.

The equations of condition which must be satisfied in this case, are

(1) Angle equations :—

(a) The sum of the angles of each of the triangles OPQ, OQR, and ORP must equal 180°

(b) The sum of the central angles must equal 360°.

(2) Side equation —

Σ (log sin L. H. angle) $=$ Σ (log sin R. H angle).

Now Σ(log sin L H. angle) $= 1\ 0900086$,
Σ(log sin R. H. angle) $= 1\cdot0900776$

$\therefore$ The difference $= -690$ and the correction $= +690$.

By condition (1a), $c_1 + c_2 + c_7 = +6''$. (1)
$c_3 + c_4 + c_8 = -3''$ (2)
$c_5 + c_6 + c_9 = -4''$. (3)
By condition (1b) $c_7 + c_8 + c_9 = -2''$ ... ... (4)

By condition (2),

$-33\cdot4c_1 + 32\cdot4c_2 - 33\ 1c_3 + 41\cdot3c_4 - 44\ 3c_5 + 36\cdot7c_6 = +690$.(5)

By the theory of least squares,

$c_1^2 + c_2^2 + c_3^2 + c_4^2 + c_5^2 + c_6^2 + c_7^2 + c_8^2 + c_9^2$
$=$ a minimum ..(6)

Differentiating the equations 1 to 6, we get

$\delta c_1 + \delta c_2 + \delta c_7 = 0$ .. . ... (1′)
$\delta c_3 + \delta c_4 + \delta c_8 = 0$ .. .. (2′)
$\delta c_5 + \delta c_6 + \delta c_9 = 0$ ... .. (3′)
$\delta c_7 + \delta c_8 + \delta c_9 = 0$ ... . (4′)

$-33\cdot4\delta c_1 + 32\ 4\delta c_2 - 33\cdot1\delta c_3 + 41\cdot3\delta c_4 - 44\ 3\delta c_5 + 36\cdot7\delta c_6$
$= 0$ .. (5′)

$c_1\delta c_1 + c_2\delta c_2 + c_3\delta c_3 + c_4\delta c_4 + c_5\delta c_5 + c_6\delta c_6 + c_7\delta c_7 + c_8\delta c_8$
$+ c_9\delta c_9 = 0$ .. (6′)

Multiplying the equations 1′ to 5′ by $-\lambda_1, -\lambda_2, -\lambda_3$, etc. respectively and adding the results to equation 6′, and then equating the coefficients of each of δc_1, δc_2, etc. to zero, we have

$$c_1 = \lambda_1 - 33.4\lambda_5 \qquad c_5 = \lambda_3 - 44.3\lambda_5$$
$$c_2 = \lambda_1 + 32.4\lambda_5 \qquad c_6 = \lambda_3 + 36.7\lambda_5$$
$$c_3 = \lambda_2 - 33.1\lambda_5 \qquad c_7 = \lambda_1 + \lambda_4$$
$$c_4 = \lambda_2 + 41.3\lambda_5 \qquad c_8 = \lambda_2 + \lambda_4$$
$$c_9 = \lambda_3 + \lambda_4$$

Substituting these values of c_1, c_2, etc in the original equations 1 to 4, we get

$$3\lambda_1 + \lambda_4 - \lambda_5 = +6'' \qquad (1')$$
$$3\lambda_2 + \lambda_4 + 8.2\lambda_5 = -3'' \qquad (2')$$
$$3\lambda_3 + \lambda_4 - 7.6\lambda_5 = -4'' \qquad (3')$$
$$3\lambda_4 + \lambda_1 + \lambda_2 + \lambda_3 = -2'' \qquad (4')$$

From equations 1" to 4" the values of λ_1, λ, λ_3, and λ_4 should be found in terms of λ_5 thus

Adding the equations 1" to 3", we get

$$3(\lambda_1 + \lambda_2 + \lambda_3) + 3\lambda_4 - 0.4\lambda_5 = -1$$

But from equation (4") $\lambda_1 + \lambda_2 + \lambda_3 = -2 - 3\lambda_4$.

$$3(-2-3\lambda_4) + 3\lambda_4 - 0.4\lambda_5 = -1 \text{ or } \lambda_4 = -\left(\frac{5}{6} + \frac{0.4}{6}\lambda_5\right)$$

Substituting the value of λ_4 in equations 1" to 3", we get

$$\lambda_1 = +\left(\frac{41}{18} + \frac{6.4}{18}\lambda_5\right) \quad \lambda_2 = -\left(\frac{13}{18} + \frac{48.8}{18}\lambda_5\right),$$
$$\lambda_3 = -\left(-\frac{19}{18} + \frac{46}{18}\lambda_5\right).$$

Now inserting the values of λ_1 λ_2, λ_3 and λ_4 in c_1, c_2 etc , we have

$$c_1 = +2.28 - 33.04\lambda_5 \qquad c_5 = -1.06 - 41.74\lambda_5$$
$$c_2 = +2.28 + 32.76\lambda_5 \qquad c_6 = -1.06 + 39.26\lambda_5$$
$$c_3 = -0.72 - 35.8\lambda_5 \qquad c_7 = +1.44 + 0.29\lambda_5$$
$$c_4 = -0.72 + 38.6\lambda_5 \qquad c_8 = -1.56 - 2.78\lambda_5$$
$$c_9 = -1.89 + 2.49\lambda_5$$

Substituting these values of c_1, c_2 etc in equation 5, we have $-141.71 + 8234\lambda_5 = +690$ or $\lambda_5 = +0.101$

Knowing the value of λ_5, the values of c_1, c_2 etc may be found

$$c_1 = -1.055 \qquad c_4 = +3.178 \qquad c_7 = +1.470$$
$$c_2 = +5.589 \qquad c_5 = -5.275 \qquad c_8 = -1.841$$
$$c_3 = -4.336 \qquad c_6 = +2.906 \qquad c_9 = -1.638$$

Whence the adjusted values are

Angle	Observed value ° ′ ″	Correction ″	Adjusted value ° ′ ″
1	32 15 30	− 1 055	32 15 28 94
2	33 2 9	+ 5 589	33 2 14 59
3	32 27 30	− 4 336	32 27 25 66
4	27 0 13	+ 3 178	27 0 16 18
5	25 26 24	− 5 275	25 26 18 73
6	29 48 13	+ 2 906	29 48 15 91
7	114 42 15	+ 1 470	114 42 16 47
8	120 32 20	− 1 841	120 32 18 16
9	124 45 27	− 1 638	124 45 25 36

Check —1 + 2 + 7 = 180°, 5 + 6 + 9 = 180

3 + 4 + 8 = 180°, 7 + 8 + 9 = 359° 59′ 59″ 99

1 + 2 + 3 + 4 + 5 + 6 = 180° 0′ 0″ 01

Adjustment of a Geodetic Quadrilateral
(Quadrilateral with Interlacing Diagonals)

In the geodetic quadrilateral, observations are made along both the diagonals and all the eight angles are measured If the quadrilateral is large it is necessary to calculate the spherical excess for the whole figure In minor work, the plane angles are derived by deducting one eighth of the spherical excess from each of the eight measured angles Fig 178 represents a plane quadrilateral ABCD in which 1, 3 5, and 7 are the Left hand angles and 2, 4 6, and 8 the Right hand ones

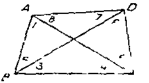


Fig 178

The conditions that must be fulfilled by the adjusted values of the angles are

Angle equations —(1) The sum of the eight angles of the quadrilateral must be exactly equal to 360°

or $$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 360° \quad (1)$$

$$1 + 2 = 5 + 6 \quad (2)$$

$$3 + 4 = 7 + 8 \quad (3)$$

Side equation —$\Sigma \log \sin (1,3,5,7) = \Sigma \log \sin (2,4,6\ 8)$ (4)

The side equation is derived as follows

The length of any side, say, AB may be calculated in two ways first through the Δs ABC and BCD, and secondly, through the Δs ABD and ACD

Then we have

(1) $AB = BC\frac{\sin 4}{\sin 1}$ and $BC = CD\frac{\sin 6}{\sin 3}$ $\quad AB = CD\frac{\sin 4 \sin 6}{\sin 1 \sin 3}$

(2) $AB = AD\frac{\sin 7}{\sin 2}$ and $AD = CD\frac{\sin 5}{\sin 8}$ $\quad AB = CD\frac{\sin 5 \sin 7}{\sin 2 \sin 8}$

Equating the two values of AB we get

$\sin 1 \sin 3 \sin 5 \sin 7 = \sin 2 \sin 5 \sin 6 \sin 8$

It may be written in the logarithmic form as

$$(\log\sin 1 + \log\sin 3 + \log\sin 5 + \log\sin 7)$$
$$= (\log\sin 2 + \log\sin 4 + \log\sin 6 + \log\sin 8)$$

Let c_1 c_2 c_3 etc be the corrections in seconds to the angles 1, 2 3, etc , d_1 d_2 d_3 etc the tabular differences for one second for log sin 1, log sin 2 log sin 3 etc

Then

By condition (1) $c_1+c_2+c_3+c_4+c_5+c_6+c_7+c_8 = \pm k_1$ (1)

By condition (2) $c_1+c_2-c_5-c_6 = \pm k_2$ (2)

By condition (3) $c_3+c_4-c_7-c_8 = \pm k_3$ (3)

By condition (4) $d_1c_1-d_2c_2+d_3c_3 - d_4c_4+d_5c_5 - d_6c_6+d_7c_7-d_8c_8 = \pm M$ (4)

By the theory of least squares

$c_1^2+c_2^2+c_3^2+c_4^2+c_5^2+c_6^2+c_7^2+c_8^2$ = a minimum (5)

The values of the corrections may be determined by the method of Correlates as follows

Angles of Equal Weight —

Differentiating the equations 1 to 5 we get

$\delta c_1+\delta c_2+\delta c_3+\delta c_4+\delta c_5+\delta c_6+\delta c_7+\delta c_8 = 0$ (1)

$\delta c_1+\delta c_2-\delta c_5-\delta c_6 = 0$ (2)

$\delta c_3+\delta c_4-\delta c_7-\delta c_8 = 0$ (3)

$d_1\delta c_1-d_2\delta c_2+d_3\delta c_3-d_4\delta c_4+d_5\delta c_5-d_6\delta c_6 +d_7\delta c_7-d_8\delta c_8 = 0$ (4)

$c_1\delta c_1+c_2\delta c_2+c_3\delta c_3+c_4\delta c_4+c_5\delta c_5 +c_6\delta c_6+c_7\delta c_7+c_8\delta c_8 = 0$ (5)

Multiplying the equations 1, 2, 3, and 4 by $-\lambda_1, -\lambda_2, -\lambda_3$, and $-\lambda_4$ respectively, adding the results to the equation (5) and equating the coefficients of each δc to zero, we get

$c_1 = \lambda_1 + \lambda_2 + d_1\lambda_4$, $\quad c_5 = \lambda_1 - \lambda_2 + d_5\lambda_4$

$c_2 = \lambda_1 + \lambda_2 - d_2\lambda_4$, $\quad c_6 = \lambda_1 - \lambda_2 - d_6\lambda_4$

$c_3 = \lambda_1 + \lambda_3 + d_3\lambda_4$, $\quad c_7 = \lambda_1 - \lambda_3 + d_7\lambda_4$

$c_4 = \lambda_1 + \lambda_3 - d_4\lambda_4$, $\quad c_8 = \lambda_1 - \lambda_3 - d_8\lambda_4$

TABLE No 1—Quadrilateral Adjustment by Method of Least Squares

	Measured angles	Weight	Log sin a c, e g	Log sin b d, f h	d	d^2	Corrections		Adjusted Angles	Check Log sines
a	45° 23 26" 2	1	9 852426		2 08	4 33	$\lambda_1+\lambda_2+d_a\lambda_4$	−0" 50	45° 23 25" 70	9 85 24 25
b	54 21 24 2	1		9 909910	1 51	2 28	$\lambda_1+\lambda_2-d_b\lambda_4$	+1 02	54 21 26 12	9 90 99 13
c	41 17 14 3	1	9 819435		2 10	5 76	$\lambda_1+\lambda_3+d_c\lambda_4$	+0 06	41 17 14 36	9 81 94 35
d	38 57 50 4	1		9 798534	2 60	6 76	$\lambda_1+\lambda_3-d_d\lambda_4$	+3 42	38 57 53 82	9 79 85 43
e	59 27 12 2	1	9 935112		1 24	1 54	$\lambda_1-\lambda_2+d_e\lambda_4$	+0 46	59 27 12 66	9 93 51 11
f	40 17 36 2	1		9 810704	2 48	6 15	$\lambda_1-\lambda_2-d_f\lambda_4$	+2 97	40 17 39 17	9 81 07 11
g	35 10 22 8	1	9 761531		2 98	8 88	$\lambda_1-\lambda_3+d_g\lambda_4$	1 18	35 10 20 82	9 76 15 27
h	44 58 45 4	1		9 849328	2 11	4 45	$\lambda_1-\lambda_3-d_h\lambda_4$	+1 95	44 58 47 35	9 84 93 32
	359° 59 51" 2		39 368504 Difference = +28	39 368476	Σd^2 = 40 15				360° 00 00" 00	Difference = 1

359° 59 51" 2
360 0 0

$e_1 = -8\ 8$

$a+b = 99°\ 44\ 50"\ 4$

$e+f = 99\ 44\ 48\ \ 4$

$e_2 = +2"$

$c+d = 80°\ 15'\ 4"\ 7$

$g+h = 80\ 15\ \ 7\ \ 7$

$e_3 = -3$

$(d_a + d_c + d_e + d_g) = 8\ 70$

$(d_b + d_d + d_f + d_h) = 8\ 70$

$(d_a - d_b - d_e + d_f) = 1\ 81$

$(d_c - d_d - d_g + d_h) = -1\ 07$

$m = +28$

$$8\lambda_1 + 0\lambda_4 = +8\ 8$$
$$4\lambda_2 + 1\ 81\lambda_4 = -2$$
$$4\lambda_3 - 1\ 07\lambda_4 = +3$$
$$0\lambda_1 + 1\ 81\lambda_2 - 1\ 07\lambda_3 + 40\ 15\lambda_4 = -28$$

$\lambda_1 = +1\ 1$, $\lambda_2 = -\ 1953$;
$\lambda_3 = +\ 5690$, $\lambda_4 = -\ 6735$

Note —a, c, e, and g are left hand angles, b, d, f, and h are right hand angles

Substituting the values of c_1 c_2 etc in the original equations 1 to 4 we have

$$
\begin{aligned}
&8\lambda_1+(d_1-d_2+d_3-d_4+d_5-d_6+d_7-d_8)\lambda_4 && = \pm\, l_1 && (1')\\
&4\lambda_2+\{(d_1-d_2)-(d_5-d_6)\}\lambda_4 && = \pm\, l_2 && (2')\\
&4\lambda_3+\{(d_3-d_4)-(d_7-d_8)\}\lambda_4 && = \pm\, l_3 && (3')\\
&\{(d_1-d_2)+(d_3-d_4)+(d_5-d_6)+(d_7-d_8)\}\lambda_1 && && \\
&+\{(d_1-d_2)-(d_5-d_6)\}\lambda_2+\{(d_3-d_4)-(d_7-d_8)\}\lambda_3 && && \\
&+(d_1^2+d_2^2+d_3^2+d_4^2+d_5^2+d_6^2+d_7^2+d_8^2)\lambda_4 && = \pm\, M && (4')
\end{aligned}
$$

The solution of these normal equations gives the values of λ_1 λ_2 λ_3 and λ_4 from which the values of the corrections c_1 c_2 etc may be obtained

The most probable values of the angles are then determined by applying the corrections to their measured values

The method is illustrated in Table No 1 in which the angles a b c etc denote the angles 1 2 3 etc of the quadrilateral ABCD (Fig 172)

Approximate Adjustment of a Geodetic Quadrilateral —

The following method which is sufficiently accurate and involves less labour may be used in adjusting a quadrilateral of moderate size or minor importance In this method it is assumed that the angles have been observed with equal care and reduced for spherical excess if necessary

Note —The angles 1 2 3 etc of the quadrilateral ABCD (Fig 172) are denoted by a b c etc respectively

The equations of condition to be satisfied are

Angle equations —

$a + b + c + d + e + f + g + h = 360°$

$a + b = e + f$

$c + d = g + h$

Side equation —

$(\log \sin a + \log \sin c + \log \sin e + \log \sin g)$
$\quad - (\log \sin b + \log \sin d + \log \sin f + \log \sin h) = 0$

The adjustment is made in four steps

(1) Make the station adjustment as follows

Adjust the angles around each point so as to make their sum equal to 360° by distributing the error equally among the several angles

(2) Using the values obtained by station adjustment, find the sum of the eight angles (a, b, c, etc) and subtract their sum from 360°. Correct each angle by one eighth of the discrepancy.

(3) From the values of the angles so obtained find the difference between the sums $(a + b)$ and $(e + f)$ Each of the four angles is then corrected by one-fourth the discrepancy. If $a + b$ is greater than $e + f$, the sign of the correction to a and b is minus, and that to e and f is plus, and vice versa Similarly, if $c + d$ is not equal to $g + h$, find the discrepancy and distribute it equally among the four angles, c, d, g, and h The sign of the correction to c and d is minus, and that to g and h is plus, if $c+d$ is greater than $g + h$, and vice versa

(4) The adjusted values of the angles are then tested to satisfy the side equation, by adding logarithmic sines of the angles in two groups as indicated in the side equation and finding the discrepancy between the two sums To reduce this discrepancy to zero, the following procedure may be adopted

Let a_1, b_1, c_1, etc. represent the measured angles as so far adjusted

(1) Record the log sines of the angles a_1, b_1 etc in each group

(2) Record the tabular differences (d) for 1" for log sin a_1, log sin b_1, etc

(3) The corrections to be applied to the several angles are

Correction to the angle $a_1 = \dfrac{d_a}{\Sigma d^2} m'$.

" " " $b_1 = \dfrac{d_b}{\Sigma d^2} m'$.

" etc. etc.

in which d_a, d_b, d_c. etc = the tabular differences for 1" for log sin a_1, log sin b_1, etc

Σd^2 = the sum of the squares of the tabular differences for 1" for log sines of the several angles

m' = the numerical value of the difference between (log sin a_1 + log sin c_1 + log sin e_1 + log sin g_1) and

TABLE No 2—Quadrilateral Adjustment by Approximate Method

	Measured Angles	Angle equation Adjustment for 360°	for opposite angles	Corrected Values	Log sin a_1 c_1 e_1 g_1	Log sin b_1 d_1 f_1 h_1	d	d^2	Side equation Adjustment	Corrected Angles
a	45°23′20″ 2	+1″ 10	0″ 70	a_1 45 23 26″ 80	9 852127		2 08	4 33	−1″ 40	45°23′25″ 40
b	54 21 24 2	+1 10	−0 50	b_1 54 21 24 80		9 900910	1 51	2 28	+1 02	54 21 25 82
c	41 17 14 3	+1 10	+0 75	c_1 41 17 16 15	9 819110		2 10	5 76	−1 61	41 17 14 ·54
d	38 57 50 4	+1 10	+0 75	d_1 38 57 52 25		9 798539	2 60	6 76	+1 75	38 57 54 00
e	59 27 12 2	+1 10	+0 50	e_1 59 27 13 80	9 935114		1 24	1 54	−0 84	59 27 12 ·96
f	40 17 36 2	+1 10	+0 50	f_1 40 17 37 80		9 810708	2 18	6 15	+1 67	40 17 39 ·47
g	35 16 22 3	+1 10	−0 75	g_1 35 16 22 65	9 761592		2 98	8 88	−2 01	35 16 20 64
h	44 58 45 4	+1 10	0 75	h_1 44 58 45 75		9 844329	2 11	4 45	+1 42	44 58 47 ·17
	359°59 51′ 2			360°00 00″ 00	39 368513 $m = +27$	39 368486	Σd =17 40	Σd^2 =40 15		360°00 00″ 00

Correction for 360° = 8 8/8 = 1″ 1 + ve

Correction for opposite angles $\left.\begin{aligned} a + b &= 99° \, 44' \, 50'' \, 4 \\ e + f &= 99 \;\; 44 \;\; 48 \; \cdot 4 \end{aligned}\right\}$ Difference = 2″, $\left.\begin{aligned} c + d &= 80° \, 15' \, 4'' \, 7 \\ g + h &= 80 \;\; 15 \;\; 7 \;\; 7 \end{aligned}\right\}$ Difference = 3″.

Correction for opposite angles = 2/4 = 0″ 50 Correction for opposite angles = ¾ = 0″ 75

Difference between the sums = m' = 27, Σd^2 = 40 15 $m/\Sigma d^2$ = 27/40 15 = 0 0726

Correction for side equation adjustment = 0 0726 × 2 08 = 1″ 40, 0 0726 × 1 51 = 1″ 02, etc

Corrections for side equation adjustment may be found more simply thus

Average change required (y) = 27/8, Average difference (x) for 1″ = 17 40/8 Correction to each angle = (y/x) = 27/17 40 = 1″·55

Note —a, c, e, and g are left hand angles, b, d, f, and h are right hand angles

$(\log \sin b_1 + \log \sin d_1 + \log \sin f_1 + \log \sin h_1)$

The signs of these corrections are determined as follows: If Σ log sin (L H angle) is greater than Σ log sin (R H angle), the corrections to the L H angles are minus and those to the R H angles are plus and vice versa

Due to the side equation adjustment the previously adjusted values of the angles may be disturbed slightly but seldom appreciably If necessary, both the adjustments should be repeated The method is illustrated in Table No 2

Adjustment of a Quadrilateral with a Central Station —

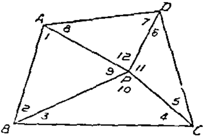


Fig 1 9

Fig 1"9 represents a quadrilateral ABCD with a central station P The central angles are denoted by 9 10 11 and 12 the Left-hand angles by 1, 3 5 and 7 and the Right hand angles by 2, 4 6, and 8

The conditions that must be fulfilled by the adjusted values of the angles are

Angle equations —

(1) $1 + 2 + 9 = 180°$, (2) $3 + 4 + 10 = 180°$;
(3) $5 + 6 + 11 = 180°$, (4) $7 + 8 - 12 = 180°$,
(5) $9 + 10 + 11 + 12 = 360°$

Side equation:

$$\Sigma \log \sin (1, 3\ 5, 7) = \Sigma \log \sin (2, 4\ 6, 8).$$

Proceeding exactly similarly as in the preceding case, we get the following equations from which the required values of the corrections may be determined by the method of Correlates.

as explained in the preceding case. The most probable values of the angles are then obtained by applying the corrections to their observed values

$$c_1 + c_2 + c_9 = \pm k_1 \quad \ldots \quad \ldots \quad (1)$$
$$c_3 + c_4 + c_{10} = \pm k_2 \quad \ldots \quad \ldots \quad (2)$$
$$c_5 + c_6 - c_{11} = \pm k_3 \quad \ldots \quad \ldots \quad (3)$$
$$c_7 + c_8 + c_{12} = \pm k_4 \quad \ldots \quad \ldots \quad (4)$$
$$c_9 + c_{10} + c_{11} + c_{12} = \pm k_5 \quad \ldots \quad \ldots \quad (5)$$
$$d_1c_1 - d_2c_2 + d_3c_3 - d_4c_4 - d_5c_5 - d_6c_6 + d_7c_7 - d_8c_8 = \pm M \quad \ldots \quad \ldots \quad (6)$$
$$\Sigma_1^{12} c^2 = \text{a minimum} \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

Example —ABCD is a quadrilateral with a central station P. The angles measured at A, B, C, and D, and P are as follows

L. H. angle	R. H. angle.	Central angle.
2 = 40° 18 6"	1 = 45° 36′ 10"	9 = 94° 5′ 48"
4 = 45 4 5	3 = 42 12 12	10 = 92 43 40
6 = 46 56 6	5 = 44 8 10	11 = 88 55 50
8 = 47 51 14	7 = 47 54 6	12 = 84 14 35

Determine the most probable values of the corrections,

(See Fig 179)

Note —The angles at A, B, C, and D are designated as L. H angles and R H angles according as they appear if we face the central station P

Let c_1, c_2, c_3, etc be the corrections to the measured values of the angles 1, 2, 3, etc.

In the $\triangle$PAB, 1 + 2 + 9 = 180° 0′ 4".

„ error = + 4" and the total correction = − 4".

„ PBC, 3 + 4 + 10 = 179° 59′ 57".

error = − 3" and the total correction = + 3".

„ PCD, 5 + 6 + 11 = 180° 0′ 6".

error = + 6" and the total correction = − 6".

„ PDA, 7 + 8 + 12 = 179° 59′ 55".

error = − 5" and the total correction = + 5".

Central angles : 9 + 10 + 11 + 12 = 359° 59′ 53".

. error = − 7" and the total correction = + 7".

The conditions which must be satisfied are

(1) Angle equations —

(a) The sum of the angles of each of the triangles PAB, PBC, PCD, and PDA must be equal to 180°

(b) The sum of the angles around the common vertex P must be equal to 360°

(2) Side equation —

Σ (log sin L H angle) = Σ log (sin R H angle)

Now Σ (log sin L H angle) = $\bar{1}$ 3945194

and Σ (log sin R H angle) = $\bar{1}$ 3944610

By condition (1a)

$$c_1 + c_2 + c_9 = -\ 4'' \qquad (1)$$
$$c_3 + c_4 + c_{10} = +\ 3'' \qquad (2)$$
$$c_5 + c_6 + c_{11} = -\ 6'' \qquad (3)$$
$$c_7 + c_8 + c_{12} = +\ 5'' \qquad (4)$$

" , (1b) $c_9 + c_{10} + c_{11} + c_{12} = +$ [illegible] (5)

By condition (2)

$$-20\ 6c_1 + 24\ 8c_2 - 23\ 2c_3 + 21\ 0c_4 - 21\ 7c_5 + 19\ 7c_6$$
$$-19\ 0c_7 + 19\ 1c_8 = -584 \qquad (6)$$

By the theory of least squares $\sum_1^{12} c^2 =$ a minimum (7)

Differentiating the equations 1 to 7, we get

$$\delta c_1 + \delta c_2 + \delta c_9 = 0 \qquad (1) \times -\lambda_1$$
$$\delta c_3 + \delta c_4 + \delta c_{10} = 0 \qquad (2) \times -\lambda_2$$
$$\delta c_5 + \delta c_6 + \delta c_{11} = 0 \qquad (3) \times -\lambda_3$$
$$\delta c_7 + \delta c_8 + \delta c_{12} = 0 \qquad (4) \times -\lambda_4$$
$$\delta c_9 + \delta c_{10} + \delta c_{11} + \delta c_{12} = 0 \qquad (5) \times -\lambda_5$$
$$-20\ 6\delta c_1 + 24\ 8\delta c_2 - 23\ 2\delta c_3 + \text{etc} = 0 \qquad (6) \times -\lambda_6$$
$$c_1\delta c_1 + c_2\delta c_2 + c_3\delta c_3 + \text{etc.} = 0 \qquad (7')$$

Multiplying the equations 1 to 6 by $-\lambda_1, -\lambda_2$ etc, and adding them to equation 7 and then equating the coefficients of each δc to 0, we have

$c_1 = \lambda_1 - 20.6\lambda_6$	$c_5 = \lambda_3 - 21.7\lambda_6$	$c_9 = \lambda_1 + \lambda_5$
$c_2 = \lambda_1 + 24.8\lambda_6$	$c_6 = \lambda_3 + 19.7\lambda_6$	$c_{10} = \lambda_2 + \lambda_5$
$c_3 = \lambda_2 - 23.2\lambda_6$	$c_7 = \lambda_4 - 19.0\lambda_6$	$c_{11} = \lambda_3 + \lambda_5$
$c_4 = \lambda_2 + 21.0\lambda_6$	$c_8 = \lambda_4 + 19.1\lambda_6$	$c_{12} = \lambda_4 + \lambda_5$

Inserting these values in the original equations 1 to 6, we get

$$3\lambda_1 + \lambda_5 + 4.2\lambda_6 = -4'' \qquad (1')$$

$$3\lambda_2 + \lambda_5 - 2.2\lambda_6 = +3'' \qquad (2')$$

$$3\lambda_3 + \lambda_5 - 2.0\lambda_6 = -6'' \qquad (3')$$

$$3\lambda_4 + \lambda_5 + 0.1\lambda_6 = +5'' \qquad (4')$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + 4\lambda_5 = +7'' \qquad (5')$$

$$-20.6(\lambda_1 - 20.6\lambda_6) + 24.8(\lambda_1 + 24.8\lambda_6) + \text{etc} = -584 \qquad (6')$$

To find the value of λ_5 in terms of λ_6, add equation ($1'$ to $4'$) and substitute for $(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)$ its value, viz $(7 - 4\lambda_5)$ as obtained from equation ($5'$). Thus we get $\lambda_5 = \dfrac{23 + 0.1\lambda_6}{8}$

Substituting this value of λ_5 in equations $1'$ to $4'$, we get the values of λ_1, λ_2, λ_3, and λ_4 in terms of λ_6

$\lambda_1 = -2.292 - 1.4\lambda_6$, $\quad \lambda_2 = 0.042 + 0.73\lambda_6$,
$\lambda_3 = -2.958 + 0.66\lambda_6$, and $\lambda_4 = 0.708 - 0.037\lambda_6$

Inserting the values of λ_1, λ_2, etc thus found in equation ($6'$), we have $\lambda_6 = -0.1614$. Knowing the value of λ_6 find the values of λ_1, λ_2, etc

$\therefore \quad \lambda_1 = -2.066$, $\lambda_2 = -0.076$; $\lambda_3 = -3.065$;
$\lambda_4 = +0.714$, $\lambda_5 = +2.878$

Substituting these values in $c_1 = \lambda_1 - 20.6\lambda_6$,

$c_2 = \lambda_1 + 24.8\lambda_6$, etc, we get

$c_1 = +1.259$	$c_7 = +3.781$
$c_2 = -6.069$	$c_8 = -2.368$
$c_3 = +3.668$	$c_9 = +0.807$
$c_4 = -3.465$	$c_{10} = +2.797$
$c_5 = +0.437$	$c_{11} = -0.192$
$c_6 = -6.245$	$c_{12} = +3.587$

Check $-c_1+c_2+c_9=-4''\ 003$ | $c_7+c_8+c_{12}=+5''\ 000$

$c_3+c_4+c_{10}=+3''\ 000$ | $c_9+c_{10}+c_{11}+c_{12}=+6''\ 99$

$c_5+c_6+c_{11}=-6''\ 000$

Adjustment of a Polygon with a Central Station —

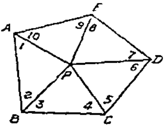


Fig 180a

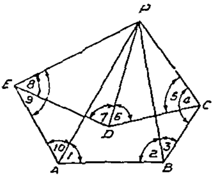


Fig 180b

1 *When the Central Station is not Occupied* —The central station P may be inside the polygon as in Fig 180 *a* or outside the polygon as shown in Fig 180 *b* In Fig 180 *a* let ABCDE be a polygon with a central station P which is not occupied (i.e the angles at P are not measured) The angles measured at the stations A B etc may be designated by the numerals 1, 3 5 7 and 9 being the left hand angles and 2 4 6 8 and 10 the right hand angles

The conditions which must be fulfilled by the adjusted angles are

Angle equation The sum of the ten interior angles must be equal to 540°

Side equation The sum of the log sines of the left hand angles must equal the sum of the log sines of the right hand angles

The side equation results from the condition that if one side (AP) be calculated from another side (BP) by two routes, the results shall be equal

Let c_1 c_2 c_3 etc be the corrections in seconds to the observed values of the angles 1, 2 3 etc respectively

d_1 d_2 d_3 etc the tabular differences for one second for log sin 1, log sin 2 log sin 3 etc

Then we have

By condition (1), $c_1 + c_2 + c_3 + c_4 + c_5 + c_6 + c_7 + c_8 + c_9 + c_{10} = \pm k$ (1)

By condition (2), $d_1c_1 - d_2c_2 + d_3c_3 - d_4c_4 + d_5c_5 - d_6c_6 + d_7c_7 - d_8c_8 + d_9c_9 - d_{10}c_{10} = \pm M$ (2)

By the theory of least squares, $c_1^2 + c_2^2 + c_3^2 + c_4^2 + c_5^2 + c_6^2 + c_7^2 + c_8^2 + c_9^2 + c_{10}^2 =$ a minimum (3)

To determine the most probable values of the corrections the method of Correlates may be used The adjusted values of the angles may then be found by applying the corrections to their observed values

II. *When the Central Station P is Occupied* —(Fig 181) Denoting the angles measured at P by 11, 12 etc, we get the following equations of condition

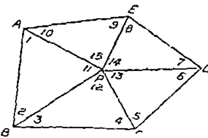


Fig 181

Angle equations —

(1) The sum of the central angles at P must be equal to 360°

(2) The sum of the three angles of each triangle must be equal to 180°

Side equation —

(3) The sum of the log sines of the left hand angles must be equal to the sum of the log sines of the right hand angles.

The side equation arises from the condition that the polygon may not be distorted

Let c_1, c_2, etc. be the corrections to the observed values of the angles 1, 2, etc.

d_1, d_2, etc. the tabular differences for one second for log sin 1, log sin 2, etc.

Then we have

By condition (1), $c_{11} + c_{12} + c_{13} + c_{14} + c_{15} = \pm k_1$. (1)

By condition (2), $c_1 + c_2 + c_{11} = \pm k_2$ (2)

$c_3 + c_4 + c_{12} = \pm k_3$. (3)

$c_5 + c_6 + c_{13} = \pm k_4$ . . (4)

$c_7 + c_8 + c_{14} = \pm k_5$. (5)

$c_9 + c_{10} + c_{15} = \pm k_6$ . . (6)

By condition (3), $d_1c_1 - d_2c_2 + d_3c_3 - d_4c_4 + d_5c_5 - d_6c_6 + d_7c_7 - d_8c_8 + d_9c_9 - d_{10}c_{10} = \pm M$ ... (7)

By the least square condition, $\Sigma_1^{15} c^2 =$ a minimum ... (8)

The rest of the procedure is exactly similar to that shown in the example on page 433

Example —ABCDE is a pentagon with an interior station P. The following are the angles observed at A, B, C, D, and E between each side and the line to P, the station P being unoccupied:

L. H. angle.	R. H. angle.
1 = 26° 12′ 6″	2 = 50° 10′ 10″
3 = 55° 24′ 20″	4 = 49° 15′ 12″
5 = 84° 48′ 24″	6 = 62° 16′ 30″
7 = 82° 54′ 36″	8 = 39° 28′ 24″
9 = 42° 6′ 48″	10 = 47° 23′ 18″.

Determine the most probable values of the angles. (Fig. 180*a*).

Let c_1, c_2, c_3, etc. be the corrections in seconds to the observed angles.

The conditions to be satisfied are

(1) The sum of the ten interior angles must be equal to 540°.

(2) Σ log (sin L. H. angle) = Σ(log sin R. H. angle).

Now the sum of the observed angles = 539° 59′ 48″.

∴ Error = − 12″; hence the total correction = + 12″.

$\Sigma \log$ (sin L H angle) $= \overline{1}\ 3818039$,

$\Sigma \log$ (sin R H angle) $= \overline{1}\ 3819260$

By condition (1), $c_1+c_2+c_3+c_4+c_5+c_6+c_7+c_8 + c_9 + c_{10} = +12''$ (1)

" (2) $42\ 8c_1 - 17\ 6c_2 + 14\ 5c_3 - 18\ 1c_4 + 1\ 9c - 11\ 1c_6 + 2\ 6c_7 - 25\ 6c_8 + 23\ 3c_9 - 19\ 4c_{10} = +1221$ (2)

where 42 8, 17 6, etc are the tabular logarithmic differences for one second for log sines of the angles 1, 2, etc

By the theory of least squares $\Sigma_1^{10} c^2 =$ a minimum (3)

Proceeding similarly as in the example on page 433, we have

$$\Sigma_1^{10} \delta c = 0 \qquad (1')$$

$$42\ 8\delta c_1 - 17\ 6\delta c_2 + 14\ 5\delta c_3 - \text{etc} = 0 \qquad (2)$$

$$\Sigma_1^{10} c\delta c = 0 \qquad (3)$$

Multiplying the equations 1 and 2 by $-\lambda_1$ and $-\lambda_2$ respectively, and adding them to equation 3, and then equating the coefficients of each δc to zero we get

$$\begin{aligned} c_1 &= \lambda_1 + 42\ 8\lambda_2 & c_6 &= \lambda_1 - 11\ 1\lambda_2 \\ c_2 &= \lambda_1 - 17\ 6\lambda_2 & c_7 &= \lambda_1 + \ 2\ 6\lambda_2 \\ c_3 &= \lambda_1 + 14\ 5\lambda_2 & c_8 &= \lambda_1 - 25\ 6\lambda_2 \\ c_4 &= \lambda_1 - 18\ 1\lambda_2 & c_9 &= \lambda_1 + 23\ 3\lambda_2 \\ c_5 &= \lambda_1 + \ 1\ 9\lambda_2 & c_{10} &= \lambda_1 - 19\ 4\lambda_2 \end{aligned}$$

Inserting these values in the original equations 1 and 2 we have

$$10\lambda_1 - 6\ 7\lambda_2 = +12 \qquad (1')$$

$$-\ 6\ 7\lambda_1 + 4387\ 65\lambda_2 = +1221 \qquad (2')$$

from which $\lambda_1 = 1\ 3879$, $\lambda_2 = 0\ 2804$

The values of the corrections are obtained by inserting the values of λ_1 and λ_2 in $c_1 = \lambda_1 + 42\ 8\lambda_2$ $c_2 = \lambda_1 - 17\ 6\lambda_2$ etc

Whence,

$c_1 = +13\ 39$ secs	$c_6 = -1\ 73$ secs
$c_2 = -\ 3\ 55$,,	$c_7 = +2\ 12$,,
$c_3 = +\ 5\ 45$,,	$c_8 = -5\ 79$,,
$c_4 = -\ 3\ 69$,,	$c_9 = +7\ 92$,,
$c_5 = +\ 1\ 92$,,	$c_{10} = -4\ 05$,,

Check $\Sigma_1^{10} c = +30\ 80 - 18\ 81 = +11\ 99$ secs

The most probable values of the angles are

Angle	Observed value	Correction	Adjusted value.
1	26° 12 6″	+ 13″ 39	26° 12 19″ 89
2	50 10 10	− 3 55	50 10 6 45
3	55 24 20	+ 5 45	55 24 25 45
4	49 15 12	− 3 69	49 15 8 31
5	84 48 24	+ 1 92	84 48 25 92
6	62 16 30	− 1 73	62 16 28 27
7	82 54 36	+ 2 12	82 54 38 12
8	39 28 24	− 5 79	39 28 18 21
9	42 6 48	+ 7 92	42 6 55 92
10	47 23 18	− 4 05	47 23 13 95
			Sum = 539° 59 59″ 99

THREE-POINT PROBLEM

Computation of the Position of a Station from Observations to Three Known Points

Three-Point Problem —When the main triangulation has been completed, it is frequently found necessary to locate additional points which are subsequently used as instrument stations as in topographic survey. This problem also arises when it is required to locate on plan the position of an observer in the boat as in hydrographic survey. The methods by which the three-point problem may be solved are (i) mechanical (ii) graphical, and (iii) analytical. The analytical method is used in more precise work.

Analytical Method.—(Figs. 182, 183, and 184.)

In minor triangulation the position of an instrument station O is determined by measuring each of the two angles subtended at it by the three stations A, B, and C whose positions are known by means of a theodolite (or by means of a sextant in hydrographic survey) and by solving the triangles AOB and BOC. Since the positions of stations A, B, and C are known, the sides AB and BC, and the angle ABC (B) of the triangle ABC are known. From this data and from the observed values of the angles AOB and BOC, the angles BAO (x) and BCO (y) can be computed. Knowing the angles x and y, the distances OA, OB, and OC may be determined by the application of the sine rule.

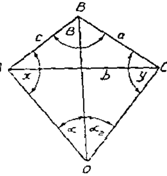


Fig. 181

In Fig. 182, let A, B, and C = the stations of known positions.

c = the distance AB, $\quad a$ = the distance BC.
B = the angle ABC, $\quad \alpha_1$ = the angle AOB
α_2 = the angle BOC,
x = the angle BAO, $\quad y$ = the angle BCO
$\phi = 360° - (\alpha_1 + \alpha_2 + B) = x + y$

Then from the $\triangle$ OAB, $\quad OB = \dfrac{c \sin x}{\sin \alpha_1}$

,, ,, OBC, $\quad OB = \dfrac{a \sin y}{\sin \alpha_2}$

$$\therefore \quad \frac{c \sin x}{\sin \alpha_1} = \frac{a \sin y}{\sin \alpha_2} \quad \text{or} \quad \sin y = \frac{c \sin \alpha_2 \sin x}{a \sin \alpha_1}$$

Substituting the value of y ($= \phi - x$), we get

$$\sin(\phi - x) = \frac{c \sin \alpha_2 \sin x}{a \sin \alpha_1}$$

or $\sin\phi\cos x - \cos\phi\sin x = \dfrac{c\sin\alpha_2\sin x}{a\sin\alpha_1}$

Dividing both members of the equation by $\sin\phi\sin x$, we have

$$\cot x - \cot\phi = \frac{c\sin\alpha_2}{a\sin\alpha_1\sin\phi}$$

$$\text{or } \cot x = \cot\phi\left\{1 + \frac{c\sin\alpha_2\sec\phi}{a\sin\alpha_1}\right\} \quad \ldots \quad \ldots \quad (1)$$

Knowing x, the angle y may be obtained from the relation

$$y = \phi - x$$

Whence, from the $\triangle$ OAB, $OA = \dfrac{c\sin ABO}{\sin\alpha_1}$; $OB = \dfrac{c\sin x}{\sin\alpha_1}$;

from the $\triangle$ OBC, $OB = \dfrac{a\sin y}{\sin\alpha_2}$; $OC = \dfrac{a\sin OBC}{\sin\alpha_2}$;

in which $ABO = 180° - x - \alpha_1$ and $OBC = 180° - y - \alpha_2$.

Alternative Method:—The unknown angles BAO (x) and BCO (y) may be determined from the formula

$$\tan\psi = \cot(\theta + 45°)\tan\frac{\phi}{2} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

where $\psi = \frac{1}{2}(x - y)$; $\phi = 360° - (\alpha_1 + \alpha_2 + B) = x + y$;

and $\tan\theta = \dfrac{c\sin\alpha_2}{a\sin\alpha_1}$

Having found the value of ψ the values of x and y may be determined from the equations

$$\frac{1}{2}(x + y) = \frac{\phi}{2} \text{ and } \frac{1}{2}(x - y) = \psi$$

The derivation of the formula is as follows:

$$\frac{\tan\frac{1}{2}(x - y)}{\tan\frac{1}{2}(x + y)} = \frac{\sin\frac{1}{2}(x - y)\cos\frac{1}{2}(x + y)}{\sin\frac{1}{2}(x + y)\cos\frac{1}{2}(x - y)} = \frac{\sin x - \sin y}{\sin x + \sin y}$$

$$= \frac{1 - \dfrac{\sin y}{\sin x}}{1 + \dfrac{\sin y}{\sin x}} \quad . \quad . \quad \ldots \quad (a)$$

Now $\dfrac{\sin y}{\sin x} = \dfrac{c \sin \alpha_2}{a \sin \alpha_1}$ as derived in the first method.

Let $\dfrac{c \sin \alpha_2}{a \sin \alpha_1} = \tan \theta \; \therefore \; \dfrac{\sin y}{\sin x} = \tan \theta.$

The equation (a) may, therefore, be written as

$$\frac{\tan \frac{1}{2}(x-y)}{\tan \frac{1}{2}(x+y)} = \frac{1 - \dfrac{\sin y}{\sin x}}{1 + \dfrac{\sin y}{\sin x}} = \frac{1 - \tan \theta}{1 + \tan \theta}$$

But $\dfrac{1 - \tan \theta}{1 + \tan \theta} = \cot(\theta + 45°)$

Hence $\dfrac{\tan \frac{1}{2}(x-y)}{\tan \frac{1}{2}(x+y)} = \cot(\theta + 45°)$

Substituting the values of $\frac{1}{2}(x-y)$ and $\frac{1}{2}(x+y)$, we get

$$\tan \psi = \cot(\theta + 45°) \tan \frac{\phi}{2}.$$

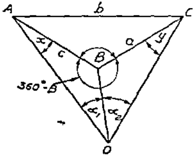


Fig. 183

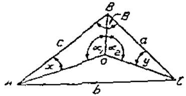


Fig 184

Three cases arise according to the position of station O with respect to three stations A, B, and C.

Case I: Stations B and O are on opposite sides of AC. (Fig 182)

Case II Stations B and O are on the same side of AC. (Fig 183).

Case III: Station O is within the triangle ABC. (Fig. 184).

It may be noted that in case II, $360° - \angle ABC$ must be used instead of $\angle$ ABC (B) in finding the value of ϕ from the relation

$$\phi = 360° - (\alpha_1 + \alpha_2 + B)$$

The solution of the problem is indeterminate when station O lies on the circle passing through A, B, and C, i e when $\alpha_1 + \alpha_2 + B = 180°$.

Examples on Three-Point Problem

Example 1 —A, B, and C are three visible stations in a hydrographical survey The sides AB and BC are 3325 m and 3712·9 m respectively and the angle ABC is 100° 20 30″ The angles observed with a sextant between A and B, and B and O from a sounding boat at O are 39° 12 20″ and 52° 48′ 40″ respectively. The points B and O are on opposite sides of AC. Find the distances OA, OB, and OC

(See Fig 182)

(i) Let $\angle BAO = x$; $\angle BCO = y$, $\angle AOB = \alpha_1 = 39° 12' 20''$, $\angle BOC = \alpha_2 = 52° 48' 40''$; $AB = c = 3325$ m ; $BC = a = 3712 \cdot 9$m.

Now the angle x may be obtained from

$$\cot x = \left\{ \cot \phi + \frac{c \sin \alpha_2}{a \sin \alpha_1 \sin \phi} \right\}$$

where
$$\begin{aligned} \phi &= 360° - (\alpha_1 + \alpha_2 + \angle ABC) \\ &= 360° - (39° 12' 20'' + 52° 48' 40'' + 100° 20' 30'') \\ &= 167° 38' 30'' = x + y \end{aligned}$$

$$\therefore \quad \cot x = \left\{ \cot 167° 38' 30'' + \frac{3325 \sin 52° 48' 40''}{3712 \cdot 9 \sin 39° 12' 20'' \sin 167° 38' 30''} \right\}$$

$$= \{-4 \cdot 5640863 + 5 \cdot 2734061\} = 0 \cdot 7093198$$

or $\quad x = 54° 39' 4''\cdot 13$; $x + y = 167° 38' 30''$

$\therefore \quad y = 112° 59' 25''\cdot 87$

(ii) In the $\triangle$ AOB, $\angle BAO = x = 54° 39' 4''\cdot 13$;
$\angle AOB = \alpha_1 = 39° 12' 20''$;

$$\begin{aligned} \angle ABO &= 180° - x - \alpha_1 \\ &= 180° - 54° 39' 4'' \cdot 13 - 39° 12' 20'' = 86° 8' 35'' \cdot 87 \end{aligned}$$

The distances OA and OB may be obtained by the sme rule.

$$\therefore \quad OA = \frac{AB \sin ABO}{\sin \alpha_1} = \frac{3325 \sin 86^\circ 8' 35''\cdot 87}{\sin 39^\circ 12' 20''}$$

$$= 5248\ 295 \text{ m}$$

$$OB = \frac{AB \sin x}{\sin \alpha_1} = \frac{3325 \sin 54^\circ 39' 4''\cdot 13}{\sin 39^\circ 12' 20''}$$

$$= 4290\ 46 \text{ m}$$

In the $\triangle BOC$, $\angle OBC = \angle ABC - \angle ABO$
$= 100^\circ 20' 30'' = 86^\circ 8' 35''\ 87 = 14^\circ 11' 54''\cdot 13.$

Then $$OB = \frac{BC \sin y}{\sin \alpha_2} = \frac{3712\ 9 \sin 112^\circ 59' 25''\cdot 87}{\sin 52^\circ 48' 40''}$$

$$= 4290\ 46 \text{ m}$$

$$OC = \frac{BC \sin OBC}{\sin \alpha_2} = \frac{3712\ 9 \sin 14^\circ 11' 54''\cdot 13}{\sin 52^\circ 48' 40''}$$

$$= 1143\ 165 \text{ m}.$$

Alternative method —The angles x and y may be obtained from

$$\tan \psi = \cot (\theta + 45^\circ) \tan \frac{\phi}{2},$$

where $\psi = \frac{1}{2}(x - y)$, $\tan \theta = \frac{c \sin \alpha_2}{a \sin \alpha_1}$

$$\frac{\phi}{2} = \frac{1}{2}(x + y) = \frac{1}{2}(360^\circ - \alpha_1 - \alpha_2 - \angle ABC)$$

Now $\tan \theta = \frac{3325 \sin 52^\circ 48\ 40''}{3712\ 9 \sin 39^\circ 12\ 20''}$ or $\log \tan \theta = 0\cdot 0525556$

$\theta = 48^\circ 27\ 30''\ 08$, and $\theta + 45^\circ = 93^\circ 27' 30''\cdot 08;$

Now $\quad \phi \quad 167^\circ 38\ 30'' \qquad \frac{\phi}{2} \quad 83^\circ 49' 15''.$

Now $\quad \tan \psi = \cot 93^\circ 27' 30''\ 08 \tan 83^\circ 49' 15''$
or $\quad \log \tan \psi = -1\ 7467786$

$\psi = -29^\circ 10' 10''\cdot 64$

Now $\frac{1}{2}(x + y) = \quad 83^\circ 49' 15''$ .. .. .. (1)

$\frac{1}{2}(x - y) = -29^\circ 10' 10''\cdot 64$ .. .. .. (2)

By adding equations 1 and 2, we get $x = 54° 39' 4''·36$

By subtracting equation 2 from equation 1, we have
$y = 112° 59' 25''·64$.

Example 2 —The following observations were made on three stations A, B, and C from station O, stations B and O being on the same side of AC

$\angle AOB = 30° 23' 12''$, $\angle BOC = 40° 36' 48''$, $AB = 2112·5$ m. $BC = 2537·5$ m, $\angle ABC = 125° 12' 20''$.

Determine the distances OA, OB, and OC (see Fig 183)

(i) Here $\alpha_1 = 30° 23' 12''$, $\alpha_2 = 40° 36' 48''$,
$c = 2112·5$ m $a = 2537·5$ m.

$\phi = 360° - \{\alpha_1 + \alpha_2 + (360° - \angle ABC)\}$

or $= \angle ABC - \alpha_1 - \alpha_2 = 125° 12' 20'' - 30° 23' 12'' - 40° 36' 48''$
$= 54° 12' 20'' = x + y$

It may be noted that since B is towards O, $360° - \angle ABC$ must be used instead of $\angle ABC$ in finding the value of ϕ from, $\phi = 360° - \alpha_1 - \alpha_2 - \angle ABC$

Then by $\cot x = \left\{\cot \phi + \dfrac{a \sin \alpha_2}{a \sin \alpha_1 \sin \phi}\right\}$, we get

$$\cot x = \left\{\cot 54° 12' 20'' + \frac{2112·5 \sin 40° 36' 48''}{2537·5 \sin 30° 23' 12'' \sin 54° 12' 20''}\right\}$$
$$= 2·0419032$$

$\therefore$ $x = 26° 5' 34''·12$ and $y = 28° 6' 45''·88$

(ii) In the $\triangle$ AOB, $x = 26° 5' 34''·12$, $\alpha_1 = 30° 23' 12''$ and $\angle ABO = 123° 31' 13''·88$.

In the $\triangle$ BOC, $y = 28° 6' 45''·88$ $\alpha_2 = 40° 36' 48''$ and $\angle CBO = 111° 16' 26''·12$

Applying the sine rule, we have

From the $\triangle$ AOB, $OA = \dfrac{2112·5 \sin 123° 31' 13''·88}{\sin 30° 23' 12''} = 3496·715$ m.

$$OB = \frac{2112·5 \sin 26° 5' 34''·12}{\sin 30° 23' 12''} = 1836·838 \text{ m.}$$

From the $\triangle$ BOC, $OB = \dfrac{2537 \cdot 5 \sin 28°6'45''\cdot 88}{\sin 40° 36' 48''} = 1830{\cdot}836$ m

$$OC = \frac{2537 \cdot 5 \sin 111° 15' 26''\cdot 12}{\sin 40° 36' 48''} = 3632{\cdot}51 \text{ m.}$$

Alternative method :—

$$\tan \theta = \frac{2112 \cdot 5 \sin 40° 36' 48''}{2537 \cdot 5 \sin 30° 23' 12''} \quad \text{or} \quad \theta = 46° 58' 22''{\cdot}29$$

$$\theta + 45° = 91° 58' 22''{\cdot}29.$$

Now $$\tan \psi = \cot (\theta + 45°) \tan \frac{\phi}{2}$$

$$= \cot 91° 58' 22'' \cdot 29 \tan 27° 6' 10''.$$

or $$\psi = -1° 0' 35'' \cdot 89$$

Now $$\tfrac{1}{2}(x + y) = 27° 6' 10''$$

$$\tfrac{1}{2}(x - y) = -1° 0' 35'' \cdot 89$$

Solving these equations, we have

$$x = 26° 5' 34'' \cdot 11 \quad \text{and} \quad y = 28° 6' 45''{\cdot}89$$

Example 3 —Calculate the distances OA, OB, and OC and their azimuths from the following data :

Line.	Azimuth	Length in m.
AB	76° 54′ 58″	1741·5
BC	186° 23′ 48″	2728·5
CA	329° 9′ 4″	2703·39

At a station O within the triangle ABC, the measured angles are $\angle AOB = 85° 40' 15''$, $\angle BOC = 146° 32' 10''$, $\angle COA = 127° 47' 35''$.

(see Fig. 184)

(i) Let $\angle AOB = \alpha_1 = 85° 40' 15''$, $\angle BOC = \alpha_2 = 146° 32' 10''$;

$\angle BAO = x$, $\angle BCO = y$

$\angle ABC$ = azimuth of BA − azimuth of BC

$= 256° 54' 58'' - 186° 23' 48'' = 70° 31' 10'' = B.$

Now $\phi = 360° - (\alpha_1 + \alpha_2 + B)$

$= 360° - (85° 40' 15'' + 146° 32' 10'' + 70° 31' 10'')$

$= 57° 16' 25'' = x + y$

$\tfrac{1}{2}(x + y) = \tfrac{1}{2}\phi = 28° 38' 12''{\cdot}5.$

Let $\tan = \theta \dfrac{c \sin \alpha_2}{a \sin \alpha_1} = \dfrac{1741\ 5 \sin 146°\ 32\ 10''}{2"25\ 8 \sin 8ɔ°\ 40\ 15''}$

or $\quad \log \tan \theta = \bar{1}\ 5477161$

$\theta = 19°\ 26\ 26''\ 18$ and $\theta + 45° = 64°\ 26\ 26''\ 18.$

Let $\quad \psi = \frac{1}{2}(x - y)$

Now $\quad \tan \psi = \cot (\theta + 45°) \tan \dfrac{\phi}{2}$

$= \cot\ 64°\ 26\ 26''\ 18\ \tan\ 28°\ 38\ 12\ \ 5.$

or $\quad \log \tan \psi = \bar{1}\ 4168872 \qquad \psi = 14°\ 38\ 8''\ 98$

Now $\frac{1}{2}(x + y) = 28°\ 38\ 12''\ 5$ and $\frac{1}{2}(x - y) - 14°\ 38\ 8''\ 98$

$x = 43°\ 16\ 21''\ 48$ and $y = 14°\ 0\ 3''\ ɔ^{\circ}$

To check the results the value of x may be obtained from

$$\cot x = \left\{ \cot \phi \quad \frac{c \sin \alpha_2}{a \sin \alpha_1 \sin \phi} \right\}$$

$$= \left\{ \cot\ 5"°\ 16\ 2ɔ'' + \frac{1"41\ ɔ \sin 146°\ 3^{\circ}\ 10''}{."28\ 5 \sin 8ɔ°\ 40\ 15'' \sin 5"°\ 16\ 25''} \right\}$$

$= 1\ 06^{\circ}1904 \qquad x = 43°\ 16\ \ ^{\circ}1''\ 4"$ and $y = 14°\ 0\ 3''\ 53$

(ii) In the $\triangle$ AOB $x = 43°\ 16\ 21''\ 48 \quad \alpha_1 = 8ɔ°\ 40\ 15''$

$\angle OBA = ɔ1°\ 3\ 23''\ 52$

In the $\triangle$ BOC $y = 14°\ 0\ 3''\ ɔ^{\circ} \quad \alpha_2 = 146°\ 32\ 10''$

$\angle OBC = 19°\ 2"\ 46''\ 48$

Check $-\angle B = \angle OBA + \angle OBC = 51°3\ 23''\ ɔ2 + 19°27\ 46''\ 48$
$= "0°\ 31\ 10''$

Now $OA = \dfrac{1"41\ ɔ \sin\ 51°3\ 23''\ 52}{\sin 85°\ 40\ 15''} = 13ɔ8\ 365$ m

$OB = \dfrac{1"41\ 5 \sin 43°\ 16\ 21''\ 48}{\sin 85°\ 40\ 15''} = 119"\ 163$ m,

Also $OB = \dfrac{2"28\ 5 \sin 14°\ 0\ 3''\ 5^{\circ}}{\sin 146°\ 32\ 10''} = 119"\ 163$ m

$GC = \dfrac{2"^{\circ}8\ 5 \sin 19°\ 2"\ 46''\ 48}{\sin 146°\ 32\ 10''} = 1648\ "2"$ m

(iii) Azimuths of the lines —

Azimuth of AB	= 76°54′58″·00	Azimuth of BC	= 186°23′48″·00
Add $\angle x$	= 43°16′21″·48	Add $\angle$OBC	= 19°27′46″·48
Azimuth of AO	= 120°11′19″·48	Azimuth of BO	= 205°51′34″·48
Add 180°	= 180°	Deduct 180°	= 180°
Azimuth of OA	= 300°11′19″·38	Azimuth of OB	= 25°51′34″·48
Angle BCO (y)	= 14° 0′ 3″·52		
Deduct azimuth of CB	= 6°23′48″	(Azimuth of CB = azimuth of BC − 180°)	
Azimuth of CO	= 7°36′15″·52	anticlockwise from North	
or „ „	= 352°23′44″·48	clockwise „ „	
Deduct 180°	= 180°		
Azimuth of OC	= 172°23′44″·48		

Check — α_1 = difference of azimuths of OA and OB.

$$300°\ 11'\ 19''{\cdot}48 - 25°\ 51'\ 34''{\cdot}48$$
$$= 274°\ 19'\ 45'' = 360° - (274°\ 19'\ 45'') = 85°40'15''$$

α_2 = difference of azimuths of OB and OC

$$= 172°\ 23'\ 44''{\cdot}48 - 25°\ 51'\ 34''{\cdot}48$$
$$= 146°\ 32'\ 10''.$$

(iv) If the co-ordinates of stations A, B, and C be given the co-ordinates of station O may be calculated by first finding the latitudes and departures of OA, OB, OC from their known lengths and azimuths, and then adding them algebraically to the respective co-ordinates of station O. The co-ordinates of station O are thus obtained in three ways.

Adjustment of Level Work

Measurements of Equal Weight :—

If the difference of elevation of two points is found a number of times under exactly similar conditions, of in the same manner and over the same length, the arithmetic mean of several measurements is the most probable value of the difference of elevation

between the given points The probable error of a single measurement of unit weight is given by the formula

$$p\ e = 0\ 6745 \sqrt{\frac{\Sigma v^2}{(n-1)}} \tag{1}$$

where v = the residual i e the difference between the measured value and the arithmetic mean
n = the number of measurements

$$p\ e \text{ of the arithmetic mean} = 0\ 6745 \sqrt{\frac{\Sigma v^2}{n(n-1)}} \tag{2}$$

Measurements of Unequal Weight —

If the difference of level of two points is determined in the same manner, and over the same length but under such conditions that the measurements must be regarded as of unequal weight the weighted arithmetic mean of several measurements gives the most probable value of this difference of level The probable error of a single measurement of unit weight is given by the formula

$$p\ e = 0\ 6745 \sqrt{\frac{\Sigma wv^2}{(n-1)}} \tag{3}$$

where w = the weight of measurement

The probable error of any measurement of weight w is given by formula

$$p\ e = 0\ 6745 \sqrt{\frac{\Sigma wv^2}{w(n-1)}} \tag{3a}$$

The probable error of the weighted arithmetic mean is givan by the formula

$$p\ e = 0\ 6745 \sqrt{\frac{\Sigma wv^2}{\Sigma w(n-1)}} \tag{4}$$

Duplicate Lines —In precise levelling a line is run twice over the same route with the same care, but in opposite directions Such a line is called a duplicate line of levels In such a case the most probable value of the difference of elevation of any two points is the average of the two results and the probable error of a single measurement is given by the formual

$$p.\ e. = \pm\ 0{\cdot}4769d \quad \dots \qquad (5)$$

$$p.\ e \text{ of the arithmetic mean} = \pm\ 0{\cdot}3373d \qquad (6)$$

where d = the discrepancy between the measurements taken in opposite directions

Example —Find the most probable value of the difference of elevation of two bench marks, given the following observed values

Observed values · 46·568, 46·546

$d = 46\ 568 - 46\ 546 = 0{\cdot}022$

$p.e$ of a single measurement $= 0\ 4769 \times 0{\cdot}022 = \pm\ 0{\cdot}0105$

$p\ e$ of the arithmetic mean $= 0\ 3373 \times 0\ 022 = \pm\ 0{\cdot}0074$

Arithmetic mean $= \dfrac{46\ 568 + 46{\cdot}546}{2} = 46{\cdot}557.$

∴ Most probable value $= 46\ 557 \pm 0{\cdot}0074.$

Sectional Lines —If a line of levels includes one or more intermediate bench marks, it is regarded as made up of a series of sections connecting these bench marks, each section being regarded as a duplicate line

Let d_1, d_2, d_3, represent the most probable values of the difference of elevation between the successive bench marks, and e_1, e_2, e_3, the probable errors of the several values

Then the most probable value (D) of the difference of elevation between the terminal bench marks is

$$D = d_1 + d_2 + d_3 + \ . \quad \dots + d_n = \Sigma d \quad \dots \quad (7)$$

and the probable error of the total difference of elevation (D) is

$$p.\ e. = \sqrt{e_1^2 + e_2^2 + e_3^2 + \dots + e_n^2} = \sqrt{\Sigma e^2} \quad \dots \quad (8)$$

General Laws of the Probable Errors and Relative Weight

"Under the same conditions of measurement the probable error of a line of levels varies as the square root of its length."

"Under the same conditions of measurement the weight of the result due to any line of levels varies inversely as the length of the line."

Multiple Lines :—(Fig. 179). A set of the two or more lines connecting the same two bench marks is called a multiple line

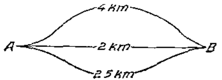


Fig 185

of levels. Each line should be weighted inversely as its length. The most probable value of the difference of elevation between the terminals of a multiple line is then the weighted arithmetic mean of the observed values, and its probable error is obtained from the formula (4).

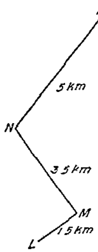


Fig 186

Intermediate Points :—(Fig 186) Points are said to be intermediate when they lie only on a single line of levels and have no influence on the general adjustment.

In this case, the discrepancy is to be distributed in direct proportion to the distances from the initial point.

Example :—The adjusted values of L and P are 25·568 m and 27·794 m respectively In the line of levels (Fig. 186), the following are the observed differences of elevation :

L to M=+0·809 m.; M to N =− 0·908 m.; N to P=+2·442 m.

Find the most probable values of the elevations of M and N.

From the given differences in elevation, find the elevations of M, N, and P, commencing from L.

Elevation of L = 25 568.

„ „ M = 25 568 + 0 809 = 26 377.
„ „ N = 26 377 − 0 908 = 25·469.
„ „ P = 25·469 + 2·442 = 27·911.

Discrepancy = 27·911 − 27·794 = + 0·117 m.

Total distance = $1\frac{1}{2} + 3\frac{1}{2} + 5 = 10$ km.

$$\text{Correction to } M = \frac{1.5}{10} \times 0.117 = 0.0176. \qquad (-\text{ve})$$

$$\text{Correction to } N = \frac{5}{10} \times 0.117 = 0.0585. \qquad (-\text{ve})$$

Then the adjusted elevations are

$L = 25.568$
$M = 26.377 - 0.0176 = 26.3594 = 26.359$ m
$N = 25.469 - 0.0585 = 25.4105 = 25.411$ m.
$P = 27.911 - 0.117 = 27.794$ m.

Closed Circuits —(Fig 187) In level work a circuit is said to be closed when a line of levels ends on the initial point or forms a single closed ring In this case, it is run only once under the same conditions The most probable values of the elevations of any points forming a closed circuit may be found by distributing the discrepancy among the observed elevations in direct proportion to the respective distances from the initial point The discrepancy or the *error of closure*, as it is called, is positive, if the observed value of the initial point is too high, and vice versa

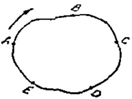


Fig 188

Example —The following are the observed differences of elevation for the points forming a closed circuit :

A to B = + 0.823	distance	= 2 km
B to C = − 1.709	,,	= 1 ,,
C to D = + 1.135	,,	= 3.5 ,,
D to A = − 0.354	,,	= 2.5 ,,

Adjust the elevations of B, C, and D, given that the elevation of A = 50.752

Elevation of A = 50.752.
,, of B = 50.752 + 0.823 = 51.575.
,, of C = 51.575 − 1.709 = 49.866
,, of D = 49.866 + 1.135 = 51.001.
,, of A = 51.001 − 0.354 = 50.647.

Discrepancy $= 50\ 647 - 50\ 752 = -0\ 105$

Total distance $= 2 + 1 + 3\ 5 + 2\ 5 = 9$ km

$\therefore$ Correction to the elevation of B $= + \frac{2}{9}\ (0\ 105) = +0\ 023$

$\quad$ „ $\quad$ „ C $= + \frac{3}{9}\ (0\ 105) = +0\ 035$

$\quad$ „ $\quad$ „ D $= + \frac{6\ 5}{9}\ (0\ 105) = +0\ 0758$

Hence elevation of B $= 51\ 575 - 0\ 023 = 51\ 598$

„ $\quad$ of C $= 49\ 866 + 0\ 035 = 49\ 901$

„ $\quad$ of D $= 51\ 001 + 0\ 076 = 50\ 925$

„ $\quad$ of A $= 50\ 647 + 0\ 105 = 50\ 752$

Adjustment of a Level Net

A level net is an interconnecting net work of level circuits formed by level lines interconnecting three or more bench marks The method of least squares may be used in adjusting a level net The most probable values of the several differences of elevation between the bench marks may be determined (1) by the method of *Correlates* or (2) by the method of *normal* equations The most probable values of the elevations of the bench marks may then be found by combining the corrected level differences Another method is to find the most probable values of the elevations of the bench marks directly from their observed values by the method of normal equations The weight that is to be assigned to the observed difference of elevation of the ends of a connecting line is taken as inversely proportional to the length of the line The method of Correlates is illustrated by following example (1)

Example 1 —In running a circuit of precise levels for four bench marks, the following level differences were obtained: (Fig 188)

A to B $= +4\ 380$ weight 2, P to A $= -16\ 760$ weight 1,
C to B $= -7\ 620$ „ 1, B to P $= +12\ 520$ „ 2
P to C $= -4\ 820$ „ 2,

The arrows show the direction in which each line of levels was run

Here there are two level circuits, ABP and BPC. Let c_1, c_2, c_3, c_4, and c_5 be the corrections to the level differences AB, CB, PC, PA, and BP respectively, and w_1, w_2, w_3, w_4 and w_5, their weights respectively. The total error in the level circuit ABP $= 4\ 380 + 12\ 520 - 16\ 760$

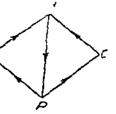


Fig 188

$= +0\ 140$

The total correction $= -0\ 140$

Similarly, the total error in the level circuit BPC

$= 12\ 520 - 4\ 820 - 7\ 620 = +0\ 080$

The total correction $= -0\ 080$

Now the equations of condition are

$c_1 + c_5 + c_4 = -0\ 140$ (1), $c_5 + c_3 + c_2 = -0\ 080$ (2)

By the theory of least squares, $\Sigma_1^5 wc^2 =$ a minimum (3)

Differentiating the three equations, we have

$\delta c_1 + \delta c_5 + \delta c_4 = 0$ (4), $\delta c_5 + \delta c_3 + \delta c_2 = 0$ (5)

and $w_1c_1\delta c_1 + w_2c_2\delta c_2 + w_3c_3\delta c_3 + w_4c_4\delta c_4 + w_5c_5\delta c_5 = 0$ (6)

Multiply the equations (4) and (5) by $-\lambda_1$ and $-\lambda_2$ respectively, and add the results to the equation (6). Equate the coefficients of each δc to zero. Then we have

$w_1c_1 = \lambda_1$, $w_2c_2 = \lambda_2$, $w_3c_3 = \lambda_2$, $w_4c_4 = \lambda_1$, and $w_5c_5 = \lambda_1 + \lambda_2$.

Now substitute these values of c_1, c_2, etc in the original equations (1) and (2)

Then $$\left(\frac{1}{w_1} + \frac{1}{w_5} + \frac{1}{w_4}\right)\lambda_1 + \frac{1}{w_5}\lambda_2 = -0\ 140 \quad (7)$$

and $$\frac{1}{w_5}\lambda_1 + \left(\frac{1}{w_2} + \frac{1}{w_3} + \frac{1}{w_5}\right)\lambda_2 = -0\ 080 \quad (8)$$

Insert the values of w_1 w_2, etc, in equations (7) and (8)

Here $w_1 = 2$ $w_2 = 1$ $w_3 = 2$, $w_4 = 1$, $w_5 = 2$

$$\therefore \quad (\tfrac{1}{2}+\tfrac{1}{2}+1)\lambda_1+\tfrac{1}{2}\lambda_2 = -0.14 \text{ or } 2\lambda_1+0.5\lambda_2 = -0.14 \quad .. (9)$$

$$\tfrac{1}{2}\lambda_1+(1+\tfrac{1}{2}+\tfrac{1}{2})\lambda_2 = -0.08 \text{ or } 0.5\lambda_1+2\lambda_2 = -0.08 \quad (10)$$

The solution of these equations gives the values of λ_1 and λ_2

$$\therefore \quad \lambda_1 = -0.064, \ \lambda_2 = -0.024$$

Finally, obtain the values of c_1, c_2 etc

$$c_1 = \frac{\lambda_1}{w_1} = -\frac{0.064}{2} = -0.032$$

$$c_2 = \frac{\lambda_2}{w_2} = -\frac{0.024}{1} = -0.024$$

$$c_3 = \frac{\lambda_2}{w_3} = -\frac{0.024}{2} = -0.012$$

$$c_4 = \frac{\lambda_1}{w_4} = -\frac{0.064}{1} = -0.064$$

$$c_5 = \frac{\lambda_1+\lambda_2}{w_5} = -\frac{0.088}{2} = -0.044$$

Whence, the adjusted differences of elevation are

A to B = + 4.380 − 0.032 = + 4.348
C to B = − 7.620 − 0.024 = − 7.644
P to C = − 4.820 − 0.012 = − 4.832
P to A = − 16.760 − 0.064 = − 16.824
B to P = + 12.520 − 0.044 = + 12.476

Check — + 4.348 + 12.476 − 16.824 = 0

+12.476 − 4.832 − 7.644 = 0

If in the above example, the true level difference of A to C is given, we have another equation of condition Suppose, for instance, C is known to be 11.970 m above A.

Then the equations of condition are

$$c_1+c_5+c_4 = -0.140, \ c_5+c_3+c_2 = -0.080, \text{ and } c_1-c_2 = -0.030$$

It may be noted that the correction to the level difference in BC is opposite in sign to that in CB The rest of the procedure is exactly similar to that in the above example

Following the above procedure exactly, we get

$$\delta c_1+\delta c_5+\delta c_4=0,\ \delta c_5+\delta c_3+\delta c_2=0,\ \delta c_1-\delta c_2=0, \text{ and } \sum_1^5 wc\,\delta c=0$$

Multiplying the first three equations by $-\lambda_1, -\lambda_2$, and $-\lambda_3$ respectively, adding the results to the last equation and then equating the coefficients of each δc to zero we have

$$c_1=\frac{\lambda_1+\lambda_3}{w_1} \quad c_2=\frac{\lambda_2-\lambda_3}{w_2} \quad \Bigg| \quad c_3=\frac{\lambda_2}{w_3} \quad c_4=\frac{\lambda_1}{w_4} \quad \Bigg| \quad c_5=\frac{\lambda_1+\lambda_2}{w_5}$$

Substituting the values of c_1, c_2 etc in the original equations, we have

$$\left(\frac{1}{w_1}+\frac{1}{w_4}+\frac{1}{w_5}\right)\lambda_1+\frac{1}{w_5}\lambda_2+\frac{1}{w_1}\lambda_3=-0\ 140$$

$$\frac{1}{w_5}\lambda_1+\left(\frac{1}{w_2}+\frac{1}{w_3}+\frac{1}{w_5}\right)\lambda_2-\frac{1}{w_2}\lambda_3=-0\ 080$$

$$\frac{1}{w_1}\lambda_1-\frac{1}{w_2}\lambda_2+\left(\frac{1}{w_1}+\frac{1}{w_2}\right)\lambda_3=-0{\cdot}030$$

The solution of these equations gives the values of λ_1, λ_2, and λ_3 and hence the values of c_1, c_2, c_3, c_4 and c_5

The correct level differences may then be determined by applying these corrections to the observed level differences

Note —If the line is levelled twice in opposite directions, the average of the two results should be taken as the observed level difference If the lengths of the lines are given, the observed differences of elevation must be weighted *inversely* as the lengths of the lines

The method of normal equations is best shown by the following example

Example 2 :—The field notes give the following results

K to L = + 10·769	Distance = 2 km
L to M = − 5·268	„ = 2 „
M to N = + 7·986	„ = 2 5 „
N to P = − 6·012	„ = 4 „
P to K = − 7 242	„ = 5 „
L to P = − 3 506	„ = 2 „
M to P = + 2 178	„ = 2 5 „

The arrow-heads show the direction in which each line is run. The elevation of the bench mark K is 250 780 m It is required to determine the most probable values of the elevations of the other bench marks (Fig 189)

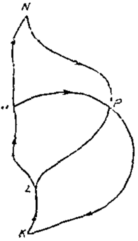


Fig 189

Here the lines KL LM, MN, and NP are taken as the independent unknowns

Let c_1, c_2, c_3 and c_4 be the corrections to the corresponding observed differences of elevation Then the most probable values of the respective differences of elevation are

$$\begin{aligned} K \text{ to } L &= -10\ 769 + c_1 \\ L \text{ to } M &= -\ \ 5\ 268 - c_2 \\ M \text{ to } N &= +\ \ 7\ 986 + c_3 \\ N \text{ to } P &= -\ \ 6\ 012 + c_4 \end{aligned}$$

Substituting these values in the observation equations, we have

$$\begin{aligned} c_1 &= 0 \quad (\text{weight } \tfrac{1}{2}) \\ c_2 &= 0 \quad (\text{weight } \tfrac{1}{2}) \\ c_3 &= 0 \quad (\text{weight } \tfrac{2}{5}) \\ c_4 &= 0 \quad (\text{weight } \tfrac{1}{4}) \\ -c_1 - c_2 - c_3 - c_4 &= +0{\cdot}233 \ (\text{weight } \tfrac{1}{5}) \\ c_2 + c_3 + c_4 &= -0{\cdot}212 \ (\text{weight } \tfrac{1}{2}) \\ c_3 + c_4 &= +0\ 204 \ (\text{weight } \tfrac{2}{5}). \end{aligned}$$

These equations are formed by first finding the most probable values of the differences and then adding algebraically the values so obtained for the differences between the respective points, and then comparing them with the observed values

e g observed value, L to P $= -3.506$

Most probable value L to M $= -5.268 + c_2$

" " " M to N $= +7.986 + c_3$

" " " N to P $= -6.012 + c_4$

Adding algebraically, we get

$$\text{L to P} = -3.294 + c_2 + c_3 + c_4$$
$$= -3.506$$
$$c_2 + c_3 + c_4 = -0.212$$

Forming the normal equations from the above reduced observation equations, we have

$$\begin{aligned}
7c_1 + 2c_2 + 2c_3 + 2c_4 &= -0.0466 \\
2c_1 + 1.2c_2 + 7c_3 + 7c_4 &= -0.1526 \\
2c_1 + 7c_2 + 1.5c_3 + 1.1c_4 &= -0.0710 \\
2c_1 + 7c_2 + 1.1c_3 + 1.35c_4 &= -0.0710
\end{aligned}$$

The solution of these equations gives

$c_1 = -0.0345$, $c_2 = -0.1363$, $c_3 = +0.01$,
$c_4 = +0.0161$

therefore, the most probable values of the differences are

K to L $= +10.7345$, L to M $= -5.4043$,
M to N $= +7.9960$, N to P $= -5.9959$,
P to K $= -7.3303$, L to P $= -3.4042$,
M to P $= +2.0001$

The most probable values of the elevations are

K $= 250.780$ m L $= 261.514$ m
M $= 256.110$ m N $= 264.106$ m
P $= 258.110$ m

Alternative Method —In this method the most probable values of the elevations of the bench marks are found directly

as illustrated below The approximate values of the elevations of the points L, M, and P are

$$
\begin{array}{rlrl}
K = & 250\ 780 & M = & 256\ 281 \text{ approximate} \\
 & +10\ 769 & & +7\ 986 \quad ,, \\
\hline
L = & 261\ 549 \text{ approx} & N = & 264\ 267 \quad ,, \\
 & -5\ 268 & & -6\ 012 \\
\hline
M = & 256\ 281 \quad ,, & P = & 258\ 255 \quad ,,
\end{array}
$$

Let c_1 c_2, c_3, and c_4 be the corrections to the above approximate values

Therefore, the most probable values are

$L = 261\ 549 + c_1$, $N = 264\ 267 + c_3$,

$M = 256\ 281 + c_2$, $P = 258\ 255 + c_4$

Substituting these values in the observation equations, we get

$$
\begin{array}{lll}
K \text{ to } L = +10\ 769 + c_1 & & = +10\ 769 \\
L \text{ to } M = -\ \ 5\ 268 - c_1 + c_2 & & = -\ \ 5\ 268 \\
M \text{ to } N = +\ \ 7\ 986 - c_2 + c_3 & & = +\ \ 7\ 986 \\
N \text{ to } P = -\ \ 6\ 012 - c_3 + c_4 & & = -\ \ 6\ 012 \\
P \text{ to } K = -\ \ 7\ 475 - c_4 & & = -\ \ 7\ 242 \\
L \text{ to } P = -\ \ 3\ 294 - c_1 + c_4 & & = -\ \ 3\ 506 \\
M \text{ to } P = +\ \ 1\ 974 - c_2 + c_4 & & = +\ \ 2\ 178
\end{array}
$$

These equations should then be reduced and weighted inversely as their distances

Then

$$
\begin{array}{lclr}
c_1 & = & 0 & (\text{weight } 5) \\
-c_1 + c_2 & = & 0 & (\ ,,\ 5) \\
-c_2 + c_3 & = & 0 & (\ ,,\ 4) \\
-c_3 + c_4 & = & 0 & (\ ,,\ 25) \\
-c_4 & = & +0\ 233 & (\ ,,\ 2) \\
-c_1 + c_4 & = & -0\ 212 & (\ ,,\ 5) \\
-c_2 + c_4 & = & +0{\cdot}204 & (\ \cdot 4)
\end{array}
$$

Forming the normal equations from these reduced observation equations by the usual rule, we have

$$1\ 5c_1 - 0\ 5c_2 \qquad - 0\ \ 5c_4 = +0\ 1060$$
$$-0\ 5c_1 + 1\ 3c_2 - 0\ \ 4c_3 - 0\ \ 4c_4 = -0\ 0816$$
$$-0\ 4c_2 + 0\ 65c_3 - 0\ 25c_4 = \quad 0$$
$$-0\ 5c_1 - 0\ 4c_2 - 0\ 25c_3 + 1\ 35c_4 = -0\ 0710$$

Check —The coefficients in the first row and first column are exactly the same in value, sign, and order The same is true in the case of other rows and columns

Solving these equations we get

$c_1 = -0\ 0348$, $c_2 = -0\ 1706$, $c_3 = -0\ 1611$,
$c_4 = -0\ 1452$

Hence the most probable values of the elevations of the bench marks are

K = 250 780	N = 264 106
L = 261 514	P = 258 110
M = 256 110	

Effect of Curvature of the Earth on Surveys

There are two effects of the curvature of the earth on surveys, viz (1) spherical excess and (2) convergence of meridians. The former is appreciable only when the triangles are very large, while the latter has a very appreciable effect on surveys Due to the effect of the curvature of the earth, a straight line is constantly changing its azimuth A line having the same azimuth throughout is not a straight line, but a parallel of latitude is such a line It is well to note here the distinction between the azimuth of a line and its bearing

Azimuth of a Line —The azimuth of a line AB may be defined as the angle between the plane of the meridian at A and the plane of the great circle passing through the line AB, while the Reverse azimuth, i e the azimuth of A from B is the angle between the plane of the meridian at B and the plane of the great circle containing the line AB Now the meridians through A and B are parallel, only when A and B lie upon the equator

in which case the azimuth of A from B is equal to the azimuth of B from A $\pm$ 180°. In general, however, the meridians through A and B are not parallel, but they converge to the earth's poles Consequently, the azimuth of A from B is not the same as the azimuth of B from A $\pm$ 180°

Bearing of a Line —The bearing of a line AB may be defined as the angle between the plane of reference at each station and the plane of the great circle through AB the plane of reference being parallel to some standard plane preferably near the centre of the area surveyed The fore bearing and the back bearing of AB are, therefore supplementary angles (or B B of AB =F B of AB$\pm$180°) *Convergence or change in azimuth* is the angle between the true meridian through B and the line through B parallel to the original meridian through A By computing the change in azimuth, we can check the accuracy of work in the case of a long open traverse Suppose for instance a traverse is run from A to B The azimuth of the first line Aa is determined by an astronomical observation The bearing of the last line say, pB with reference to an axis parallel to the original meridian through A is computed by means of observed angles The azimuth of the last line pB is then determined by an astronomical observation The computed bearing of the last line pB will not agree with its observed azimuth, since the meridian through B converges and meets the original meridian through A at the poles The difference between the computed bearing of the last line pB and its azimuth is equal to the convergence or change in azimuth

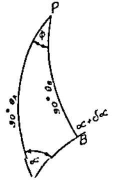


Fig 190

In Fig 190 let P denote the pole, A and B any two points on the earth s surface AB the great circle arc, PA and PB the meridians through A and B respectively Then the azimuth of AB at A $= \angle$PAB while the azimuth of AB at B $= 180° - \angle$PBA The difference between these two azimuths is known as the convergence of the meridians Denoting the angles PAB and PBA of the spherical triangle PAB by A and B respectively, the convergence of meridians is equal to 180°$-$(A$+$B)

The formula for the convergence of meridians (change in azimuth) may be derived as follows (Fig 184),

Let θ_A = the latitude of A, ϕ_A = the longitude of A
θ_B = " " of B, ϕ_B = " " of B
C_M = the convergence of the meridians $= 180° - (A+B)$

Then in the $\triangle$ PAB, $PA = 90° - \theta_A = b$, $PB\ 90° - \theta_B = a$

APB = the difference of longitude $= \phi_A - \phi_B = \phi_D$

$$\text{Now } \tan \tfrac{1}{2}(A+B) = \frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} \cot \frac{P}{2}$$

$$\frac{\cos \frac{1}{2}(\theta_A - \theta_B)}{\sin \frac{1}{2}(\theta_A + \theta_B)} \cot \tfrac{1}{2}\phi_D$$

$$\text{i e } \cot\left\{90° - \tfrac{1}{2}(A+B)\right\} = \frac{\cos \frac{1}{2}(\theta_A - \theta_B)}{\sin \frac{1}{2}(\theta_A + \theta_B)} \cot \tfrac{1}{2}\phi_D$$

But $\left\{90° - \tfrac{1}{2}(A+B)\right\} = \tfrac{1}{2}C_M$

$$\cot \tfrac{1}{2}C_M = \frac{\cos \frac{1}{2}(\theta_A - \theta_B)}{\sin \frac{1}{2}(\theta_A + \theta_B)} \cot \tfrac{1}{2}\phi_D$$

Whence

$$\tan \tfrac{1}{2}C_M = \frac{\sin \frac{1}{2}(\theta_A + \theta_B)}{\cos \frac{1}{2}(\theta_A - \theta_B)} \tan \tfrac{1}{2}\phi_D \qquad (1)$$

When AB is small compared with the radius of the earth we may write $\tan \frac{1}{2}C_M = \frac{1}{2}C_M$ (in circular measure)
and $\tan \frac{1}{2}\phi_D = \frac{1}{2}\phi_D$ (" ")

Substituting these values in equation (1) we have

$$C_M = \frac{\sin \frac{1}{2}(\theta_A + \theta_B)}{\cos \frac{1}{2}(\theta_A - \theta_B)} \times \phi_D \qquad (2)$$

where both C_M and ϕ_D must be expressed in the same units (minutes or seconds) When the difference of latitude of the adjacent stations is small $\cos \frac{1}{2}(\theta_A - \theta_B)$ is equal to unity The equation (2) may then be written as

$$C_M = \sin \tfrac{1}{2}(\theta_A + \theta_B) \times \phi_D = \phi_D \times \sin \tfrac{1}{2}(\theta_A + \theta_B) \qquad (3)$$

or expressed in words

Convergence of meridians = difference of longitude × sine of average latitude

It may be noted that the convergence of meridians is zero, (i) when the points A and B lie on the equator ($\theta_A = 0 = \theta_B$) and the two meridians are then parallel, and (ii) also when the points A and B lie on the same meridian ($\phi_D = 0$). It increases in value as the poles are approached. There is no effect of convergence of meridians on lines running north and south since they form part of the meridian of longitude, while it is greatest on lines running east and west.

We shall now derive the formula for convergence of meridians when the *linear difference* of latitude and *linear difference* of departure are given.

Let R = the radius of the earth

θ_A = the latitude of A

θ_B = " " of B

C_M = the convergence of meridians

ΣL = the linear difference of latitude (or total latitude) $(l_1 \cos \alpha_1 + l_2 \cos \alpha_2 + \quad + l_n \cos \alpha_n)$

ΣD = the linear difference of departure (or total departure) $(l_1 \sin \alpha_1 + l_2 \sin \alpha_2 + \quad + l_n \sin \alpha_n)$

where l_1, l_2, etc = the lengths of the sides of the traverse

α_1, α_2, etc. = the reduced bearings of the sides of the traverse.

Then the difference of latitude $= \theta_A - \theta_B = \dfrac{\Sigma L}{R \tan 1'}$ minutes.

of A and B

the difference of longitude $= \phi_D = \dfrac{\Sigma D}{R \cos \frac{1}{2}(\theta_A + \theta_B)}$ radians.

of A and B

The parallel of middle latitude is a circle whose radius is equal to $R \cos \frac{1}{2}(\theta_A + \theta_B)$

Substituting the value of ϕ_D in equation (2), we get

$$C_M = \frac{\sin \frac{1}{2}(\theta_A + \theta_B)}{\cos \frac{1}{2}(\theta_A - \theta_B)} \times \frac{\Sigma D}{R \cos \frac{1}{2}(\theta_A + \theta_B)}$$

i. e. $$C_M = \frac{\Sigma D \tan \frac{1}{2}(\theta_A + \theta_B)}{R \cos \frac{1}{2}(\theta_A - \theta_B)} \text{ radians} \tag{4}$$

or $$C_M \text{ (in minutes)} = \frac{\Sigma D \tan \frac{1}{2}(\theta_A + \theta_B)}{R \tan 1' \cos \frac{1}{2}(\theta_A - \theta_B)} \tag{4a}$$

When the difference of latitude is small $\cos \frac{1}{2}(\theta_A - \theta_B) = 1$

$$\therefore \qquad C_M = \frac{\Sigma D \tan \frac{1}{2}(\theta_A + \theta_B)}{R} \text{ radians} \tag{5}$$

$$\text{or } C_M \text{ (in minutes)} = \frac{\Sigma D \tan \frac{1}{2}(\theta_A + \theta_B)}{R \tan 1'} \text{ (approximate)} \tag{5a}$$

or expressed in words convergence of meridians in minutes

$$= \frac{\text{Total departure} \times \tan \text{average latitude}}{\text{Radius of the earth} \times \tan \text{one minute}}$$

It may be observed that in the derivation of the above formula the earth has been assumed to be a sphere The formula for convergence of meridians is used in checking the angles of long open traverses as in route surveys by determining the azimuths of the first and last lines by an astronomical observation

Computation of Geodetic Positions

The co-ordinates (latitude and longitude) of a station B may be computed from (i) the known latitude and longitude of station A (ii) the distance from A to B, and (iii) the azimuth of B from A by the Mid Latitude formula which is simple and gives sufficiently accurate results (correct to 0" 01) It should be used for lines less than 40 km in length and in latitudes less than 60°

Notation —θ_A = the latitude of A, ϕ_A = the longitude of A
θ_B = the latitude of B ϕ_B = the longitude of B
α = the azimuth of AB at A
$\delta\alpha$ = the increase of azimuth
$\alpha + \delta\alpha$ = the azimuth of AB at B
$\delta\theta$ = the difference of latitude of A and B
$\delta\phi$ = the difference of longitude of A and B
l = the length of the line AB
m = the length of 1" of latitude, in metres at the mean (or average) latitude of A and B
ι = the length of 1" of longitude in metres at the mean (or average) latitude of A and B

The values of m and n may be obtained from Geodetic Tables in which they are given at intervals of 5 of latitude from 0° to 60°.

In Fig 191, the lines AA_1 and AB_1 are drawn at right angles to each other to represent the meridian and parallel through A

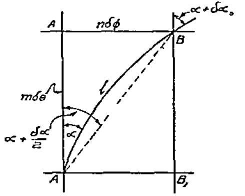


Fig 191

Similarly, the lines BB_1 and BA_1 are drawn at right angles to each other to represent the meridian and parallel through B

The distance between the two parallels = the difference of latitude between A and B in seconds × the length of 1″ of latitude at the mean latitude of A and B, or $AA_1 = m\delta\theta$ Similarly, the distance between the two meridians = the difference of longitude between A and B in seconds × the length of 1″ of longitude at the mean latitude of A and B, or $AB_1 = n\delta\phi = A_1B$

The average azimuth of AB (dotted line) is $\alpha + \frac{1}{2}\delta\alpha$

Then $$m\delta\theta = l\cos(\alpha + \tfrac{1}{2}\delta\alpha)$$

or $$\delta\theta = \frac{l\cos(\alpha + \frac{1}{2}\delta\alpha)}{m} \text{ seconds} \qquad (6)$$

$$n\delta\phi = l\sin(\alpha + \tfrac{1}{2}\delta\alpha)$$

or $$\delta\phi = \frac{l\sin(\alpha + \frac{1}{2}\delta\alpha)}{n} \text{ seconds} \qquad (7)$$

From formula (1), $\tan\frac{1}{2}\delta\alpha = \tan\frac{1}{2}\delta\phi\sin(\theta_A + \frac{1}{2}\delta\theta)\sec\frac{1}{2}\delta\theta$ (8)

or $$\delta\alpha = \delta\phi \times \text{sine of mean latitude} \qquad (8a)$$

$$\tan(\alpha + \tfrac{1}{2}\delta\alpha) = \frac{n\,\delta\phi}{m\delta\theta} \qquad (8b)$$

When the latitude and longitude of station A, the length of AB, and its azimuth at A are given, the latitude and longitude of station B, and the azimuth of AB at B can be computed by the application of the above formulæ To do this, we proceed by successive approximations, the order of procedure being as follows: As a first approximation, we ignore $\frac{1}{2}\delta\propto$ in formula (6) and (7) and then calculate $\delta\theta$, taking the mean latitude as the latitude of A to determine the value of m in formula (6), and then determine $\delta\phi$ Knowing $\delta\theta$ and $\delta\phi$, $\delta\propto$ may be found from formula (8a) Having found $\delta\propto$, all the three calculations are repeated in the above order to obtain the final values of $\delta\theta$, $\delta\phi$, and $\delta\propto$.

Procedure —(1) Find the value of m at the latitude of A from the tables

(2) Using formula (6) and omitting $\frac{\delta\propto}{2}$, find the difference of latitude $\delta\theta$.

(3) Knowing $\delta\theta$, find the mean latitude from mean latitude = latitude of A + $\frac{1}{2}\delta\theta$.

(4) Find the value of n at the mean latitude thus obtained.

(5) Compute $\delta\phi$, from formula (7).

(6) Finally, obtain the value of $\delta\propto$ from formula (8a)

(7) Using the value of $\delta\propto$, repeat the three calculations in the above order to find the final values of $\delta\theta$, $\delta\phi$, and $\delta\propto$.

(8) Knowing $\delta\theta$, $\delta\phi$, and $\delta\propto$, find the latitude and longitude of station B and the azimuth of AB at B.

The method is exemplified in example (6)

Inverse Problem —When the latitudes and longitudes of two stations A and B are given, the length and azimuth of the line AB may be computed by the application of the formulae 6 to 8b

Procedure —(1) Find the mean latitude from the known latitudes of A and B

(2) Obtain the values of m and n at the mean latitude.

(3) Knowing $\delta\theta$, $\delta\phi$, m, and n, find the value of $\propto + \frac{1}{2}\delta\propto$ from formula (8b).

(4) Substituting the value of $\alpha + \frac{1}{2}\delta\alpha$ in formula (6), calculate the length of AB.

(5) Find the increase of azimuth ($\delta\alpha$) from formula (8 or 8a).

(6) Find the azimuth of AB at A from the computed values of ($\alpha + \delta\alpha$) and $\delta\alpha$.

The method is exemplified in example (7).

If the earth is considered as a sphere, the length of AB may be determined by solving the triangle PAB (Fig. 184), P being the north pole. In the ΔPAB, $PA = 90° - \theta_A$; $PB = 90° - \theta_B$; APB = diff. of long. $= \phi_A - \phi_B$. Using the cosine rule, we find the value of AB in degree.' Arc AB may then be computed from the relation arc = R × central angle in circular measure.)

$$\therefore \quad \text{arc } AB = \frac{\pi R \times \angle AB}{180°}.$$

Example 1 :—Determine the convergence of meridians for a departure of 30 km, given that the mean latitude is 52° 45'. Take the radius of the earth (R) as 6370290 m

$$\text{Convergence in minutes} = \frac{\text{departure} \times \tan \text{mean latitude}}{R \tan 1'}$$

$$= \frac{30 \times 1000 \times \tan 52° 45'}{6370290 \tan 1'}$$

$$= 21 \cdot 2904$$

or convergence of meridians $= 21' \, 17'' \cdot 42$.

Example 2 :—Find the convergence of meridians from the following data :

Departure in a traverse $= 20809$ m ; $R \sin 1'' = 30 \cdot 88$ m
Mean latitude $= 20° \, 15'$.

$$\text{Convergence in seconds} = \frac{\text{dep.} \times \tan \text{mean lat.}}{R \sin 1''} = \frac{20809 \tan 20° 15'}{33 \cdot 88}$$

$$= 226 \cdot 584$$

or convergence of meridians $= 3' \, 46'' \cdot 35$.

Example 3 :—Given the following latitudes and longitudes of two stations A and F ; obtain the convergence of meridians through A and F.

Latitude of A = 40° 45′ 20″ N, longitude of A = 100° 48′ 22″ W,
Latitude of F = 41° 10 36″ N, longitude of F = 101° 12′ 28″ W

(See Fig 184)

Let θ_A and θ_F be the latitudes of A and F
ϕ_A and ϕ_F be the longitudes of A and F

Then half the difference of latitude of A and F

$$= \tfrac{1}{2}(\theta_F - \theta_A)$$
$$= \tfrac{1}{2}(41^\circ 10\ 36'' - 40^\circ 45\ 20'')$$
$$= 12\ 38''$$

half the sum of the „ „ $= \tfrac{1}{2}(\theta_F + \theta_A)$
$$= \tfrac{1}{2}(41^\circ 10\ 36'' + 40^\circ 45'\ 20'')$$
$$= 40^\circ 57\ 58''$$

Difference of longitude between A and F
$$= 101^\circ 12\ 28'' - 100^\circ 48' 22'' = 24' 6''$$

Now convergence of meridians in seconds

$$= \text{longitude difference in seconds} \frac{\sin \text{ mean lat.}}{\cos \tfrac{1}{2} \text{ (diff in lat.)}}$$

$$= \frac{1446'' \sin 40^\circ 57\ 58''}{\cos 12\ 38''} = 948\ 03$$

or convergence of meridians = 15′ 48″ 03

Example 4—Given the following particulars of a traverse

Line	Length	W C Bearing
AB	10 km	56°15′
BC	7 „	60°48′
CD	6 „	48°36′

The latitude of A was 50° 30 N Take the radius of the earth as 6370 km Determine the correction to be applied to the bearing at D to allow for the convergence of the meridians

(i) The latitudes and departures of the lines may be calculated by the formulæ lat $= l \cos \alpha$ and dep $= l \sin \alpha$.

Lat of AB = 10 cos 56°15 = 5 5557020
„ of BC = 7 cos 60°48 = 3 4150179
„ of CD = 6 cos 48°36 = 3 9678714
Sum = 12 9385913

Dep. of AB $= 10 \sin 56°15' = 8\ 3146960$
" of BC $= 7 \sin 60°48' = 6\ 1104547$
" of CD $= 6 \sin 48°36' = 4\ 5006666$
sum $= 18\ 9258173$

(ii) Then the linear difference of latitude between A and D $= \Sigma L = 12\ 9385913$ km

The linear difference of departure between A and D $= \Sigma D = 18{\cdot}9258173$ km

(iii) Now let θ_A and θ_D be the latitudes of A and D, and ϕ_A and ϕ_D the longitudes of A and D

$$\text{Now } \theta_D - \theta_A = \frac{\text{linear lat diff}}{R \tan 1'} \text{ minutes}$$

$$= \frac{12\ 9385913}{6370 \tan 1'} = 6\ 9827 \text{ mins.}$$

Hence mean latitude = latitude of A $+ \frac{1}{2}$ lat diff.

$$= 50°30' + \tfrac{1}{2}(6\ 9827)$$

i e. $\frac{1}{2}(\theta_A + \theta_D) = 50°33'\ 4914$

(iv) Now convergence of meridians (in minutes)

$$= \frac{\Sigma D \tan \frac{1}{2}(\theta_A + \theta_D)}{R \cos \frac{1}{2}(\theta_A - \theta_D) \tan 1'}$$

$$= \frac{18\ 9258173 \tan 50°33'\ 27''{\cdot}48}{3956 \cos 3'29''\ 48 \tan 1'} = 12{\cdot}416 \text{ mins.}$$

∴ The required correction $= 12'\ 24''{\cdot}96$

Example 5 :—The following notes refer to a traverse from station A to station D in a route survey

Line.	True bearing	Length	
AB	N 50° E	8·0 km	Latitude of A = 52°30'N.
BC	N 70° E	9 6 "	Longitude of A = 78°15'E.
CD	N. 40° E	6 4 "	

Find the latitude and longitude of D and the azimuth of CD at D given that 1' of meridian is 1·852 km and 1' of longitude is 1·855 km on the equator.

The latitudes and departures of the lines are:

Line	Latitude in km	Departure in km.	Remarks
AB	5 1423008	6 1283552	Lat. $= l \cos \measuredangle$
BC	3 2833930	9 0210490	Dep $= l \sin \measuredangle$
CD	4 9026842	4 1138506	
	Total 13 3283780	19 2632548	= Coordinate of D

(ΣL and ΣD) with respect to A

(i) Let θ_A, ϕ_A, and θ_D and ϕ_D be the latitudes and longitudes of stations A and D respectively, $\delta\theta$ and $\delta\phi$, the latitude difference and longitude difference between A and D respectively

Then

$$\delta\theta = \theta_D - \theta_A = \frac{13\ 133283780}{R \tan 1'} \text{ mins}$$

$$= \frac{13\ 3283780}{1\ 852} \text{ mins}$$

$$= 7'\ 19671.$$

$$= 7'\ 11''\ 80 \text{ (approximate)}$$

$\therefore$ Mean latitude $= \theta_A + \frac{1}{2}\delta\theta = 52°\ 30' + 3'\ 37''\cdot 9.$

$= 52°\ 33'\ 35''\ 9$

If the exact value of 1′ of meridian at the mean latitude 52° 33′ 35″ 9 is 1 8542 km, then

$$\delta\theta = \frac{13\ 3283780}{1\ 8542} = 7'\ 18817 = 7'\ 11''\ 29$$

$\therefore$ Mean latitude $= 52°\ 33\ 35''\ 64$

(ii) Now $\delta\phi = \phi_D - \phi_A = \dfrac{19\ 2632548 \sec 52°\ 33'\ 35''\ 64}{1\ 855}$

$= 17\ \ 0815 = 17'\ 4''\ 89$

(iii) Whence, latitude of D $= \theta_D = \theta_A + \delta\theta$

$= 52°\ 30' + 7'\ 11''\ 65 = 52°\ 37'\ 11''\cdot 65$ N

longitude of D $= \phi_D = \phi_A + \delta\phi$

$= 78°\ 15' + 17'\ 4''\cdot 89$

$= 78°\ 32'\ 4''\cdot 89$

(iv) Convergence of meridians $= \delta\phi' \times$ (sin mean lat.) minutes

$= 17\ \ 0815 \sin 52°\ 33'\ 35''\ 64$

$= 13\ 5626$ mins

$= 13'\ 33''\cdot 76.$

$\therefore$ Azimuth of CD at D $= 40° 13' 33'' 76$

(N $40° 13' 33'' 76$ E.)

Example 6 —Determine the latitude and longitude of D and the reverse azimuth of CD at D from the following data:

Latitude of C $= 45° 1' 40''$ N, longitude of C $= 92° 36' 12''$ E.
Azimuth of CD $=$ N $56° 22' 30''$ E, length of CD $= 19057\ 62$ m.

Latitude	1″ of lat in m	1″ of long in m
45° 0′	30 8703	21 9032
45° 5′	30 8707	21 8714

(1) First approximation —

Let $\delta\theta$ = the difference in latitude between C and D.
$\delta\phi$ = „ „ „ longitude „ „ „ „
$\delta\alpha$ = the change in azimuth at D
= (convergence of meridians)

(i) Now 1″ of latitude at the latitude of station C (i e 45°1′40″) should be obtained by interpolation

$$\therefore\ 1'' \text{ of lat.} = 30\ 8703 + \frac{100}{300}(0\ 0004) = 30\ 8704 \text{ m.}$$

Now the linear latitude of CD

$$= \frac{19057\cdot62 \cos 56° 22' 30''}{30\ 8704} = 341''\ 86 = 5' 41''\ 86.$$

Hence mean latitude $(\overline{\theta})$ = lat of C $+ \dfrac{\delta\theta}{2}$

$= 45° 1' 40'' + \frac{1}{2}(5' 41''\ 86) = 45° 4' 30''\ 93$ (approximate)

(ii) 1″ of longitude at $\overline{\theta}$ (45° 4′ 30″ 83)

$$= 21\ 9032 - \frac{270\ 93}{300}(\cdot0318) = 21\ 8745 \text{ m}$$

Now the longitude difference $= \delta\phi = \dfrac{19057\ 62 \sin 56° 22' 30''}{21\ 8745}$

$= 725''\ 451 = 12' 5''\ 451.$

(iii) Change in azimuth $\delta\alpha = \delta\phi'' \times \sin\overline{\theta}$ seconds.

$= 725''\cdot451 \sin 45° 4' 30''\cdot93$
$= 513''\ 645 = 8' 33''\ 645.$

(2) Second approximation :—

(i) Now mean latitude $(\overline{\theta}) = 45° \; 4' \; 30''\cdot93$.

$$1'' \text{ of lat at } 45° \; 4' \; 30'' \; 93 = 30\cdot8703 + \frac{270 \; 93}{300} \; (\cdot0004)$$

$$= 30\cdot87066 \text{ m}$$

$$\therefore \quad \delta\theta = \frac{19057\cdot62 \cos 56°26'46\cdot8''}{30 \; 87066} = 341''\cdot442 = 5'41''\cdot44$$

Hence mean lat. $(\theta) = 45° \; 1' \; 40'' + \frac{1}{2} \; (5' \; 41''\cdot44)$
$= 45° \; 4' \; 30''\cdot72$. (exact).

Since the difference $(0''\cdot21)$ between the approximates and exact values of the mean latitude (θ) is too small to affect the values of $\delta\phi$ and $\delta\alpha$ obtained as a first approximation, we need not calculate them again.

Hence $\delta\phi = 12' \; 5''\cdot451$ (exact) and $\delta\alpha = 8' \; 33''\cdot645$ (exact).

Now latitude of D $= \theta_A + \delta\theta = 45° \; 1' \; 40'' + 5' \; 41''\cdot44$
$= 45° \; 7' \; 21''\cdot44$ N

longitude of D $= \phi_A + \delta\phi = 92° \; 36' \; 12'' + 12' \; 5''\cdot451$
$= 92° \; 48' \; 17''\cdot451$ E.

Azimuth of CD at D $= 56° \; 22' \; 30'' + 8' \; 33''\cdot645 = 56° \; 31' \; 3''\cdot645$

" " DC at D $= 236° \; 31' \; 3''\cdot645$ = S. $56° \; 31' \; 3''\cdot645$ W.

Example 7 —Given the following co-ordinates of two stations P and Q.

Latitude of P $= 54° \; 51' \; 28''$N ; longitude of P $= 83° \; 12' \; 40''$ E.
" of Q $= 54° \; 55' \; 42''$N. " of Q $= 83° \; 58' \; 20''$ E.

Latitude.	1″ of latitude in m	1″ of longitude in m.
54° 50′	30·9234	17·8509
54° 55′	30·9238	17·8141

Find the azimuth of PQ at P and the azimuth of QP at Q, and also the length of PQ.

(i) Average latitude $= \theta = \frac{1}{2} \; (54° \; 51' \; 28'' + 54° \; 55' \; 42'')$
$= 54° \; 53' \; 35''$.

Difference of latitude $= \delta\theta = (54° \; 55' \; 42'' - 54° \; 51' \; 28'')$
$= 0° \; 4' \; 14'' = 254''$.

" of longitude $= (83° \; 58' \; 20'' - 83° \; 12' \; 40'')$
$= 0° \; 45' \; 40'' = 2740''$.

Now change in azimuth (in seconds) = difference in long (in secs) $\times$ Sin mean lat. $= \delta\phi'' \sin\bar{\theta} = 2740 \sin 54^\circ\, 53' 35'' = 2241 \cdot 54$.

or $\qquad \delta \propto\ = 37'\ 21''\cdot 54$.

(ii) Length of 1" of lat. at $\bar{\theta}$ (54° 53′ 35″)

$$= m = 30\cdot 9234 + \frac{215}{300}\ (\cdot 0004)$$

$= 30\cdot 92369$ m.

Length of 1" of long. at $\bar{\theta}$ (54° 53′ 35″)

$$= n = 17\cdot 8509 - \frac{215}{300}\ (\cdot 0368)$$

$= 17\cdot 82453$ m.

(iii) Let $\propto$ be the azimuth of PQ a. P and $\delta \propto$ increase in azimuth at Q. Then

$$\tan\left(\propto + \frac{\delta \propto}{2}\right) = \frac{n \times \text{difference of longitude}}{m \times \text{difference of latitude}} = \frac{n\delta\phi}{m\delta\phi}$$

$$= \frac{17\cdot 82453}{30\ 92369} \times \frac{2740}{254}$$

$$\log \tan\left(\propto + \frac{\delta \propto}{2}\right) = 0\ 7936429$$

$$\therefore \qquad \propto + \frac{\delta \propto}{2} = 80^\circ\, 51'\, 48''\ 9\text{; but } \delta \propto\ = 37'\, 21''\cdot 54.$$

$$\propto\ = 80^\circ\, 51'\, 48''\cdot 9 - \tfrac{1}{2}(37'\, 21''\cdot 54)$$
$$= 80^\circ\, 33'\ \ 8''\cdot 13.$$

$$\because \qquad \propto + \delta \propto\ = 80^\circ\, 33'\ \ 8''\cdot 13 + 37'\, 21''\cdot 54$$
$$= 81^\circ\, 10'\, 29''\cdot 67.$$

Whence, Azimuth of PQ at P = 80° 33′ 8″·13.

" " at Q = 81° 10′ 29″·67.

Azimuth of QP at Q = 261° 10′ 29″·67.

(vi) Length PQ $= m\delta\theta \sec\left(\propto + \frac{\delta \propto}{2}\right)$

$= 30\cdot 92369 \times 254 \times \sec 80^\circ\, 51'\, 48'\cdot 9$

$= 49467.0$ m.

Also, length of PQ $= n\delta\phi \operatorname{cosec}\left(\propto + \frac{\delta \propto}{2}\right)$

$= 17\cdot 82453 \times 2740 \operatorname{cosec} 80^\circ\, 51'\, 48''\cdot 9$

$= 49466\cdot 8$ m.

Parallel of Latitude —To set out a portion of a parallel of latitude for a distance d in latitude θ :—

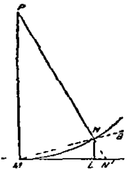


Fig 192

In Fig 192, let P be the pole; M and N the two points on the parallel; *MNQ*, the great circle through M and N; ML the great circle perpendicular to the meridian MP; ML = d; R = the radius of the earth.

Now $PNQ = PMN + \delta\alpha$,
where $\delta\alpha$ = change in azimuth
$= 180° - PNM$

But by symmetry $PMN = PNM$

$$\therefore \quad PMN + \delta\alpha = 180° - PMN \text{ or } PMN = 90° - \frac{\delta\alpha}{2}.$$

whence, $PNQ = 90° + \dfrac{\delta\alpha}{2}$

$$NML = PML - PMN = 90° - \left(90° - \frac{\delta\alpha}{2}\right) = \frac{\delta\alpha}{2}.$$

Now the offset (NL) to the great circle ML perpendicular to the meridian MP at a distance ML (d) = ML $\tan\dfrac{\delta\alpha}{2}$

$$= d\tan\frac{\delta\alpha}{2} = d \times \frac{\delta\alpha}{2}\tan 1'' \quad \text{But } \delta\alpha = \frac{d\tan\theta}{R\tan 1''},$$

$$\therefore \quad NL = d \times \frac{d\tan\theta}{2R\tan 1''} \times \tan 1'' = \frac{d^2\tan\theta}{2R}.$$

in which NL, d, and R are expressed in the same units.

The offset NL may also be shown equal to $\dfrac{d^2\sin\theta\tan 1''}{2M}$, where M is the length of $1''$ of parallel in latitude θ

e.g suppose the latitude $(\theta) = 50°$, the distance $(d) = 10$ km; and R = 6370 km.

Then the offset $= \dfrac{100 \tan 50^\circ}{2 \times 6370}$ km $= 9.35$ km

Strictly speaking, the offset should be along the meridian NP and not at right angles to ML. NN′ is the correct offset.

The above formula is to be used for short distances not exceeding 15 km. To determine the next point on the parallel, the meridian is determined at N and a line is set out at right angles to the meridian NP and the required offset is then calculated.

Example.—At a point B in latitude 48° N, a straight line BC 50 nautical miles long, is ranged at 90° to the meridian (due east). It is proposed to travel north from C so as to reach the 48° parallel at D. Find the angle BCD at which we must set out and the distance CD, assuming the earth to be a sphere.

Draw the meridians through B and C intersecting at the pole P. BC is at right angles to the meridian BP at B so that angle PBC is a right angle (Fig. 192a).

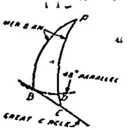


Fig. 192 (a)

Now in the spherical triangle PBC, PB $= 90^\circ - 48^\circ = 42^\circ$, BC $= 50'$, since 1 nautical mile subtends one minute at the centre of the earth, and the angle B $= 90^\circ$. Using the cosine formula, we get

$\cos PC = \cos PB \cos BC + \sin PB \sin BC \cos B$

Since $B = 90^\circ$, $\cos B = 0$

$\therefore \cos PC = \cos 42^\circ \cos 50'$, whence, $PC = 42^\circ\ 0'\ 24''$.

Now $PD = 90^\circ - 48^\circ = 42^\circ$, since D is on the parallel of latitude.

$\therefore$ $CD = PC - PD = 42^\circ 0' 24'' - 42^\circ = 24'' = \cdot 4' = 0\cdot 4$ nautical mile.

By the sine rule, $\sin BCP = \dfrac{\sin PBC \sin PB}{\sin PC} = \dfrac{\sin 90^\circ \times \sin 42^\circ}{\sin 42^\circ\ 24''}$

$\therefore \angle BCP = 89^\circ\ 5'$

Hence the angle (BCD) at which we must set out $= 89^\circ\ 5'$

Adjustment of a Closed Traverse

Crandall's Method.—The Crandall's method is most commonly used in the adjustment of a closed traverse when the *bearings* are to remain *unaltered*. If there be any angular error, it

should be distributed among the several angles before the latitudes

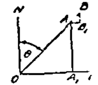


Fig 193

and departures of the sides of the traverse are computed The assumption made in this method is that the closing error (error of closure) is solely due to linear sources

In Fig 193, let l = the length of the side OA; L = the latitude of OA , D = the departure of OA, AB $= xl$, BB_1 = the correction (e) of the latitude $= x$L, AB_1 = the correction (f) of the departure $= x$D, in which x varies with the different lengths of the sides

Then

$$\text{The total correction in latitude} = E = \Sigma x L \quad \ldots \quad (1)$$

$$\text{The total correction in departure} = F = \Sigma x D \quad \ldots \quad (2)$$

$$\text{By the theory of least squares, } \Sigma\left(\frac{x^2 l^2}{l}\right) = \text{a minimum} \quad (3)$$

Differentiating the equations 1 to 3, we have

$$\Sigma L \delta x = 0 \quad (1'), \quad \Sigma D \delta x = 0 \quad (2'); \quad \Sigma (x l \delta x) = 0 \quad (3')$$

Multiplying the equations 1′ and 2′ by $-\lambda_1$ and $-\lambda_2$, adding the results to equation (3) and equating the coefficient of each δx to zero, we get

$$x_1 = \frac{\lambda_1 L_1 + \lambda_2 D_1}{l_1}, \quad x_2 = \frac{\lambda_1 L_2 + \lambda_2 D_2}{l_2}, \text{ etc}$$

Substituting these values in the original equations(1 and 2),we have

$$E = \lambda_1 \Sigma\left(\frac{L^2}{l}\right) + \lambda_2 \Sigma\left(\frac{LD}{l}\right), \text{ and } F = \lambda_1 \Sigma\left(\frac{LD}{l}\right) + \lambda_2 \Sigma\left(\frac{D^2}{l}\right)$$

The solution of these two equations gives the values of λ_1 and λ_2.

Then the corrections in latitude are

$$e_1 = \frac{\lambda_1 L_1^2 + \lambda_2 L_1 D_1}{l_1}, \text{ etc}$$

and the corrections in departure are

$$f_1 = \frac{\lambda_1 L_1 D_1 + \lambda_2 D_1^2}{l_1}, \text{ etc.}$$

Example —The following are the lengths and bearings of the sides of a closed traverse ABCDEF

Line	Length in m	R B
AB	902 6	S 45° 20 12″ E.
BC	816 4	N 62° 15′ 20″ E
CD	425 5	N 20° 40′ 10″ E.
DE	627 9	N 74° 26′ 30″ E
EF	1225 3	N 58° 36′ 24″ W
FA	1423 2	S 48° 15 0″ W

Adjust the traverse without altering the bearings of the lines

(i) Latitudes and departures · From the known lengths and bearings, the latitudes and departures of the sides should be calculated by the formulæ lat. $= l \cos \alpha$, and dep $= l \sin \alpha$.

Line.	Latitude.	Departure
AB	-634 47	$+$ 641 97
BC	$+380$ 06	$+$ 722 54
CD	$+398$ 11	$+$ 150 19
DE	$+168$ 42	$+$ 604·89
EF	$+638$ 27	-1045 93
FA	-953 67	-1068 50
	$\Sigma L = -8$ 28	$\Sigma D = +5$ 16

∴ Total error in lat $= -$ 8 28, total error in dep $= +$ 5 16

(ii) Now the correction to latitude of a side $= \lambda_1 \left(\frac{L^2}{l}\right) + \lambda_2 \frac{LD}{l}$

„ to departure of a side $= \lambda_1 \left(\frac{LD}{l}\right) + \lambda_2 \frac{D^2}{l}$.

in which L = latitude of a side
D = departure of a side
l = length of a side
λ_1 and λ_2 = the values obtained from

$$\left\{\Sigma\left(\frac{L^2}{l}\right)\right\}\lambda_1 + \left\{\Sigma\left(\frac{DL}{l}\right)\right\}\lambda_2 = \text{total correction in lat} \quad (1)$$

and $$\left\{\Sigma\left(\frac{LD}{l}\right)\right\}\lambda_1 + \left\{\Sigma\left(\frac{D^2}{l}\right)\right\}\lambda_2 = \quad „ \quad „ \quad \text{in dep} \quad (2)$$

(iii) $\dfrac{(\text{Lat.})^2}{\text{length}} = \dfrac{L^2}{l}$:—

$$AB; \frac{(-634\cdot47)^2}{902\cdot6} = 445\cdot99$$

$$BC; \frac{(+380\cdot06)^2}{816\cdot4} = 176\cdot93$$

$$CD; \frac{(+398\cdot11)^2}{425\cdot5} = 372\cdot48$$

$$DE; \frac{(+168\cdot42)^2}{627\cdot9} = 45\cdot18$$

$$EF; \frac{(+638\cdot27)^2}{1225\cdot3} = 332\cdot48$$

$$FA; \frac{(-958\cdot67)^2}{1432\cdot2} = 635\cdot03$$

$$\Sigma\left(\frac{L^2}{l}\right) = \text{sum} = 2008\cdot09$$

$\dfrac{\text{Lat.} \times \text{Dep.}}{\text{length}} = \dfrac{L \times D}{l}$:—

$$\frac{(-634\cdot47)(+641\cdot97)}{902\cdot6} = -451\cdot26$$

$$\frac{(+380\cdot06)(+722\cdot54)}{816\cdot4} = +336\cdot36$$

$$\frac{(+398\cdot11)(+150\cdot19)}{425\cdot5} = +140\cdot52$$

$$\frac{(+168\cdot42)(+604\cdot89)}{627\cdot9} = +162\cdot27$$

$$\frac{(+638\cdot27)(-1045\cdot93)}{1225\cdot3} = -544\cdot84$$

$$\frac{(-958\cdot67)(-1068\cdot50)}{1432\cdot2} = +711\cdot49$$

$$\Sigma\left(\frac{L.D}{l}\right) = 354\cdot54$$

$\dfrac{(\text{Departure})^2}{\text{length}} = \dfrac{D^2}{l}$:—

$$AB; \frac{(+641\cdot97)^2}{902\cdot6} = 456\cdot60$$

$$BC; \frac{(+722\cdot54)^2}{816\cdot4} = 639\cdot47$$

$$CD; \frac{(+150\cdot19)^2}{425\cdot5} = 53\cdot01$$

$$DE; \frac{(+604\cdot89)^2}{627\cdot9} = 582\cdot73$$

$$EF; \frac{(-1045\cdot93)^2}{1225\cdot3} = 892\cdot82$$

$$FA; \frac{(-1068\cdot5)^2}{1432\cdot2} = 797\cdot16$$

$$\Sigma\left(\frac{D^2}{l}\right) = 3421\cdot79$$

(iv) Substituting these values in equations 1 and 2, the values of λ_1 and λ_2 should be obtained.

$$\therefore \quad 2008 \cdot 09\lambda_1 + 354 \cdot 54\lambda_2 = +3 \cdot 28$$
$$354 \cdot 54\lambda_1 + 3421 \cdot 79\lambda_2 = -5 \cdot 16$$

From which we get $\lambda_1 = +0 \cdot 001935$; $\lambda_2 = -0 \cdot 0017085$.

(v) Correction to latitude :—

$$\text{By correction} = \lambda_1 \left(\frac{L^2}{l}\right) + \lambda_2 \left(\frac{LD}{l}\right)$$

From the known values of λ_1, λ_2, $\left(\frac{L^2}{l}\right)$, and $\left(\frac{LD}{l}\right)$, the corrections to the lines should be calculated.

	$\lambda_1 \left(\frac{L^2}{l}\right)$	$\lambda_2 \left(\frac{LD}{l}\right)$
AB ;	(+0 001935) (445 99) = +0 8630	(−0 0017085) (−451 26) = +0 7710
BC ;	(„) (176 93) = +0 3424	(„) (+336 36) = −0 5747
CD ;	(„) (372 48) = +0 7208	(„) (+140 52) = −0 2401
DE ;	(„) (45 18) = +0.0874	(„) (+162 27) = −0 2772
EF ;	(„) (332 48) = +0 6434	(„) (−544 84) = +0 9308
FA ;	(„) (635 03) = +1 2288	(„) (+711 49) = −1 2156

Hence the corrections to the latitudes of the sides are

AB ; correction	$= +1 \cdot 634$	DE ; correction	$= -0 \cdot 190$
BC ; „	$= -0 \cdot 232$	EF ; „	$= +1 \cdot 574$
CD ; „	$= +0 \cdot 481$	FA ; „	$= +0 \cdot 013$

Check :—The total correction = the algebraic sum of the corrections $= +3\ 28$.

(vi) Correction to departure :—

$$\text{By correction} = \lambda_1 \left(\frac{LD}{l}\right) + \lambda_2 \left(\frac{D^2}{l}\right).$$

	$\lambda_1 \left(\frac{LD}{l}\right)$	$\lambda_2 \left(\frac{D^2}{l}\right)$
AB ;	(+0·001935) (−451 26) = −0 8732	(−0 0017085) (456 60) = −0 7801
BC ;	(„) (+336 36) = +0 6509	(„) (639 47) = −1·0925
CD ;	(„) (+140 52) = +0·2719	(„) (53 01) = −0·0906
DE ;	(„) (+162 27) = +0 3140	(„) (582 73) = −0·9956
EF ;	(„) (−544 84) = −1 0543	(„) (892 82) = −1 5254
FA ;	(„) (+711 49) = +1 3768	(„) (797·16) = −1·3619

Hence the corrections to the departure of the lines are

AB, correction	= − 1 653	DE; correction	= − 0 682
BC, "	= − 0 442	EF, "	= − 2 580
CD, "	= + 0 182	FA, "	= + 0 015

Check —The total correction = the algebraic sum of the corrections = − 5 16

Applying the respective corrections to the latitudes and departures of the lines, we have

Line	Latitude	Correction	Corrected latitude	Departure	Correction	Corrected departure
AB	−634 47	+1 634	−632 836	+ 641 97	−1 653	+ 640 317
BC	+380 06	−0 232	−379 828	+ 722 54	−0 442	+ 722 098
CD	+398 11	+0 481	+398 591	+ 150 19	+0 182	+ 150 372
DE	+168 42	−0 190	+168 230	+ 604 89	−0 682	+ 604 208
EF	+638 27	+1 574	+639 844	−1045 93	−2 580	−1048 510
FA	−953 67	+0 013	−953 657	−1068 50	+0 015	−1068 485
Algebraic sum.	− 3 28	+3 28	0000 000	+ 5 16	−5 16	0000 000

PROBLEMS

1 Write short notes on :—

(a) Satellite station, (b) Phase of signal, (c) Closing the horizon, and (d) Axis Signal correction.

Adjust the following angles closing the horizon at a station :

(a) = 122° 05′ 58″ 6, weight 2
(b) = 86° 45 16″ 4 " 1
(c) = 72° 50′ 31″ 2 " 3
(d) = 78° 18 16″ 6 " 1 (U B)

(Ans. Corrections −0″ 50, −0″ 99, −0″.33, −0″.99)

2 What do you understand by 'a geodetic quadrilateral ?' Explain how you would adjust it by the approximate method. (U. B)

3 (a) you are asked to measure very accurately the horizontal angles in triangulation Describe the types of instruments you would use and the methods you would adopt to guarantee the requisite degree of accuracy What errors will be eliminated by each of the methods ?

(b) The following values were recorded for a triangle ABC, the individual measurements being uniformly precise

A = 62° 28 16″, 6 obs

B = 56° 44 36″, 8 obs

C = 60° 46 56″, 4 obs

Find the correct values of the angles (U B)

(Ans Corrections +3 69″, +2 77″ +5 54″)

4 Explain clearly any two of the following —

(a) Convergence of meridians,

(b) Corrections to be applied in base line measurements, and

(c) Phase of sun signals. (U B)

5 What is meant by the term 'side equation' ? State the equations of conditions in each of the following cases

(a) a polygon comprising the triangles having a common vertex

(b) a geodetic quadrilateral

Explain clearly the approximate method of adjustment of a geodetic quadrilateral (U B)

6 Explain the effects of curvature of the earth on surveys and derive an expression for convergence of meridians (U P)

7 What are the different "Triangulation systems" in a geodetical survey ? Which is the most accurate and why ? (U P)

8 State the equations of conditions that must be satisfied in the adjustment of the following figures

(i) a triangle with a central station, (ii) a polygon with a central station and (iii) a geodetic quadrilateral (U P)

9 Two points A and B have the following co-ordinates —

Point	Latitude	Longitude
A	44° 52′ N	42° 24′ E
B	45° 10′ N	43° 9′ E

Find the convergence of the meridians through A and B and the length of the side AB, assuming the earth to be a sphere with a radius of 20889000 ft Take cos 37 = 0 9999 (U P.)

(Ans 42 5806 miles)

10 (a) What is meant by "Convergence of Meridians" ? Derive an expression for the same.

(b) Determine the approximate increase in azimuth in a traverse which has total northings and eastings, each of 42,500 ft. from a station in latitude 59° 10′ N, given that the radius of the earth = 2,08 90 000 ft and log tan 1′ = 4 4637. (U. B)

(Ans 11′ 44″ 65.)

11. The following notes refer to a traverse survey for a proposed railway

Line	Length in km	Bearing
AB	16 09	N 75° E
BC	19 308	N 70° E
CD	24 135	N 65° E

The latitude of A was 50°N Determine the latitude of D and the correction which must be applied to the reduced bearing of CD at D to allow for convergence of meridians

Take 111 3428 km = 1° at the centre of the earth

(Ans. 50° 11′ 18″ N., 35′ 48″)

12 A traverse is run as follows —

Station	Length in ft.	Deflection Angle	Azimuth
A			42° from North
	19000		
B		32° R	
	20000		
C		43° L	
	21000		
D			

The latitude of A is 45° N Find the azimuth of CD at D and the latitude of D, given the following

Latitude	1″ of Latitude	1″ of Longitude
45° 0	101 2804 ft	71 8607 ft
45° 5	101 2819 ft	71 7566 ft

(U P)

(Ans 31° 7 5″ 85, 45° 6 5″ 83 N)

13 The angles of a geodetic triangle have been read each being weighted differently, and the length of one side is known Explain in correct sequence how you would compute the lengths of the remaining sides

14 The following is the data for three stations A, B and C as determined by triangulation

Line	Azimuth	Length in ft
AC	327° 7 49″	9011
CB	74° 56′ 52″	5795
BA	184° 25 52″	9099

A station P is established within the triangle ACB The angles CPB and BPA are measured and found to be 87° 38 and 144° 34′ respectively Determine the lengths and azimuths of PA, PB, and PC (U P)

(Ans PA 5601 ft, 169° 54′ 22″, PB 3937 ft, 25° 20 22″)
PC 4418 ft, 297° 42 22″)

15 (a) The elevation of an instrument at A is 219 3 ft Find the minimum height of signal required at B, 27 6 miles distant, where the elevation of the ground is 301 4 ft The intervening ground may be assumed to have a uniform elevation of 155 ft, and the line of sight must nowhere be less than 10 ft above the surface

(b) Find the most probable values of the angles A and B from the following observations at a station O,

A = 49° 48′ 36″ 6, weight
= 54° 37 48″.3, weight 3
A + B = 104° 26 28″.5, weight 4 (U B)

(Ans. (a) 47 ft., (b) Corrections + 1″.66, + 1″ 11, —0″ 83)

16. The following are the latitudes and longitudes of two stations

Station	Latitude	Longitude
A	38° 48′ 16″ N	68° 15′ 36″ E
B	39° 14′ 24″ N	68° 40′ 39″ E

Determine the angular convergency of the meridians through A and B
(Ans. 15′ 41″ 99)

17. Below are given the notes of part of a traverse in a preliminary survey

Line	Length in km	Bearing
AB	20 117	62° 30
BC	29 29	65° 24
CD	22 933	60° 48

The latitude of A = 45° 35′ N and the mean radius of the earth is 6366067 m. Find the increase in azimuth at the station D
(Ans 35′ 43″ 34)

18. Two stations A and B have the following co-ordinates

Station	Latitude	Longitude
A	46° 11′ 40″ N	86° 42′ 30″ E
B	46° 14′ 50″ N	86° 58′ 45″ E

Calculate (a) the length of the line AB, (b) the azimuth of AB at A, and the azimuth of BA at B, given the following values for the spheroid

Latitude	1″ of Latitude in m	1″ of Longitude in m
46° 10	30·8767	21 4396
46° 15	30 8771	21 4073

(Ans. (a) 21691 558 m (b) 74°12′ 38″ 85, (c) 254° 24′ 22″ 81)

19. The azimuth of a line AB 10929 616 m in length is N 50° 18′ W at A in latitude 50° 32′ 30″ N and longitude 90° 48′ 12″ E Determine (a) the latitude and longitude of B and (b) the reverse azimuth of AB at B given the following values for the spheroid

Latitude	1″ of Latitude in m	1″ of Longitude in m
50° 30′	30 9002	19 7100
50° 35′	30 9007	19 6756

(Ans (a) 50° 36′ 55″ 84 N 90° 39′ 48″ 2 E (b) 129° 35′ 30″ 81 clockwise from north)

20. The following traverse is run for a proposed railway :

Station	Length in m	Deflection Angles
A		
B	7485 89	12° L
C	6842 76	20° R
D	6367 27	

The latitude of A = 55° 1′ 15″ N and the azimuth of AB is 60° 36′ clockwise from north Obtain (a) the latitude and longitude of B, (b) the azimuth of CD, and (c) the azimuth of DC at D given the following values

Latitude	1' of Latitude in m	1' of Longitude in m
55° 0'	30 9242	17·7773
55° 5	30 9247	17 7405

(Ans (a) 55° 7 34' 72 N , 16 30' 83 east of A (b) 63° 49 32".441, (c) 243° 49 32" 41)

21 (a) What is meant by " convergence of meridians ?

Obtain the convergence of the meridians through A and B from the following data

Station	Latitude.	Longitude
A	47° 30' 20" N	116° 50' 12" W.
B	47° 54 40" N	117° 14' 6" W.

(b) The angles of a geodetic triangle were recorded as follows

A = 48° 20' 17"·2 weight 2

B = 63° 17' 32" 8 „ 1

C = 63° 22' 13" 4 „ 3

If the area of this triangle is 760 sq miles, adjust the angles A, B, and C, given that the spherical excess for 76 square miles is 1". (U. P.)

(Ans. (a) 17 40' 73; (b) 48° 20' 19", 63° 17' 36" 4, 63° 22' 14".6)

22 Find the convergence of meridians for (i) a departure of 45 06 km and (ii) a departure of 19696 18 m in a mean latitude of 59° 45'. Take R = 6366967 2 m

(Ans. (i) 41' 43" 21, (ii) 18' 14"·14.)

23 A line of levels was run from a bench mark A of R. L. 625 475 to a bench mark D of R.L. 640 520 to establish two intermediate points B and C with the following results.

	Observed level difference	Length in km.
A to B	− 6 345 m	12 5
B to C	+ 9 463 m	20
C to D	+11 492 m	25

Determine the most probable values of the reduced levels of B and C

(Ans R L. of B = 619 225 , R L. of C = 628 839)

24. In running a closed line of levels, the following results were obtained

B M.	Difference of level in m	Distance in km	Remarks.
A to B	+5 372	9	Elevation of A
B to C	−6 465	12	= 825 654
C to D	+7 216	18	
to	+4 138	15	
E to A	−1 727	6	

Calculate the most probable elevations of the bench marks

(Ans. A, 825 654 , B, 830 937 , C, 824 471 D, 831 603 , E, 827 407)

\+ + +

CHAPTER IX

HYDROGRAPHIC SURVEYING

Hydrographic surveying is that branch of surveying which deals with any body of still or running water, such as a lake, harbour, stream, or river It comprises all surveys made for (1) the determination of (i) shore lines, (ii) soundings, (iii) characteristics of the bottom, (iv) areas subject to scouring and silting, (v) depths available for navigation, (vi) velocity and characteristics of the flow of water, and (2) the location of buoys, lights, rocks, sand bars, etc

Control:— In hydrographic surveying the same mode of procedure is adopted as in topographic surveying except that the depths of water must be determined and the points on a body of water have to be located The first step in making a hydrographic survey is to establish control both horizontal and vertical In an extensive survey the primary horizontal control is established by triangulation and the secondary one by running a transit and tape traverse between the triangulation stations, the traverse lines being run to follow the shore line approximately In surveys of less extent the primary horizontal control only is required and is established by running a transit and tape traverse sufficiently close to the shore line For rough work the control may be established by running a transit and stadia traverse or plane table traverse Vertical control is based upon a series of bench marks established near the shore line by spirit levelling

Shore Line Survey—Having established the control, the next step is (i) to determine the shore line, (ii) to locate the shore details, prominent topographical features light houses, points of reference, and (iii) to determine the high and low water lines for average spring tides both in plan and elevation in the case of tidal waters All irregularities in the shore line, as well as the shore details are located by means of offsets measured with a tape from the traverse lines, by stadia or plane table The points of reference should be those which are clearly visible from the water

surface and which are near enough, such as church spires, windmills, flag poles, etc. Sometimes buoys anchored off the shore, and lighthouses are used as reference points and should be located by triangulation. The position of the high water line may be judged roughly from deposits and marks on rocks. However, in order to locate it with sufficient accuracy, the elevation of mean high water is determined and the points are located on the shore at that elevation as in direct method of contouring. The line connecting the points so obtained represents the high water line. Since the low water line is bare for a short time only, it is usually located by interpolation from soundings.

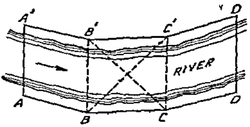


Fig. 194

River Surveys.—The survey of a shore line of a river is usually made by running a transit and tape traverse on the shore at a convenient distance from the edge of the water. The points where there is appreciable change in the direction of the shore line are then located by tape offsets from the traverse lines, by stadia or plane table methods. If the river is narrow, a single transit and tape traverse is run on one bank and both banks located by tacheometric or plane table method. If the river is wide, it is necessary to run traverses on both banks and locate each shore line by tacheometry or plane tabling from its traverse. For checking purposes, the two traverses should be tied to each other at intervals by cross bearings or angles as in Fig. 194. For example, stations B' and C' on the opposite bank are connected to the stations B and C by measuring the angles B'BC and C'BC when the instrument is at B and the angles BCB' and BCC' while the instrument is at C. From these angles and the measured length of BC, the length of B'C' may be computed. If it is in

close agreement with the measured length of B C', the figure BCC B is completely checked

If the river is too much crooked, no attempt need be made to follow it closely, but the traverse may be run in the most favourable location and subsidiary traverses run around the bends to locate the necessary details Where the shore lines of rivers and lakes are obscured by woods, it is not economical to locate them by traversing, but it will be found desirable to use a system of triangulation as in Fig 195

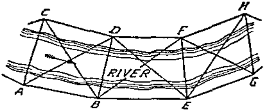


Fig 195

Thus in Figure 189 AB is the base line at the beginning of the survey, C, D, E F, etc are the triangulation stations As a check upon the survey a base line is measured at the end of the survey, and also, additional check base lines are measured at intervals of 10 or 15 miles

Soundings —The measurements of depths below the water surface are called *Soundings* The object of making soundings is to determine the configuration (or relief) of the bottom of the body of water This is done by measuring from a boat the depths of water at various points This operation of sounding is most commonly required in hydrographic surveying and is similar to that of levelling Soundings are required for (i) the preparation of charts for navigation, (ii) the determination of the quantity of the material dredged, and of the area where the material is to be dredged or where the dredged material may be dumped, and (iii) the design of works such as break waters, sea walls, wharves, etc

Since the elevation of the water surface which is taken as level surface of reference is continually varying in tidal waters, it is necessary to ascertain the water level at the time each sounding is made by taking tide gauge readings at regular intervals of

time during the period of soundings so that the observed soundings can be reduced to the datum

Gauges —The gauges may be divided into two classes

(1) non self registering and (2) self registering An observer is required to read the former while the latter are automatic, and are generally used when an accurate and continuous record of the fluctuations of the water surface is required There are various types of non self registering gauges viz (i) the staff gauge, (ii) the float gauge, and (iii) the chain or weight gauge. The gauge should be established at a convenient place where it is unaffected by the action of waves and sheltered from storms.

The Staff Gauge —The type of the gauge which is in most common use is the staff gauge shown in Fig 196 It consists of a board 15 cm to 25 cm wide painted white and graduated to metre and cm It should be of such a length that the readings at the highest and lowest tide can be taken. The graduations and figures are painted in black and are very bold so that they can be read from a distance It should be firmly fixed in a vertical position in the water and secured to a stationary object such as a quay wall pier, stake etc The zero of the gauge should be connected to a permanent bench mark on shore by levelling

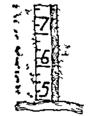


Fig 196

Datum —Mean sea level at a certain place is adopted as a datum for levelling and is accurately established by taking observations extending over a period of several years However, for ordinary purposes the observations extending over one lunar month will give sufficiently accurate results The levels of high and low water are read daily for one lunar month and the mean of an equal number of high and low water readings is taken as the value of mean sea level Knowing the gauge reading for mean sea level the elevation of the bench mark on shore may be determined

Equipment —The instruments required for taking soundings and their location are

Sounding Boat —The sounding boat should be sufficiently roomy and stable A flat bottomed boat is suitable in quiet

water, while round bottomed one is particularly convenient in rough water A power boat (steam or motor launch) is most suitable even when the wind is blowing and the currents are strong

Sounding Rods (or Poles) —The sounding rods or poles are convenient in shallow and smooth waters up to depths of about 4 to 6 m They are made of sound straight grained well seasoned tough timber and are circular in section about 5 cm in diameter, and usually 3 0 m to 7 5 m long For convenience in carrying, they are usually made in 1 m sections and are fitted at the lower end with an iron or lead shoe of sufficient weight to hold them upright in the water and to facilitate plunging and of sufficient area to prevent them from sinking into the mud or sand If samples of material of the bottom are required the shoe is provided with a cup shaped cavity which is smeared with tallow or grease to which the material will adhere The rods are painted white and graduated to m and cm the graduations being marked on two opposite faces for convenience in reading and the zero being at the bottom of the shoe

Lead Lines —The lead lines, also called sounding lines, are usually used for depths over about 6 m The lead line consists of a line of hemp cotton, or a brass chain having at its end a weight called a lead (because of that material of which it is made) The line of hemp or cotton is commonly used but is liable to stretching due to prolonged use and does not therefore maintain its length It is, therefore, necessary to stretch it thoroughly before it is graduated To do this the line is stretched tightly between two posts or coiled tightly around a tree or post It is then wetted thoroughly and allowed to dry This operation is repeated several times until there is no appreciable stretch The line is then thorougly wetted stretched taut and graduated to metres The zero of the graduations is at the bottom of the lead and each metre marked with a cloth or leather tag Each 1 m interval is marked with a tag of different colour, and each 5 m interval with a leather tag similar to the brass tag of the measuring chain The line should be kept dry when not in use but should be soaked in water for about an hour before it is used for taking soundings in order that it should assume its tested length It should be tested at frequent intervals by comparing it with a steel tape

Sounding Chain —For regular sounding, a brass sash-chain is most satisfactory, since its length is practically constant. The links are welded or brazed The brass tags are attached at 0 2 m intervals, but leather or cloth tags are preferable as the brass ones are likely to injure the hands of the leadsman. The chain should be tested periodically because of the wear of the links, and tags reset

Sounding Lead —The weight (Fig 197) attached to a lead line is conical in shape and varies from 2 5kg to 12 5kg depending upon the depth of water and the strength of the current For shallow still water a weight of 2 5 kg is sufficient For moderate depths up to about 10 m, in fairly quiet water a weight of 5 kg is satisfactory, while for greater depths and where the currents are strong a 10 kg weight will suffice The weight is circular in cross section, and its length about three to four times its average diameter, and slightly tapers towards the top end The line is attached to an eye fastened in the top

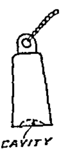


Fig 197

Sounding machine —A sounding machine is very useful when much sounding is to be done The type which is commonly used is hand driven and consists of (i) a piano wire carrying a 7 kg lead and wound round a drum, and (ii) two dials, the outer one indicating the depth in m and the inner one tenths of a m, connected to the drum by means of gears It is mounted in a sounding boat and can be used up to a maximum depth of 30 m.

Fathometer —For ocean soundings an instrument known as a fathometer is used It is an electric device and measures the time required for the sound (impulses) to travel to the bottom of the water and back

Signals —Shore signals are required to mark the ranges, i e lines along which soundings are to be taken, and the reference points to which angular observations are to be taken from a boat. They should be sufficiently conspicuous so that they are clearly visible for considerable distances The shore signal may be either 10 cm × 10 cm mast painted white and firmly braced at the bottom or 2 5 cm × 2 5 cm pole fitted with iron shoe The tripod signal commonly used in triangulation may used For angular observations, objects,

such as church spires, wind mills, lighthouses chimneys etc are used as signals The range signals should be white-washed or painted and should have flags fastened at their tops or discs nailed at their tops For identification they should be distinguished from each other by flags of different colours or by nailing strips of wood to form various geometrical figures (Fig 198a), such as a triangle, square, cross etc They are sometimes marked by nailing strips of wood arranged in the form of Roman numerals as shown in Fig 198b which serve to designate the number of the range when read laterally Sometimes it is required to place one of the range signals in the water If the water is shallow, the ordinary pole signal may be used But if it is deep, *buoys* are used as signals A buoy is a float made of light wood or a hollow air tight vessel properly weighted at the bottom and anchored in a vertical position by means of guy wires In the top of a buoy is bored a hole in which is inserted a short flag pole Temporary signals may be piles of stone, white-washed marks on rocks or poles

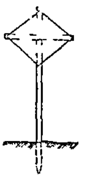


Fig 198 a

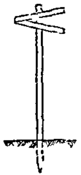


Fig 198b

The Sextant —The sextant is a portable and very accurate hand instrument It is mainly used for measuring angles from a boat in hydrographic surveying It is also used for astronomical observations and for measuring vertical angles Unlike the theodolite it measures the oblique angles when the observed objects are at different altitudes The sextant shown in Fig 199 consists essentially of (1) an index mirror or glass I rigidly fixed to a movable arm called an index arm The index arm rotates about a pivot placed at the centre of the graduated arc and carries a vernier reading to 10" at the end and is fitted with a clamp and

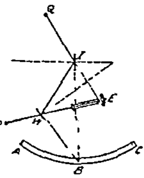


Fig 199

tangent screw (2) The horizon glass H the lower half of which is silvered and the upper half unsilvered Both index glass and horizon glass are perpendicular to the plane of the instrument and are parallel to each other when the index of the vernier is at the zero of the graduated arc (3) A telescope rigidly attached to the frame and pointing to the horizon glass (4) A graduated arc called the limb which is one sixth of circle (60°) The arc is divided into degrees and 10 minutes and measures angles upto 120° It is read by the vernier to 1 minute or 10 seconds (5) Coloured glasses which may be interposed when bright objects are sighted The sextant is identical in principle with the box sextant

Measuring Angles with the Sextant —Suppose it is required to measure a horizontal angle between two objects

(1) Hold the instrument by its handle in the right hand so that the plane of the limb coincides with the plane of the eye and the two objects (2) Look through the telescope and sight the left hand object directly through the unsilvered portion of the horizon glass (3) Move the index arm until the image of the right hand object seen in the silvered portion of the horizon glass is coincident with the object sighted directly Clamp the index arm and bring the two images into exact coincidence by means of the tangent screw (4) Read the vernier The vernier reading is then the required angle

It may be noted that unless the three points lie in a horizontal plane the observed angle is an oblique angle and not a true horizontal angle

Measuring Vertical Angles —On land an *artificial horizon* is required in observing the altitude of a celestial body (the sun or a star) It consists of a shallow vessel (tray) filled with mercury, water or oil At sea the visible (sea) horizon is sighted The altitude of the sun or a star is measured above the visible horizon In this case it is necessary to apply a correction for dip To observe the altitude of a celestial body (i) hold the instrument in the hand so that its arc lies in a vertical plane (ii) Bring the image of the celestial body as seen by reflection in the mirror into exact coincidence with its image viewed directly in the artificial horizon with the tangent screw (iii) Read the vernier The vernier reading is double the required altitude

Thus in Fig. 200, let AH be the surface of mercury; E the position of the eye; EK the horizontal line drawn through E; M the object; N its reflection in the mercury; MEN the observed angle (i. e. the angle subtended at the eye by the object and its image); KEM the true altitude (the required angle).

Since the distances EM and NM are very great as compared with EN, EM is parallel to NM. ∴ ANM = KEM. By the laws of reflection, ANM = HNE. Since AH and KE are both horizontal, HNE = KEN ∴ MEN = twice KEM.

Hence the observed angle MEN is double the angle KEM or the required altitude is half the observed angle.

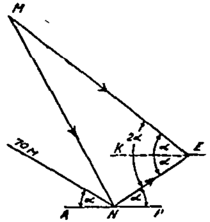


Fig. 200

Adjustments of the Sextant:—The adjustments of the sextant are : (1) To make the index glass perpendicular to the plane of the graduated arc.

(2) To make the horizon glass perpendicular to the plane of the graduated arc.

(3) To make the horizon glass parallel to the index glass when the vernier reads zero.

(4) To make the line of sight of the telescope parallel to the plane of the graduated arc.

The adjustments 2 and 3 may be made as for the box sextant.

Sounding Party —The personnel of the sounding party depends upon the method used in locating soundings When the soundings are located from the boat, the sounding party consists of

(1) The surveyor or the chief of the party He directs and supervises all operations sees that the boat is kept on the range and usually acts as signalman Sometimes he acts as an instrument man

(2) The instrument man who takes angular observations on the shore objects

(3) The recorder who books the soundings as they are called out by the leadsman, the results of angular of observations, and records the times when soundings are made

(4) The leadsman who makes the soundings and calls out the readings in feet and tenths to the recorder

(5) The boat crew comprising two or three experienced oarsmen to steer the boat and keep it on the range.

(6) The signalman who makes signals When the signal is to be given he holds up the flag for about 10 seconds and drops it suddenly at the instant the sounding is made.

When soundings are located by angular observations from the shore, one or two instrument men are required and stationed on the shore Prior to the commencement of the sounding work the instrument man should set his watch to correspond with that of the recorder and compare it at the close of the work. A staffman is added when soundings are located by stadia observations In tidal waters a gauge reader is stationed at the gauge to note the readings at 10 to 15 minutes intervals He must be reliable and should set his watch to agree with that of the recorder. If soundings are to be plotted as they are made, a draughtsman is added

Ranges —The lines on which soundings are taken, are called *ranges or range* lines They are laid on the shore paralled to each other and at right angles to the shore line or radiating from a prominent natural object such as a church spirea when the shore line is very irregular as shown in Fig 201 Each range line should be marked by means of signals erected at two points on it

which should be a considerable distance apart The positions of the signals defining ranges should be carefully located by direct measurement, stadia, or triangulation In the case of rivers or

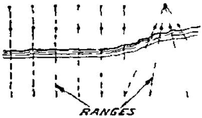


Fig 201

streams of great magnitude, the ranges are usually run at right angles to the axis of the stream, the signals being erected on either one bank or both banks The spacing of the range lines ranges from 6 m to 30 m depending upon the object of the survey and the nature of the bottom

Making the Soundings —Up to depths of 20 m, the soundings are usually made while the boat is in motion If the sounding is made by a sounding rod, the leadsman stands in the bow and plunges it in the forward direction far enough so that when it reaches the bottom, it will be in a vertical position He then reads the rod quickly to the nearest tenth of a metre and calls out the observed reading (depth of each sounding) to the recorder who repeats it and records it, and also the time and the number of the sounding The nature of the bottom is observed and recorded at intervals in the note book When the sounding line is used, the leadsman casts the lead forward at such a distance that the line will become vertical at the point where the sounding is to be taken when the lead reaches the bottom When the depth of water is less than about 9 m, the lead is withdrawn from the water after the reading is taken. But if the depth is greater, the lead is lifted between soundings just enough to clear the obstructions as the boat moves onward. If the water is very deep and still, soundings are taken by stopping the boat for each sounding For ordinary engineering purposes soundings are usually taken at 8 to 15 m intervals.

But for special work they may be taken at as close as 2 to 3 m interval

Methods of Locating Soundings —Soundings may be located by the following methods which are in most common use

(1) By transit and stadia, (2) by range and time intervals (3) by range and one angle from shore, (4) by range and one angle from boat (5) by two angles from shore, (6) by two angles from boat (7) by intersecting ranges, (8) by distances along a wire or rope stretched across a stream between stations, and (9) by cross rope

Location by Transit and Stadia —In this method a transit is set up at a point on the range and the stadia readings are taken on a stadia rod held on the bottom of the boat at the instant the sounding is taken. The transit station should be near the water level so that there will be no need to read vertical angles The transit may be set up at any shore point whose position has been previously fixed In this case, the azimuth must be observed and recorded In shallow waters the stadia rod may be dispensed with and the stadia readings taken on the sounding rod The method is rapid and sufficiently accurate but is suitable only in smooth and shallow waters It is unsuitable when soundings are taken far from shore Suppose AB is the range and B the transit station. P_1 P_2 P_3 etc are the points where soundings are taken Knowing the stadia intercepts the distances BP_1 BP_2 BP_3 etc may be calculated.

Location by Range and Time Intervals —In this method the sounding boat is rowed at a uniform speed along the range and the soundings are taken at regular intervals of time. The method is particularly applicable in still water and for short distances and when great accuracy is not required It is however, best used in conjunction with other methods In such a case, the first and last soundings on a line of soundings are located by angular observations from the shore The intermediate soundings are then located by interpolation according to time intervals.

Location by Range and One Angle from Shore—In this method the boat is kept on the range and the angle between the

base line and the boat is observed with a theodolite set up over one end of the base line, at the instant the sounding is made when a signal is given from the boat The instrument stations should be so chosen that the lines of sight will cut the ranges as nearly at right angles as practicable, and that their positions previously determined Thus in Fig 202, a theodolite is set up at A on the base line AB at right angles to the range, and with both plates clamped at zero the station B is bisected Loosening the upper plate the telescope is directed to the boat or leadsman The signalman in the boat raises the flag a few seconds before the sounding is taken to warn the instrument man to be ready and lowers it at the instant the sounding is made when the instrument man reads the angle ($\propto_1$) to the nearest 5 minutes, records it and the time in his note book The distance (BC) of the position of the sounding = $d \tan \propto_1$, where d is the perpendicular distance of A from the range line and $\propto_1$ the observed angle It is customary to locate every tenth sounding by an angle, the intermediate soundings being fixed by time intervals The method is useful and gives accurate results

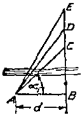


Fig 202

Location by Range and One Angle from Boat —In this method instead of measuring the angle from the shore, the angle between the range line and some shore signal of known position (Fig 203) is observed with a sextant from the boat The routine is the same as in the preceding method The distance of the position of the sounding at a is given by $d \cot \propto_1$, where d is the perpendicular distance of the shore signal from the range line The method is not in common use, since it increases the office work The only advantage which this method possesses is that there is better control over the entire work as the instrument-man is in the boat and all work is done under the direction of the surveyor

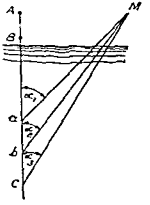


Fig 203

Location by Two Angles from Shore —In this method the position of a sounding is located by taking simultaneous angular observations to the boat with a theodolite from two shore stations. For this purpose, two instruments and two instrument men are required The instrument stations should be so chosen that the lines of sight will intersect as nearly at right angles as possible They should be previously connected to the shore traverse or triangulation system, and the distances between

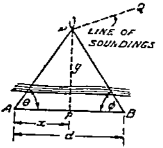


Fig 204

them should be accurately measured or determined by triangulation. Thus in Fig 204, A and B are the two instrument stations on shore The instrument is set up at each, with both plates clamped at zero, the instrument man at A bisects the station B. Similarly, the instrument man at B bisects the station A Unclamping the vernier plate, each one directs the telescope to the boat and follows the sounding rod or the lead line with the vertical cross hair When the signal flag in the boat is lowered, both men simultaneously read and record both the horizontal angle and the time The intersection of the two lines of sight determines the position of the sounding The co-ordinates of the position P of the sounding may be computed from the relations

$$x = \frac{d \tan \phi}{\tan \theta + \tan \phi} \quad \text{and} \quad y = \frac{d \tan \theta \tan \phi}{\tan \theta + \tan \phi}$$

It is very laborious to locate each sounding in this way Consequently, it is the general practice to locate the first and last soundings on the line of soundings in this manner, and the positions of intermediate soundings located by time intervals The method is convenient and gives sufficiently accurate results, if the work is done carefully It is commonly used where it is not possible to keep the boat exactly on a range, or where it is not convenient to set out the range lines on account of topography of the shore. The disadvantage of the method is that two instrument men are required on the shore

Location by Two Angles from Boat :—(Fig. 205.) In this method the positions of soundings are located by measuring two angles simultaneously with a sextant from the boat (P) to three shore signals, or any points (A, B, and C) whose positions have been previously known. The points sighted should be well defined, clearly visible, p minent natural objects, such as church spires, chimneys, lighthouses, flagstaffs, buoys, etc , but if they are not available, the range poles may be used. In this work it is important that the angles must be measured simultaneously and, therefore, observations are taken both by the surveyor and the instrument man (one observes the angle APB and the other the angle BPC). If the observations are taken by the surveyor alone, he should use two instruments in order that very little time is lost between two observations. The angles are read afterwards. In order to minimise the error in measuring the angles and plotting them, the nearer objects should be preferred to distant ones. This method is the application of the well known *three-point* problem and is commonly used where no ranges are employed.

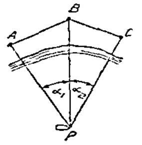


Fig. 205

Location by Intersecting Ranges :—In this method fixed ranges are so located on the shore that they intersect as nearly at right angles as practicable and are permanently marked by means of range signals. The boat is rowed to the several intersections of these ranges (Fig. 206), and the soundings are taken in the usual way. This method is used when repeated soundings are to be made at the same points at different periods to determine whether the bottom of the channel in a given place is silting or scouring, or to determine the quantity of material removed by dredging.

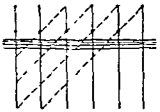


Fig 206

Location by Distanc along a Wire or Rope stretched cross Stream between Stations :—(Fig 207). In this method

a wire or rope is stretched taut between fixed points on opposite banks and is marked by means of cloth or metal tags at equal

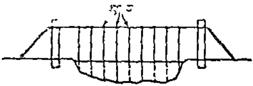


Fig 20"

intervals along the wire or rope The boat is rowed to these points and soundings are then taken This is the most accurate, but most expensive method It is used when soundings are to be taken along the cross sections of a canal or narrow river It is also used when it is required to determine the quantity of material removed by dredging, the soundings being taken before and after the dredging work is done If a contour survey of the river bed over a considerable distance is required a traverse is run along one bank, and at definite distances along the traverse lines sections are taken across the stream in suitable directions

Location by Cross Rope —In this method a steel strand wire rope with brass or leather tags fastened to it at intervals of 2 m to 5 m is stretched across the line of soundings, the zero end of the rope being secured to a spike on the range The reel boat proceeds along the line of soundings unwinding the rope as it moves The sounding boat is steered to each of the tags and the soundings are taken opposite each tag On the completion of the section, the sounding boat is rowed to the starting point of the next line and the reel boat moves back along the line, winding the rope This is the most accurate method and is well adapted to soundings in harbours and across rivers of less than about 400 m in width

Reduction of Soundings —The datum commonly adopted for reduction of soundings is the *mean low water* (*or mean level of low water*) of spring tides written as M L W S or L W O S T and all soundings are reduced to this datum This is done by applying gauge corrections algebraically to the observed soundings In tideless waters the correction is equal to the difference of level between the actual water surface and the datum, and is

constant, while in tidal waters the correction is not constant as the level of the water surface is constantly changing The amount of correction for each sounding may be determined by finding the difference between the appropriate gauge reading and the gauge reading of the datum The correction is positive if the value of the datum as indicated on the gauge is greater than the gauge reading and negative if it is less than the gauge reading.

Illustration —Let the gauge reading at 9 30 a m and 9 40 a m be 3 65 m and 3 75 m respectively, the gauge reading of the datum 1 5 m, the soundings 1 2 3 25 and 8 50 m at 9 35 a m

$$\text{The mean height at 9 30 a m} = \frac{3\ 65 + 3\ 75}{2} = 3\ 70\text{m}$$

The correction = − (3 70 − 1 50) = − 2 20 m

The reduced soundings are

$1 - 2\ 20 = -1\ 20$ m $\quad 2 - 2\ 20 = -0\ 20$ m,

$3\ 25 - 2\ 20 = +1\ 05$ m , $8\ 50 - 2\ 20 = +6\ 30$ m

The minus sign of the first reduced sounding indicates that the point is above the datum

Plotting Soundings —To begin with the shore survey is plotted on the plan The reference points instrument stations, range lines are then plotted Having plotted these control points the reduced soundings are plotted by means of the measured angles or distances the angles being plotted with a big size paper protractor The values of the reduced soundings are then written at the points which represent their positions and contours interpolated in the usual manner In addition to the contours the following information should be shown on the plan in conventional symbols

(i) Datum, (ii) High and low water lines (iii) Land features and lighthouses, buoys, etc

When soundings are located by two sextant angles from the boat their positions may be plotted as explained below

The Three-point Problem —Given three known points A,B, and C on the shore (Fig 205) and the values α_1 and α_2 of the angles APB and BPC subtended by them at the sounding boat P It is required to plot P The problem may be solved (i) mechanically, (ii) graphically, and (iii) analytically

Mechanical Solution —(1) The point P may be plotted very easily by the use of a *station pointer* shown in Fig 208 The

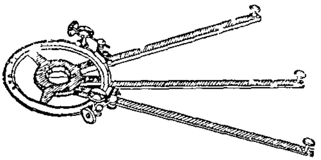


Fig 208

station pointer also called a three-arm protractor consists of (i) a circle graduated in both directions from 0° to 360°, and (ii) three arms radiating from a common centre which is the centre of the graduated circle The middle arm is fixed and its fiducial edge coincides with the zero of the graduations, while the other two arms are movable and can be revolved around the centre of the instrument They are fitted with verniers reading to one minute and also provided with clamps and tangent screws for accurate adjustment Lengthening pieces are supplied with the instrument to extend the arms To use the instrument, the left arm is accurately set at the observed angle α_1 by means of the vernier and then clamped Similarly, the right arm is set at the observed angle α_2 and then clamped The instrument is then moved over the plan until the bevelled edges of the three arms simultaneously pass through the plotted positions of the three points A, B, and C The centre of the instrument then locates the position of the required point P, which is marked on the plan with a pricker or a hard pencil Alternatively, the position of the required point P is obtained by the intersection of lines drawn along the edges of the arms

(2) *Tracing paper method* —The point P may be quickly plotted by the tracing paper method The observed angles APB (α_1) and BPC (α_2) are plotted on a piece of tracing paper The tracing paper is then placed on the plan and moved about until the lines PA, PB, and PC simultaneously pass through the

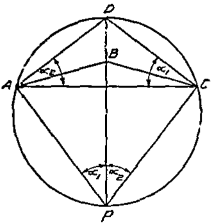


Fig. 209

plotted positions of A, B, and C respectively. The point P is then pricked through.

Graphical Solutions :—(a) In Fig. 209, let A, B, and C be three known points Join AC. At A draw a line AD, making an angle equal to α_2, and at C draw a line CD making an angle equal to α_1. Let D be the point of intersection of these two lines. Now draw a circle passing through A, D, and C. Join DB and produce it to cut the circle in P which gives the position of the required point P.

Proof :—$\angle APD = \angle ACD = \alpha_1$ and $\angle DPC = \angle DAC = \alpha_2$

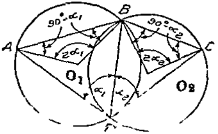


Fig 210

(b) In Fig. 210, A, B, and C are three known points. Join AB and BC. From A and B draw lines AO_1 and BO_1, each making an angle of $90° - \alpha_1$ with AB on the side towards P and

intersecting at O_1. Similarly at B and C draw lines BO_2 and CO_2 making angles with BC each equal to $90 - \alpha_2$ and intersecting at O. With centre O_1, describe a circle through A and B and with centre O_2 draw a circle through B and C. The point of intersection P of the two circles is the required point.

Proof —$\angle APB = \frac{1}{2} \angle AO_1B = \alpha_1$ $\angle BPC = \frac{1}{2} \angle BO_2C = \alpha_2$

(c) Join AB and BC (Fig 211). At B draw BE making an angle of $90° - \alpha_1$ and at A draw a perpendicular to AB, meeting BE at E. Similarly from B draw BD so that the angle CBD is equal to $90° - \alpha_2$. From C erect a perpendicular to CB cutting BD in D. Join ED and drop a perpendicular on ED from B. The foot of the perpendicular is then the required point P.

Proof —The quadrilaterals AEPB and BPDC being cyclic,

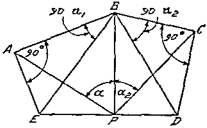


Fig 211

$\angle APB = \angle AEB = \alpha_1$ and $\angle BPC = \angle BDC = \alpha_2$

Note —(i) If the observed angles (α_1 and α_2) are obtuse the angles $\alpha_1 - 90°$ and $\alpha_2 - 90°$ should be set off on the side of AB or BC remote from P.

(ii) The problem is indeterminate when the point P of observation and the three observed points A, B, and C are concyclic.

Analytical Solution —The analytical solution is given on page 456

PROBLEMS

1. What is meant by "Soundings"? State the equipment and personnel for locating soundings How are they taken? Describe briefly the various methods of locating soundings (U P)

2. A river survey is to be conducted with a view to determine the bed level by means of soundings Explain how you would carry out this work and collect all the data for plotting the survey (L B)

3. A, B, and C are three points in a hydrographical survey and all these points are charted and visible Angles APB and BPC are observed with a sextant from a sounding boat at P Describe how you would plot the point P in the survey by each of the following three methods. (a) mechanical method (using an instrument), (b) analytical methods and (c) graphical construction method (U B)

4. The sides AB and BC of a triangle ABC with stations in clockwise order are 2001 m and 3144 m respectively and the angle ABC is 150° 24′ Outside this triangle, a station O is established, the stations B and O being on the opposite sides of AC The position of O is to be found by three point resection on A, B, and C, the angles AOB and BOC being respectively 24° 12 and 36° 6 Determine the distances OA and OC (U P)
(Ans OA = 4640 73 m , OB = 5228 23 m)

5 In order to locate the position (P) of a sounding boat, the angles APB and BPC subtended at P by three points A, B, and C on the shore were measured with a sextant and found to be 28° 42 40″ and 30° 28 20″ respectively, the points B and P being on opposite sides of AC The lengths of AB and BC scaled from a map were 918 m and 1074 m respectively, and the angle ABC was 60° 50′ 40″ Compute the distances PA, PB, and PC
(Ans PA = 1133 63 m , PB = 1733 28 m , PC = 880 13 m)

6 Observations were made with a sextant at a point P to three point A, B, and C on the shore, the point P being outside the triangle ABC and on the same side of AC as B

The observed angles APB and BPC were 28° 46 25″ and 47° 30 50″. The lengths of AB, BC, and CA were scaled from a map and found to be 1638 6, 2002 2, 2999 4 m respectively Find the distances of P from A, B, and C

(Ans PA = 2361 14 m , PB = 889 13 m , PC = 2192 32 m)

7. Below are given the co ordinates of three stations A, B, and C

Station	North co ordinate	East co-ordinate
A	5000	4000
B	9575	10360
C	5000	15580

In order to locate a secondary station P inside the triangle ABC, the angles APB and BPC were measured at P and found to be 130° 48′ 12 and 86° 32′ 48″ respectively. Determine the co ordinates of P.

Log AP = 3 786484;	Bearing of AP = N. 71° 20′ 20″·5	
Log BP = 3 451242,	,, of BP = S 22° 8′ 32″·5 W.	
Log CP = 3 785854,	,, of CP = N 71° 18′ 39″ 5 W.	
Latitude of AP = +1956 99,	Departure of AP = +5794 71	
,, of BP = −2618 00,	,, of BP = −1065 33	
,, of CP = +1956 99,	of CP = −5785 34	

(Ans North co ordinate of P = 6956 99, East co ordinate of P = 9794·68)

8. In the course of a hydrographical survey, an observer takes the sextant angles APB and BPC subtended at the boat P by the points A, B, and C on the shore, the points B and P being on the opposite sides of AC. The angles APB and BPC are found to be 36° 24′ and 48° 12′ rsespectively. The lengths of AB and BC are 984 m and 1339.5 m respectively. The angle ABC is 142° 36′. Determine the distances PA and PC.

(Ans PA = 1576·60 m., PC = 1695·32 m)

+ + +

CHAPTER X

TOPOGRAPHIC SURVEYING

By topography is meant the shape or configuration of the earth's surface, called the relief, together with the works constructed thereon by man Topographic surveying is the process of determining the positions, both in plan and elevation, of the natural and artificial features of a region, and delineating them by means of conventional symbols upon a map called a topographic map The distinguishing feature of a topographic survey is the location and sketching of contours A topographic map shows (1) the relief including hills and valleys (2) the natural features, such as streams, rivers, lakes, trees, etc , (3) the artificial features, such as roads, railways, canals, houses fences, cultivation, etc In topographic surveying methods of surveying (methods of horizontal location) are combined with methods of levelling and, therefore, every surveying instrument may be used to advantage in topographic work

Topographic maps are necessary and very valuable in the design and location of engineering projects, such as railways, highways, irrigation, watersupply, drainage, reservoir, etc. They are of great importance to the geologist, industrialist etc , and are of very great aid to the military commander for military operations in times of war Such maps are prepared by government organizations (In India by the Survey of India department)

The scales recommended range from 1 cm to 2 5 km (R. F $\frac{1}{2,50,000}$) to 1 cm to 0 25 km (R F $\frac{1}{25000}$)

The scale of the map depends mainly upon the purpose of the map and must be known before the field work is commenced, since the choice of the instruments and methods to be employed in order to ensure the desired degree of precision depends to a great extent upon the map scale

The position of a point in space is fixed by its three co-ordinates the two horizontal co-ordinates fix it in a horizontal plane, while the vertical one fixes it in a vertical plane

Representation of Relief —Relief may be represented on a map by hachures contours shading form lines or relief models However there are two general systems of representing the relief on a map viz (1) by hachures and (2) by contours or contour lines In the first system short lines called hachures are always drawn in the direction of the steepest slope The lines are fine and widely spaced for a gentle slope while for a steep slope they are thick and closely spaced This system gives the relative idea of the form of the ground but does not give the actual elevations of the surface of the ground On the other hand the contour lines not only give the relative idea of the topography, but also the actual elevations of the ground surface For this reason and because they have the widest use the system of representing the relief by the contours is the best and is in the most general use

Procedure —The primary object of a topographic survey is the preparation of a topographic map A topographic survey consists in locating a sufficient number of critical or representative points by means of three co-ordinates so as to enable the intervening surface of the ground to be known The field work may be done in the following steps (1) Establishing control both horizontal and vertical (2) Locating contours, and (3) Locating the details such as streams rivers roads railways, houses etc When the area of survey is small the entire work may be done simultaneously and by one party But in the case of extensive surveys it is usually done in correct sequence by several parties one party establishing horizontal control another party establishing vertical control while other parties locating contours and filling in the details

Horizontal Control —The purpose of establishing horizontal control points is to prevent excessive accumulation of error The control is established with such precision that the errors of their positions cannot be shown on the map The horizontal control forms the skeleton of the survey from which the contours and the details are located There are two general methods of esta

blishing the horizontal control (i e a system of control points located in plan) (1) Triangulation and (2) Traversing triangulation being the best and most accurate In very extensive surveys two systems of horizontal control are used —(i) primary and (ii) secondary, the primary control being usually established by triangulation But in flat and densely wooded country where triangulation is impracticable or very expensive the primary horizontal control may be established by precise traversing

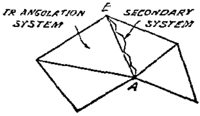


Fig 212

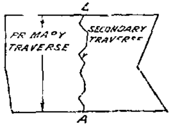


Fig 213

Sometimes a combination of the two is required To begin with, the primary control points or stations are established Within this system of primary control other stations are established with less precision forming the secondary control These secondary control stations from which the details are located are established by running transit and tape traverses each traverse starting from any one primary control station and closing on some other primary station as shown in Figs 212 and 213 In this way the framework on which the survey is built is developed so as to cover every portion of the ground Secondary traverses are sometimes run with the plane table On surveys of small areas only one system of control points is required and is established by traversing In topographical surveys of large areas the reference meridian commonly adopted is the true meridian, and it is established by means of astronomical observations The framework of the survey is then tied to the true meridian by measuring the angle between the true meridian and any main line of the survey

Vertical Control —The object of the vertical control is to determine the elevations of the primary control stations or to establish bench marks near them and at convenient intervals over

the entire area so that the levelling operations of the parties may be started from and ended on them, and also they may serve as reference points for future work On extensive surveys the primary vertical control is established by precise levelling, and the elevations of the primary stations located by triangulation are ascertained by trigonometrical levelling The secondary vertical control i e the elevations of traverse stations or bench marks near them is established by ordinary spirit levelling For rough work barometric levelling may be used The degree of accuracy desired in establishing primary vertical control varies from $4\sqrt{K}$ mm to $12\sqrt{K}$ mm according to the scale of the map and that for secondary control $12\sqrt{K}$ mm to $24\sqrt{K}$ mm where K is the distance in km

The following are the instruments and methods used for establishing horizontal as well as vertical control

Control	*Instrument*	*Method*
Horizontal Primary	Transit Transit and tape	Triangulation Precise traversing
Secondary	Transit and tape	Traversing by the fast needle method
Tertiary	Plane table Compass and chain	Traversing Traversing by the loose needle method
Vertical Primary	Transit Precise level	Trigonometric levelling Precise levelling
Secondary	Engineer s level Barometer	Ordinary Spirit levelling Barometric levelling

Locating Contours —There are two general methods of locating contours (1) Direct and (2) Indirect In the direct method the points on the contours (contour points) near enough together are found in the field and then located These points

are then plotted on the map and appropriate contours are drawn through them Enough points should be located on each contour so that accurate contours can be drawn by connecting the points when plotted on the map, since the accuracy of the contoured map depends upon the number and proper distribution of the selected points, which, in turn, depends upon the nature of the ground whether regular of irregular, and also upon the scale of the map At places of sharp curvature or abrupt changes in direction points should be close together while they should be farther apart when the ground is even or gently sloping Salient points on ridge lines and valley lines should be located This method, although most accurate is slow and tedious and is usually used (i) where great accuracy is required, e g close contouring of small areas, (ii) where a few contours are to be located, or (iii) where the ground surface is even or has gentle slopes

In the indirect method points at random (ground points) are located and contours interpolated after they are plotted In this method critical or representative points, i e points at which the ground surface changes its slope appreciably either in amount or direction, points on ridge and valley lines, are chosen and located The preceding remarks regarding the number and disposition of the points hold good in this case also If the scale is large points should be close together, while if it is small, they should be farther apart The indirect method is well adapted to locating contours when the ground is rough (very irregular) or when many contours are to be located It is in most general use In either method the elevations of the points (contour or ground points) are usually determined by spirit levelling, using the engineer s level, or hand level They are located with respect to the control stations by the angle and distance method The angles are measured with a transit (or graphically with a plane table) and the distances measured either with a tape or by stadia However, when the ground is too irregular, the transit-stadia method is well suited to locating contours and filling in the details

Methods of Locating Contours —The methods of locating contours are (1) the Direct location method, also known as the Trace contour method, (2) the Controlling point method,

(3) the method by Cross sections, also called the Cross profile-method, and (4) the method by Squares also termed, as the Checkerboard method

1 **Direct-location Method** —This method is commonly adopted on large scale surveys It is suitable when the topography is to be determined with considerable precision or when the contour interval is small In this method the plane table is commonly used for horizontal control (the transit may be used) and the engineer's level is used to determine the elevations of the contour points (i e points actually on the contours) The party comprises (i) a topographer, (ii) a levelman, (iii) a computer, (iv) two or more staffmen (v) one or more axemen, if required

Procedure —The plane table is set up at one of the control points which have been previously plotted on the plane table sheet, and is then properly oriented. Having set up the level at a convenient position the levelman finds the elevation of the plane of collimation (H I) by taking a backsight on the nearby bench mark He then obtains the staff reading required to locate the point on a given contour by deducting the elevation of the contour from the H I, and directs the staffman up or down the slope until the required reading is obtained The topographer immediately sights this point, draws a ray and plots it on the plane table sheet by scaling its distance from the plane table station The staffman then proceeds to another point on the same contour which is similarly located It may be noted that one contour is located at a time However, on rough ground, points on the next higher or lower contour are located The distances to the contour points from the instrument station may be determined by stadia or measured with the tape The contours may be located more rapidly but less accurately by means of a hand level

2 **Controlling-point Method** —In this method points are taken at random in the field and located with respect to the control stations Ground points on ridge and valley lines, tops and bottoms of slope representative points where the surface of the ground changes its slope either in direction or amount are chosen and located The instruments employed are the tacheometer (transit and stadia) the plane table, or both together

The topography party includes (i) a transitman, (ii) a recorder, (iii) two staffmen, and (iv) one or more axemen, if needed

(a) *By Transit and Stadia* —The transit is set up at either a primary or secondary control station and oriented by sighting on the nearest adjacent station The details in the neighbourhood of the station are located by measuring the angles, and the distances by stadia To locate a point, three observations are necessary : (i) the horizontal angle, (ii) the vertical angle, and (iii) the staff-readings of the top, middle, and bottom wires (or hairs) The observations taken on the detail points are termed as " side shots " The recorder enters the notes in the field book and describes all the points by appropriate remarks and sketches. Where the details are numerous, a draughtsman is stationed near the instrument and the points are plotted to a smaller scale than that of the map as they are located, and the topographical features are then sketched

(b) *By the Plane Table* —The instruments required are (i) the plane table (with telescopic alidade), (ii) a scale, pencil, and stadia tables. The topography party consists of (i) the plane-tabler, (ii) the computer, and (iii) two staffmen

Prior to the field work, the control points are plotted on the plane table sheet and the elevations of the bench marks are also recorded on the sheet.

The plane table is set up at a convenient station (either the primary or secondary control station) and oriented by taking a back sight on the nearest adjacent station The plane-table man then directs the staffman to the critical or representative point. He then sights the staff with the alidade, draws a ray, reads the vertical angle, and the three cross wires The computer now computes the distance and the elevation of the point He then plots the point by setting off to scale along the ray the distance as computed by the computer, and records the elevation near the plotted point Other points are similarly located Contours are then drawn Inaccessible points are located by intersection and their elevations determined by trigonometrical levelling The advantages of the plane table in locating the details are : (1) Since the points are plotted in the field, mistakes or omissions can be easily detected, and (2) the plane table can be set up at any advantageous station

and its position on the sheet determined by the solution of the three-point or two point problem Its elevation is determined by trigonometrical levelling To do this vertical angles are observed to signals of known height above the stations whose positions have been previously plotted on the sheet and whose elevations are known The horizontal distances from the instrument station to these three points are scaled from the plane table sheet The vertical angle should be observed on both faces to eliminate instrumental errors The computed differences of elevation should be corrected for curvature and refraction The elevation of the alidade is then computed from each of the three observations and the average of the three values is adopted as the elevation of the alidade The elevation of the instrument station is then obtained by subtracting the height of the alidade above the ground from the mean value of the elevation of the alidade (see Approximate method, page 55)

(c) *By Transit and Plane Table* —Both the transit and the plane table are advantageously employed when numerous details are to be located The topography party consists of (i) the transitman, (ii) the plane-table man, (iii) the computer, and (iv) two staffmen As before, the transit is set up at one of the control stations and oriented Similarly, the plane table is set up near transit station and properly oriented, its position being plotted on the sheet A staff is then held at the selected ground point The transitman then sights the staff, reads the vertical angle and all the three cross hairs With the alidade centred over the station point on the sheet the plane table man bisects the staff and draws a ray along the fiducial edge of the alidade The computer computes the distance and elevation of the point The plane table man then plots the distance to scale along the ray, thus locating the point, and records its elevation near the plotted point The advantage of the combined use of the transit and plane table is that the field work is more rapid

3 Method By Cross-Sections —This method is most commonly used for route surveys as well as surveys of a hilly country. The instruments required for the work are (i) the engineer's level, (ii) the hand level or the Abney level, (iii) two levelling staves, (iv) the steel or metallic tape and (v) the field book and sketch book

The topography party consists of (i) the topographer, (ii) two staffmen, (iii) two chainmen

The engineer's level is used for large scale surveys or for flat country, while in other cases, the hand level is used

Procedure —The traverse line is first staked at 30 m stations (called the Full Stations) and the elevations of these stations are determined by profile levelling Lines are set out at right angles to and on each side of the traverse line at each of the 30 m stations The representative (or critical) points, i e points of change in slope are chosen on the transverse lines and their elevations determined to the nearest tenth of a foot with either the level or the hand level The distances to these ground points are then measured with the tape Sometimes the contour points are determined on the transverse lines and are located in a similar manner For small scale maps the slopes of the ground surface may be determined by the Abney level and the distances to the ground points (critical points) by pacing

4 **Method By Squares** —This method is most suitable for large scale surveys and for areas of moderate extent The equipment consists of (i) the transit (ii) the engineer s level (iii) two levelling staves (iv) the steel tape and a number of stakes The topography party consists of (i) the topographer, (ii) the levelman, (iii) two staffmen, and (iv) two chainmen

In this method the area is divided into a series of squares (or rectangles), the sizes of the squares ranging from 3 m to 30 m side, depending upon the nature of the ground The squares are usually 15 m side

Procedure —A rectangle enclosing the area to be mapped is first set out with the transit and tape setting stakes at 30 m intervals, and a line of levels is run along the sides of the rectangle to determine the elevations of the ground at each of these stakes The area is further subdivided into 15 m or 30 m squares with stakes set at each corner The level is then set up at a convenient position and the elevation of the ground at each stake is determined Also the points of change in slope within the squares may be located on the diagonals or by measurements from the sides of the squares Other details such as roads, fences, etc should be located from the sides of the squares

Location of Details —The details are located from the nearest or most convenient stations by either linear or angular methods of locating objects

Roads streams fences buildings are located by offsets Irregular features indefinite lines, such as irregular roads shore lines banks of rivers are located by the method of intersection

Dam Surveys —The following surveys are necessary in connection with the design and construction of a dam

(1) *Triangulation Survey* —The purposes of the triangulation survey are (i) to establish control both horizontal and vertical for the topographic and hydrographic surveys, and (ii) to determine the length of the dam

A number of transit stations reference points and bench marks are accurately established both upstream and downstream of the dam Since the construction of the dam usually begins after some years, they should be permanently marked and carefully referenced so that they may be readily available for future use

(2) *Topographic Survey* (a) A topographic survey of the reservoir site is made with a view to determine the topography in detail To do this, the centre line of the dam is laid out on the ground and two lines are set out at right angles to and near each end of it The lines on which the cross sections of the

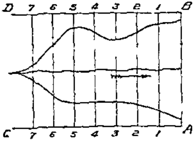


Fig 214

reservoir are to be taken are marked by measuring equal distances, usually one chain along both lines The lines so ranged are obviously parallel to each other and to the centre line of the

dam Levels are then taken on each of these lines in the usual way In addition, the cross sections are taken at intermediate points of appreciable change in slope Thus in Fig 214 AB is the centre line of the dam, AC and BD are the lines perpendicular to AB, 1 — 1, 2 — 2 3 — 3 etc are the cross sectional lines

(b) A topographic survey of the site of the dam is made by running a line of levels along the centre line of the dam and by taking cross sections along the dam site in the usual manner

(3) *Hydrographic Survey* —A hydrographic survey of the river is made over a sufficient distance along the river Extensive soundings and borings are taken to ascertain the character of the foundations

(4) *Property line Survey* —A property survey is made in order to determine the area submerged by the reservoir and the areas to be acquired from the individual owners

(5) *Route Survey* —Surveys for a road or railway are necessary to connect the site of the dam with the existing lines of communication

The shore line is marked by the direct location method (trace contour method) of contouring and stakes are set ad interval Monuments are set above the shore line by running traverse around the reservoir and also bench marks are established at points above the shore line The area to be flooded is determined by a planimeter and the capacity of the reservoir is calculated from the contour map by either the trapezoidal formula or prismoidal formula

+ + +

CHAPTER XI

ROUTE SURVEYS

Route surveys are surveys conducted along a narrow strip or belt of territory for the location and construction of transportation and communication lines, such as roads, railways, canals, pipe lines, transmission lines, etc They are conducted in the following series of surveys —(1) Reconnaissance, (2) Preliminary, (3) Location and (4) Construction Survey

(1) Reconnaissance Survey —A reconnaissance survey is rapid and rough but thorough survey or examination of the entire area covering various possible routes between selected termini with a view to ascertain the best general route and the approximate cost of the projected line It, therefore, involves the determination of all possible routes between termini and a very careful consideration as to which of the several routes may be advantageous and consequently, subjected in later surveys to a more careful, detailed and accurate study

The reconnaissance is the most important of the series of surveys conducted for the location of the line and forms the basis and the key to the project A very thorough and exhaustive examination of the whole area should, therefore, be made to ensure that no possible route has been overlooked The survey should not be restricted to special or easy routes only

A reconnaissance survey is not an *elementary* survey and mistakes made in the selection of route will ruin the entire project, and if not immediately detected and rectified before commencement of construction, the error is likely to continue for many years and the line will thus be put to serious disadvantage in relation to its competitors, this is specially so in the case of railways The reconnaissance must, therefore, be entrusted to a very experienced engineer who should be resourceful, unbiassed, painstaking, and gifted with engineering and business judgment as well as wide powers of observation and a natural aptitude for the job

To begin with, the reconnaissance engineer should secure the best available maps, (such as the Survey of India maps) of the territory under investigation, since they are of greatest help and study them carefully and sketch on them various possible routes With these maps in hand, the area under consideration should be traversed on foot, horse back, or automobile, and the various routes should be gone over and examined in detail in order to rightly assess their feasibility and relative merits The difficult routes should be gone over in both directions, because a route that does not appear to be feasible from one direction may be very feasible when approached from the other direction

The rapid observations for heights, distances, and gradients are made with an aneroid barometer, clinometer, pedometer, odometer, etc, and directions obtained by means of a compass. The field notes are usually recorded on the available maps and in a narrative form in a note book In the absence of maps, the results of this survey are made use of in preparing a rough topographic map of the area under investigation

In the course of reconnaissance, information should be collected on (1) the topography of the country, (2) possible ruling gradients (3) obligatory points, such as intermediate towns, markets, or production centres, saddles, river crossings, tunnel sites etc, and denied areas, (4) geological characteristics of the areas affecting foundations for bridges and stability of the line, (5) extent of waterway required for nalla and river crossings, (6) maximum flood levels, (7) availability of building materials and labour, (8) special structures, (9) number and length of more important bridges, (10) value of land, (11) in the case of railways, the total probable curvature, minimum probable radius and suitable sites for stations, (12) total length of the line, (13) probable amount of earthwork, and (14) the approximate cost of construction

The reconnaissance report should include (1) a summary of collected information, (2) a description of various alternative routes and a recommendation as to the feasible route or routes, (3) an approximate cost of construction, (4) time required for construction, (5) an analysis of economic values, (6) appended maps and photographs

As a result of reconnaissance only one or two routes will be selected as the most suitable and consequently deserving a further detailed study Now a days excellent topographic maps can be prepared from aerial photographs taken with special cameras and such maps are very valuable for the purpose of selecting routes in unexplored and unmapped regions They are invaluable in surveying inaccessible regions and forbidden property and supplementing in a few hours lack of data as well as bringing out any inaccuracy of available ground maps particularly in regard to topography which is of paramount importance in the location of highways and railways

(2) **Preliminary Survey** —A preliminary survey is a detailed instrumental examination or survey of a belt or narrow strip of country along a route or routes selected as a result of reconnaissance with a view to prepare an accurate topographic map of the belt of country along the selected route and thus arrive at a fairly close estimate of the cost of the projected line It therefore consists in fixing a series of straight lines as in open traversing along the selected route and determining with accuracy the various distances heights and angles in order to map out precisely the topography of a strip or belt of territory within which it is expected that the located line will lie The strip or belt should be sufficiently wide to embrace any possible variations in the position of the line as finally located this width is approximately 120 m for highways and 400 m for railways and depends very largely upon the character of the country

The preliminary survey is made with the same degree of precision as that required for the location survey It is a survey containing all data and details required for planning the paper location of the projected line Under this survey the actual setting out of the proposed alignment on the ground is *not at all required* In the case of railways the results of this survey are normally considered along with the results of the traffic survey in order to decide whether to build or not to build the line at all

The instruments generally employed for the preliminary survey are (1) a transit (2) an engineer s level (3) a hand level or an Abney level (4) 20 and 30 m tapes (5) two levelling staves (6) a plane table (7) subtense bar, etc

The survey work is done by *three* parties under the control of the locating engineer viz (1) the transit party (2) the level party and (3) the topography party On small projects the entire work is done by one party only

Transit Party —The transit party works under the direct control and instructions of the locating engineer who directs all movements and chooses the route for the survey The party consists of (i) a transitman who is the chief of the party (ii) two chainmen (iii) a flagman (iv) sufficient axemen to clear the line and set stakes and (v) sometimes a note keeper

The survey work consists in open traversing with a transit along the selected route The traverse lines are therefore run only approximately in the position of the finally located line all main stations being carefully marked with stakes In the case of highways the traverse is usually run by the method of deflection angles and in the case of railways by the method of back angles The azimuths of the first and last lines of the traverse are determined by astronomical observations and as a check upon the work in long traverses azimuths are taken at about 15 km apart or wherever a system of control points exists the traverse is tied into them Also observations are taken upon a prominent lateral object such as a church spire prominent tree gable end of a house etc from each of several traverse stations in order to check the accuracy of the work

Distances are measured with a 30 m steel tape and stakes are driven at 30 m intervals and at the ends of traverselines (by the method of lining in) All stakes are numbered and the chainage is carried forward continuously from the beginning of the survey which is designated as *zero* station and is expressed in terms of stations The 30 m stations are called *full* stations and any intermediate stations are called *plus* stations Stakes are also driven at intersections with streams roads railways etc and their chainages recorded No curves are introduced at this stage of the survey Each day s work is plotted at the close of the day to detect gross mistakes and omissions

Level Party —The levelling party comprising a levelman and one or two staffmen follows immediately behind the transit party and runs a longitudinal section of the traverse lines The

elevations of the ground at all stakes, points of change in slope, and at the intersections with roads, streams, railways, etc. are determined in the usual way. When a stream is intersected by the traverse line, the line of levels is run along the bed of the stream and the elevation of the water surface is determined; also, the high flood levels of streams are ascertained The levelling party establishes bench marks on objects, such as tree roots, rocks, etc along the line at about a 400 m intervals and describes their location in order to make them useful for subsequent work. Readings are taken to the nearest 0·01 m at full stations (except transit points) and at points of change in slope, and to the nearest 0 005 m at change points, transit stations, and bench marks The levelling work is checked by taking observations on existing permanent bench marks and G T S bench marks The profile of the ground along the traverse line is then plotted to a horizontal scale of 1 in 2000 m in railway surveys, and 1 in 1000 in highway surveys, the corresponding vertical scales being 1 in 200 and 1 in 100 Each days' work is plotted at the close of the day from the level notes recorded in the usual way

Topography Party —The topography party consisting of a topographer, two staffmen, and two chainmen follows the level party The duties of the party are (1) to locate the natural and artificial features, such as rivers, streams, buildings, property lines, roads, railways, etc, (2) to locate contours (3) to collect information in regard to the character of land, cultivation, excavation, rock outcrops, etc The artificial features are located by any of the several methods of locating objects from the traverse lines, and, if necessary, triangles and other figures are built upon the traverse lines to pick up the surface features The elevation of 30 m stations and transit stations are obtained from the level party At every 30 m station lines are set out at right angles to the traverse lines and on either side of it by an optical square These transverse lines extend far enough to cover the width of the belt of territory wherein the final location may lie. They are also run wherever the ground is rough and broken, and can be dispensed with in very flat country where any displacement of the line will not affect the profile to any large extent The cross sectional lines are usually 30 m, apart, but

in hilly and mountainous country they are as close together as 5 m and in level country as far apart as 90 to 150 m The contour points are directly located on the ground by means of a hand level, or the elevations of points of appreciable change in slope and their distances are determined and the contours are then interpolated

After the survey is over, a topographic map is prepared to a scale of 1 in 1000 to 1 in 4000 The contour interval is usually 1 m but it may be 0 5 m for flat ground and 2 m for steep ground

Transit-Stadia Method —This is the more modern method of making a preliminary survey It is rapid and economical, but less precise than the method just described It requires fewer men, but more experienced personnel especially in the transitman and recorder It is particularly suitable in an open country where clear sights can be obtained and the topography is very irregular, and is impracticable in wooded country The usual procedure in this method consists in (1) running a traverse by the fast needle method, (2) observing vertical angles and taking cross-hair readings, and (3) taking side shots to locate the details Thus the alignment, elevation and topographic details are carried out in one operation by a single party Hubs are set only at transit stations

Paper Location (*office location*) —After a careful and detailed study of the preliminary map and profile, and also of the configuration of the ground, the final location of the projected line is drawn on the map in the office by a trial and error method To do this, a tentative alignment of the route which appears to be the best is drawn on the map in pencil curves are drawn tangential to straight lengths by means of curve templates of known degrees of curvature, and a profile is prepared along this new line from contour levels on the map, and the grade line marked thereon in pencil This new line projected on the preliminary plan is called the '*paper location*" The line may be anywhere in the belt surveyed and in the most favourable position In choosing its position and the grade line, due consideration must be given to all features affecting location, such as (1) minimum gradients and curvature, (2) equalization of earthwork, (3) heavy earthwork, (4) expensive bridges and other structures, etc

The line and profile are further studied in order to find whether improvements can be effected by shifting the alignment and making changes wherever necessary. The profile is then modified and the grade line adjusted accordingly. This process is repeated until the most satisfactory location is obtained. In establishing the trial alignment on the preliminary map it is advisable to use a fine silk thread and needles in order to avoid excessive erasing of the pencil lines, only the final alignment and grade line being pencilled on the map and profile.

In highway work the ruling gradient usually adopted is 1 in 20 or 25, while in railway work the ruling gradient on the straight portion depends upon (i) the greatest train load to be hauled on the section, (ii) the least speed of ascent, and (iii) the design, power and weight of the standard locomotives in use on the section. For first class railways it varies from 1 in 125 to 1 in 200 in plain country and from 1 in 40 to 1 in 80 in hilly regions. The gradient is compensated on curves, the amount of compensation depending upon the gauge and the degree of the curve, for instance the amount of compensation is 0.04%, 0.03%, and 0.02% per degree of curvature for the broad (5' 6"), metre (3' 3⅜") and narrow (2' 6") gauges respectively in India. The maximum permissible degrees of curvature are 10°, 16°, and 40° for the broad, metre, and narrow gauges respectively. The minimum radius for highway curves is 30 m (100 ft) in undulating country, and 15 m (50 ft) in hilly country, sometimes hairpin bends having a radius of 11 m (37 ft) have to be adopted in difficult country.

(3) **Location Survey** (*Field location*).—The object of the location survey is to set out on the ground the alignment which has been finally decided upon, on the preliminary plan, to make minor improvements to the line as may appear desirable on the ground, and to fix up the final grades. What is usually set out is the centre line of the projected line.

The positions of the various points to be transferred on the ground are scaled from the preliminary map, using perpendicular offsets, intersections of the line with the main traverse, or angles and distances. Thus the tangents of the field location are set

out from points of intersection with the preliminary traverse or by chaining scaled offsets from the various stations or lines of the preliminary traverse Whenever practicable, the adjoining tangents are run to an intersection and the intersection angles are carefully measured and the curve notes computed Circular curves only are set out at this time

Stakes are driven at 30 m intervals and hubs are set at all intermediate transit stations intersection points of tangents, and tangent points of curves and referenced Transit notes and notes for curve details are recorded in a field book and all unprotant features, such as roads, streams property lines etc are sketched with reference to the finally located line on the right hand side of the page of the field book

Profile levels are then run over the located line and a suitable grade line is established on the profile, making such changes as may be desirable The line as finally located on the ground known as *field location*, is plotted on the preliminary map and the profile completed Vertical curves connecting grade lines are shown on the profile

Cross sections of the located line may be plotted from the date of the preliminary contour map in order to compute the approximate quantities of earthwork But usually the final cross-sections are taken while the slope stakes are being set

All important features in the close proximity of the located line as well as all points at which hubs are set and all bench marks are shown on the preliminary map The boundaries of private properties with names of owners are surveyed very accurately and monumented for purposes of acquisition of land and securing rights of way Also data are collected for designing and estimating culverts, bridges and other structures, if not already collected on previous surveys

(4) Construction Survey —The object of the construction survey is to set out the details of the project To begin with, the construction engineer goes over the located line finds the final location stakes, and checks them If some of the stakes be missing, he resets them from the field notes and the plan He checks all levels over the line and establishes additional bench

marks if required He then sets side slope stakes for earthwork as well as grade stakes, and stakes for culverts, bridges, etc The transition and vertical curves are then set out Borrow-pits are also staked out

Final cross sections are taken to determine the quantities of earthwork Soundings and trial borings are taken for important structures Important rivers are surveyed carefully and the waterway required for bridges determined The engineers maintains records of the progress of work and prepares drawings for various structures required for the projects Measurements of work done, and of materials and labour supplied by contractors are taken at regular intervals to ensure expeditious payment and good progress during construction

\+ + +

CHAPTER XII

CITY SURVEYING

By the term city surveying is meant an extensive co-ordinated survey of the area within the limits of a municipality made for the purposes of (1) making maps, (2) laying out plots and new streets, (3) constructing streets, pipe lines, sewers, buildings, (4) establishing and monumenting reference points and bench marks, (5) locating property lines, and (6) determining the topography of the land, etc

Such a survey is valuable and indispensable particularly when large-scale improvements, such as extensions to the existing street system, water supply and sewer systems lay out and construction of new roads, development of the area in or near a city, etc are under consideration The principles of city surveying do not materially differ from those of land surveying However, a relatively high degree of accuracy (or precision) is required in a city survey because of high value of land in an urban area The maps that are made in a city survey are (1) a topographic map generally drawn to a scale of 1 in 2000 (2) a property map usually drawn to a scale of 1 in 500 (3) a wall map to a scale of 1 in 20,000 and (4) an underground map usually drawn to a scale of 1 in 500

Horizontal Control —The first step in a city survey is to establish control both horizontal and vertical The primary horizontal control (i e primary control stations) is established by triangulation, precise traversing, or both The secondary control (i e secondary control stations) is established by running transit and tape traverses of the desired precision connecting the triangulation stations In small towns there is only one horizontal control, and it is established by precise traversing Thus the main skeleton consisting of a closed traverse or a series of closed traverses is established From the sides of these traverses the details, such as the outlines of streets,

gullies buildings etc are located In the triangulation for a city survey the sides are necessarily shorter in lengths, and objects, such as church spires, flag poles, tops of hills, chimneys, high buildings, water tanks etc are generally used as triangulation stations in order to form well shaped triangles and to avoid building high towers as far as possible to reduce the cost of the survey. Since the angles at such stations cannot be determined by direct measurement, they are obtained by reduction to centre The triangulation system consists of either quadrilaterals or polygons with a central station The angles are measured with either a direction instrument or a repeating instrument reading to 10" More usually they are measured by the method of repetition, 5 or 6 sets consisting of six repetitions with the telescope direct and equal number of repetitions with the telescope reversed being taken to ensure the required precision Because of shorter lengths of the sides of the triangles the signals should be exactly centred over the station marks Astronomical observations for azimuth should be made at two or more stations In the survey of a city it is customary to refer all points to the plane co-ordinate system Some triangulation station within the area is chosen as the origin and the true meridian through this point and the line at right angles to it are taken as the axes of co-ordinates All reference points are plotted by means of plane rectangular co-ordinates with reference to these axes The azimuths of the initial line and some intermediate lines should be determined by an astronomical observation The advantage of the co-ordinate system is that in case a point of known co ordinates is lost it may be readily and precisely replaced by means of the co ordinates of other points The entire triangulation system is adjusted by the method of least squares The precision required in triangulation is that the average error of closure of a triangle should not exceed 1 5" and the maximum one should not exceed 5" In a primary traverse the average angular error of closure should not exceed $4\sqrt{N}$ seconds and the maximum error should not exceed $6\sqrt{N}$ seconds where N is the number of the sides of the traverse In a secondary traverse, the corresponding figures are $10\sqrt{N}$ seconds and $15\sqrt{N}$ seconds The maximum error of closure in a primary traverse should not exceed 1 in 20 000 (for large-sized city work), in a secondary traverse it

should not exceed 1 in 10,000 (for medium sized city work), while in tertiary traverse it should not exceed 1 in 5000 (for small town work)

Vertical Control —The next step is to establish vertical control The purpose of vertical control is to establish bench marks at convenient intervals all over the area The first order bench marks are established by precise levelling at about half a km or so apart on permanent objects, such as walls of buildings, abutments, piers, parapet walls of bridges etc y running levels in closed circuits, the permissible error of closure b ing 4 $\sqrt{K}$ mm where K is the length of the closed circuit in kilometres The entire level net (consisting of several closed circuits) is adjusted by the method of least squares to determine the most probable values of the elevations of the bench marks Additional bench marks are established at traverse stations and also on permanent objects, such as curb stones, gate pillars fire hydrants, plinths and walls of buildings, etc by running a line of second order levels, beginning from any rsfit order bench mark and closing on some other first order bench mark or beginning from and ending on such a bench mark, the maximum closing error being 8 $\sqrt{K}$ mm where K is the length of the line in kilometres

Equipment —The instruments used in a city survey are (1) a direction instrument reading to seconds or a transit reading to 10" for the measurement of angles (2) a precise level for establishing first order bench marks, (3) a dumpy level for ordinary levelling work, (4) one standardized invar tape for measuring a base line and check bases, and the sides of a primary traverse, (5) a standardized 30 m steel tape graduated to mm provided with a thermometer scale, or compensatory handles, instead, a light 30 m steel tape provided with tension handles, for making the linear measurements, (6) steel tripods for supporting the tape, (7) three thermometers for the measurement of the actual temperature of the tape (8) a plane table, compass and 15 m steel tape for filling in the details

In some cities a Standard of Length (usually 30 m) is established in some convenient place The field tape (i e the tape used in the field for taking the linear measurements) should

be frequently compared with the *City Standard of length* or with the standardized tape, if it is not available One or more tapes should be standardized and kept in the office for the sole purpose of checking the field tape.

Monuments —It is the general practice to define street lines by establishing by traversing a system of reference points at street intersections, street corners, angle points, and curve points, and then to monument them By the term *monument* is meant an object placed to mark the established reference point The monuments should be of a permanent character They may be (1) iron pipes with a bronze spherical cap embedded in a concrete post about 30 cm or more in diameter, the exact point being marked with a drill hole or a cross etched in the top of the plug, and (2) stone or concrete posts 15 cm square and about 1 m long, the exact point being marked with a drill-hole, metal plug, or cross in the top Very often the hole is filled with lead, and a copper nail inserted in it defines the exact point. Sometimes a copper bolt is set in lead and the exact point is marked by means of a cross scratched on it The monuments should not be placed at the intersections of the centre lines of the streets as they are likely to be disturbed by street traffic or street repairs They are usually set in the sidewalk nearly flush with the surface of the sidewalk at a standard offset distance from the street lines The monuments should be carefully referenced to permanent and well defined nearby objects, such as building corners manhole covers, firehydrants, etc the measurements being accurately recorded by means of sketches The plane rectangular co-odinates of the monuments and their elevations should be determined

Topographic Map —The topographic map of a city covers the area of the city as well as the areas in the immediate vicinity of it The map is usually drawn to a scale of 1 in 2000 and is divided into sheets The topography is shown by means of contours in brown The natural and artificial features are shown in the usual colours Streams, rivers, canals, ponds, lakes, etc are shown in blue Streets, lanes, railways, culverts, bridges, monuments, bench marks, boundaries of public properties, property lines, etc are shown in black Wooded areas, public pro-

perties, gardens, parks, etc are shown in green The names of streets, buildings as well as plot numbers are also shown in black.

The plane table is the most satisfactory instrument from the *point of accuracy and cost, provided it is manipulated by well* trained and experienced topographers, and is invariably used in making a topographic survey Prior to field work, the primary and secondary control points, i e the triangulation and traverse stations are accurately plotted on the plane table sheets by means of rectangular co ordinates Whenever necessary, additional control points are established by running plane table traverses, which are tied in to the control stations These tertiary traverses are adjusted graphically and the details are then located in the usual manner by the methods of radiation and intersection

Property Map —The property map is usaually drawn to a scale of 1 in 500 and is divided into sheets It shows (i) the lengths and bearings of all street lines, lane lines, (ii) the boundaries of public property, public buildings, private buildings, (iii) rail roads, bridges, parks, streams, rivers, etc, (iv) streets, their widths and intersections (v) the co-ordinates of the control stations and the monuments, (vi) the co-ordinates of all angle, curve, and intersection points of the street lines, (vii) the names of streets, public buildings, parks, rivers, lakes etc, and (viii) all bench marks

Wall Map —The wall map covers the entire area and is drawn to a scale of 1 in 20 000 It shows the same information as the topographic map and is reproduced from the topographic map by photographic methods of reduction

Underground Map —The underground map is drawn to a scale of 1 in 500 and the size of the sheets is the same as that for the property map It shows (a) streets lanes, with their widths easement lines, (b) monuments, bench marks, surface structures such as rail roads, pavements, footpaths, curbs, transmission line poles, trees, etc, (c) underground structures, such as subways, tunnels, etc, (d) sewers, water pipes, gas mains, electric conduits, and other utilit es, different colours and symbols being used to represent them, (e) sizes and percentages of grades of sewers, pipes etc and the elevations of the critical points, and (f) natural features interfering with underground construction

City Property Survey —The objects of the property survey are (1) to collect all available recorded information regarding the property, (2) to locate the street lines which form the boundaries of the public property, (3) to locate all intersections angle points and curve points of the streets and to monument them, and (4) to determine the co-ordinates of the monuments For accurate work, the theodolite is a necessity and it is used for measuring the angles between the survey lines of the traverse. The survey work is done in two steps (1) Surveying the streets and the main frontages of buildings only (and not railings, compound walls pavements etc) and (2) Locating the details such as outlines of buildings gullies pavements gardens manholes, fences etc

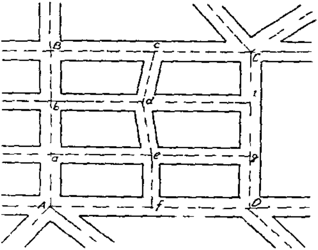


Fig 215

Procedure —To begin with the traverse lines are run as long as possible and as nearly as possible in the centre of streets by fixing stations at the junctions of all principal streets as shown in Fig 215 The intersections of the side streets with the main streets are lined in with a theodolite and from these intermediate points subsidiary lines are run to locate the side streets If the side street connecting two main streets is straight, it may be fixed by simply running a line along it and measuring it accurately But if the side street is not straight, and if it

consists of two lines, it may be fixed by measuring an angle and distance from one main line and a distance from the other main line If the side street consists of three lines the angles and distances from both main lines are necessary to locate it Thus in Fig 215, ABCD is the main traverse and a, b c, d, e, etc are intermediate stations lined in in order to fix the side streets.

Sometimes it is found advantageous to mark the traverse lines on the side walks parallel to the street lines

Marking Stations —All stations are carefully marked by driving in iron nails or spikes with their tops slightly below the surface of the road, the exact position being marked in the top of the nail or spike with a centre punch If the stations are on the pavements they are marked by means of a cross scratched in its surface Each of the stations should be carefully referenced by three measurements taken to the nearby corners of the buildings or other well defined and permanent objects the measurements being taken to the nearest mm with a steel tape These measurements should be carefully recorded by means of sketches in the field book the object being to restore the station in case the spike is knocked out

The angles between the traverse lines are then accurately measured at the main stations and at stations where there is a change in the direction of the main road by repetition, the lengths of the lines are carefully measured to the nearest tenth of a foot with a steel tape The main frontages of the buildings only are located by the method of angle and distance Corners of the streets bends angles of buildings are located by means of triangulated offsets or by perpendicular offsets and checked by check ties the offsets being measured to the nearest 3 mm to 5 mm The field notes are recorded in the field book in the usual way The best time for main survey work is in the early morning before the streets are crowded or during the night when the traffic is suspended The measurement of the quieter side streets and location of the details may be done during the remaining day

Location of Details —The location of the details is a very tedious and laborious work It is best accomplished by means of a plane table the main skeleton being plotted on the plane

table sheets Tacheometric plane tabling is well adapted to surveying the details, such as the outlines of buildings gardens, courtyards, fences, pavements, passages in front and in the rear, gullies, manholes, lamp posts, etc Another method of locating the details is by tape measurements with reference to the main frontage lines

There are two systems of keeping the detail notes In the first system the blocks of the main survey are plotted to a scale of 1 in 300 to 1 in 500 on separate sheets about 50 cm square The sheets are then mounted on a light board and the details are filled in thereon by plotting each measurement as taken. In the other system the details are sketched in by hand on the sheets and the measurements recorded on the sketch Plotting of the details is then done in the office in the usual way.

CHAPTER XIII

SETTING OUT WORKS

Setting out a Building —The object of setting out a building is to clearly define the outline of excavation on the ground for the guidance of the contractor It would be of little use, if

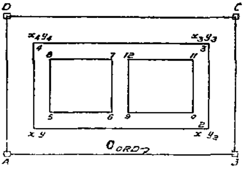


Fig 216 a

stakes are set at the exact position of each of the corners of the building, since they would be dug out as the work of excavating the foundations proceeds The best method is therefore, to set out a reference rectangle outside the limits of excavation say, about 5 m from the building line as shown in Fig 216 a so that the reference pegs A, B C and D will not be disturbed during excavation and then to locate each corner by means of co ordinates with reference to the sides of this rectangle

The contractor is usually supplied with a blue print of the plan of the foundations of the building (a plan giving the necessary dimensions for foundations) The co-ordinates of all the corners should be shown in a tabular form on this plan

The equipment required for the work consists of (i) a 30 m steel tape, (ii) two 15 m metallic tapes, (iii) a hammer, (iv) a plumb bob (v) stakes, (vi) iron and wire nails and (vii) a cord.

Procedure —Two stakes A and B are accurately driven at the required distance apart (14 5 m) A cord is then stretched, the

ends being secured to the wire nails driven in the centre of the stakes At A is set out a line perpendicular to AB, the right angle being set out with the tape by the 3–4–5 method On this line a stake is driven at D at a distance equal to the length of AD (7·6 m), and the work checked by measuring the diagonal BD and comparing it with its calculated length The error, if any, should be corrected Similar procedure is followed at B to set the stake at C As a check, the diagonal AC is measured The distance CD should now be exactly equal to the distance AB A cord is then passed round the periphery of the rectangle ABCD. Having set out the reference rectangle, each corner is fixed by measuring its co ordinates from the sides of the reference rectangle, e g. corner 1 is fixed by measuring its co ordinates x_1 and y_1 from AB and AD, and a stake is then driven in to mark its exact position When all the corners have been staked, a cord should be passed round the periphery of the figures 1234, 5678, etc and the outline of the foundations marked with lime spread along these lines outside the cord

Alternative method —In this method instead of the circumscribing rectangle, the rectangle formed by the centre lines of

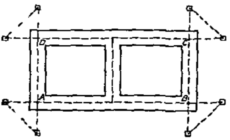


Fig 216 b

the outside walls of the building as shown in Fig 216 b is set out accurately with the tape as a reference rectangle and the corners located by measuring their co ordinates with reference to the sides of this rectangle Since the stakes put in at A, B, C, and D will be lost as the excavation proceeds reference stakes should be established on the prolongation of the sides of this rectangle well back from the work and in positions where they will remain

undisturbed (say, about 1·5 m from the building line) They should be protected by standing a drain pipe over each of them.

If the ground is uneven, the required points may be transferred to the ground by the use of a plumb bob

In the case of extensive or important buildings, a theodolite is invariably used in setting out right angles.

In English practice the batter-board method is used.

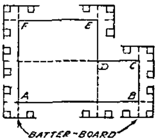


Fig 216 c

Temporary stakes are accurately set at all corners of the building, the entire work being checked by measuring the diagonals Batter-boards are then set at each end of each outside building line about 1 m outside the excavation as shown in Fig 216 c

The batter boards are 2 5 cm × 10 cm strips nailed to 5 cm × 10 cm posts which are well driven into the ground The top edges of all the batter boards are usually set at the same level. Sometimes, however, on account of the slope of the ground, it is convenient to set them at some whole number of metres above the bottom of the excavation Nails are driven in the tops of these boards on the prolongation of the building lines which are given by a theodolite. The building lines are defined by stretching a cord or wire between the nails in opposite batter boards. Reference stakes should be set on the main lines of the building in positions where they will not be disturbed as indicated, so that any batter-board, which may accidentally be disturbed, may be easily replaced. Bench marks should be established in convenient positions around the site of the work. They should be set well away from the site, so that they may remain undisturbed until the work is

completed. A stout stake with a round-headed nail driven into its top and embedded in the centre of a 60 cm square block of concrete is a convenient form of the bench mark. The head of the nail defines the elevation. It should be protected by means of a piece of drain pipe embedded in the concrete.

Setting out a Culvert —The best practical method of setting

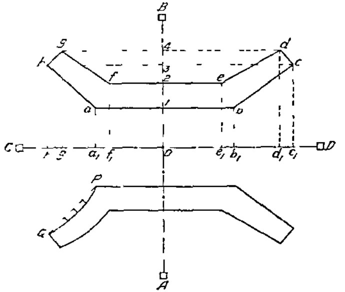


Fig 217

out the culvert foundations is to locate the corners of the abutments and wing walls by means of their co-ordinates with reference to the centre lines of a road or railway, and the nalla crossed, which are taken as the axes of co-ordinates, the origin being at the centre of the culvert. The engineer is provided with a tracing of the plan of the foundations and on this plan the co-ordinates of each of the corners of the abutments and wing walls should be indicated in a tabular form

Thus in Fig 217, AB and CD are respectively the centre lines of the road and the nalla, passing through the centre O of the culvert. The co-ordinates of a are $1a$ and a_1a; of b, $1b$ and b_1b; of d, $4d$ and d_1d, and so on

Procedure :—(i) Drive a peg at O and set up a theodolite over it Line-in carefully as many points as may be necessary on the line AOB (a few points will do in the case of fairly level ground, but a fairly large number of points will be required on rough ground), and fix chaining arrows or perforated pegs at these points The cord passing through the eyes of the arrows or through the holes in the pegs defines the line AB

(ii) Set out the line CD at right angles to AB and carefully fix as many points as may be necessary and fix arrows at these points

(iii) Next stretch cords along the lines AB and CD Set off the distances O1, O2 O3, etc on AB and Oa_1 Ob_1, Oc_1, etc. on CD, and fix arrows at these points

(iv) Now take two tapes and put the rings together Direct the chainman to pull them tight while the tapes are held by the engineer and his assistant at the arrow 1 on OB and at the arrow b_1 on OB with the respective readings $1b$ and b_1b, thus fixing the position of b which is then marked with a peg

(v) Similarly, fix other points by their co ordinates and drive pegs at each point

(vi) Fix the corners of the other abutment and wing walls in a similar manner

(vii) Pass a cord around the periphery of each abutment and two wing walls as *abcdefgh* and mark the outline of the foundations by nicking, i e cutting a narrow trench along this line

(viii) Take levels at all pegs for the purpose of determining depths of excavation and estimating the correct quantity of excavation

If the wing wall is curved, the points on the curve may be set out by offsets to the chord PQ as indicated both offsets and distances along PQ being scaled from the plan A similar procedure is followed in setting out bridge foundations When the bridge is skew the procedure is exactly similar except that the lines AOB and COD are set out at the angle between the centre lines of the road or railway and the nalla

Setting out Abutments.—Having laid the lines AOB and COD the corners of the abutments are accurately set out with a steel tape, the procedure being the same as explained above.

Bridge Survey.—A topographic survey of the bridge site and approaches to the bridge is required for long and important bridges the transit stadia method being employed for making the survey. The results of the survey are plotted to a scale of 1 in 1000 and the contours interpolated at intervals of 1 to 2 m according to the nature of the ground. The following details should be indicated on this topographic map:

(1) The north and south line (2) The name of the river and the direction of the flow of water, (3) The name of the nearest town at either end of the bridge (4) The width of the roadway over the bridge (5) The width of the existing road approaching the bridge (6) The radius of curvature of the approach road (7) The reference, description, and elevation of the bench mark used as a datum, and the ground levels along either bank for a distance of about 150 m both upstream and downstream (8) The low water level, the ordinary flood level and the highest flood level (9) The catchment area, the maximum discharge and the maximum velocity at the bridge site (10) The results of trial pits and borings, etc

Locating Piers for a Bridge.—The problem is a two fold one (1) to determine accurately the length of the center line of the bridge i e the distance between a point on one bank of a river and a point on the other bank both points being on the centre line of a road or railway and (2) to locate the central point for each pier

(1) The length of the axis (or the centre line) of a short bridge may be measured directly with the steel tape The tape should be standardized or compared with the standard one The procedure to be followed in measuring the length is similar to that followed in measuring the length of a base line of tertiary triangulation

In the case of a long bridge, the length is usually determined by triangulation

First Method —Thus in Fig 218, A and B are two points on opposite banks on the centre line of the road, AC is the base

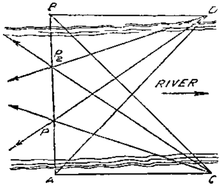


Fig 218

line laid out exactly perpendicular to the axis of the bridge AB. The base line is carefully measured and the angle ACB is accurately measured by the method of repetition the number of repetitions being six 3 with the telescope direct and 3 with the telescope reversed Then $AB = AC \tan \theta$

In this method there is no check on the work Another base line AL is therefore laid out as before on the same bank, but upstream The length of AB is then computed from the measured length of AD and the measured angle AEB The mean of the two values if in close agreement is taken as the true value of AB

Second Method —In this method a base line is laid out on each bank exactly at right angles to the centre line (axis) of the bridge extending each side of the centre line (*Fig* 219) The length of the centre line (AB) is found by triangulation as explained in the preceding method and the positions of the centre points (1 and 2) of the piers on the centre line are then determined.

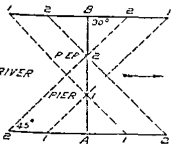


Fig 219

The procedure for locating them is as follows

axis (AB) of the bridge, and fixed the positions of the central points P_1 and P_2 of the piers (Fig 220), on the plan, the angles ADP_1 ADP_2, BCP_1, and BCP_2 are calculated from the known lengths AD, AP_1, P_1P_2, and BP_2, and BC, and from the known angles BAD, and ABC To locate the point P_1, transits are set up over A and D, the instrument at A is then directed to B, and the angle ADP_1 is turned off at D By simultaneous sighting, P_1 is established The intersection of the line of sight DP_1 with the line of sight AB along the centre line of the bridge locates the point P_1 To check the location of P_1 the transit is set up at C and the angle BCP_1 is set off The line of sight CP_1 should now pass through P_1 almost exactly A similar procedure is adopted for locating the point P_2 If the corners of the pier are to be established the corresponding angles at A and D are computed and laid off at A and D and the points established in a similar manner

For convenience targets should be set at P_1' and P_2' on the farther bank by turning off the angles ADP_1 and ADP_2 about 10 to 20 times Similarly, targets should be established at P''_1 and P''_2 on the near bank by turning off the angles P_1CB and P_2CB so that the intersecting lines may be established whenever required without turning off the angles

Setting out Tunnels —Tunnels are constructed in order (i) to meet the requirements of rapid transportation in big cities (ii) to connect by the shortest route two terminals separated by a mountain or ridge on a projecting spur, (iii) to reduce grades as in the case of "development" of a line (iv) to avoid the excessive cost of maintenance of an open cut subjected to land slides, snow drifts and avalanches, (v) to avoid the expensive acquisition of valuable built up land, tearing up pavements and holding up traffic for long periods in large cities and (vi) when the depth of ordinary cutting exceeds 20 m and the ground rises rapidly for a considerable distance afterwards The chief considerations in the location of a tunnel are (i) that it should follow the best line adapted to the proposed traffic (ii) that it should be most economical in construction and operation and also (iii) convenience of ingress and egress A very careful study of the actual topography of the tunnel site is necessary in order to

select the best alignment for a proposed tunnel Tunnels being very expensive, they must as far as practicable be avoided. Utmost care and experience are necessary in deciding upon the final alignment

The setting out of a tunnel comprises four operations —

(1) Surface survey or setting out,

(2) The connection of surface and underground surveys,

(3) Setting out underground,

(4) Levels in tunnels

(1) **Surface Survey** —(a) A preliminary survey should be carried out by means of a transit and stadia for at least three to four km on either side of the suggested alignment and plotted to a small scale, say, 1 in 20,000 with contours drawn at 5 m intervals (b) From this plan, the final alignment may be selected and a detailed survey should follow the selected alignment as closely as possible over the hill

The surface survey also includes a very detailed study of the geological structure of the land as the cost of tunnelling depends upon the nature of the materials to be encountered

The proposed route having been decided upon, the following points require consideration —

(a) Alignment of the centre line of the tunnel

(b) Gradient to be adopted

(c) Determination of the exact length of the tunnel

(d) Establishment of permanent stations marking the line.

Instruments for Setting out Tunnels —(1) *Theodolite* — Some of the longest tunnels have been set out with a theodolite reading to 20 seconds However, a micrometer theodolite reading to 5 or 10 seconds is preferable When the sights are long (about 5 to 6 km), a special transit known as a tunnel transit is most suitable Its essential features are : (i) The telescope can be rotated in the vertical plane only It cannot be used for measuring horizontal angles (ii) The telescope is fairly long, the size of the object glass varies from 4 cm to 9 cm, the focal length from 60 cm to 90 cm, and the power of the

eyepiece is 30 or 40 (iii) One of the trunnions is perforated for illuminating the cross hairs by means of a lamp placed on the standard (iv) It has a cast iron base fitted with levelling screws (v) The trunnion axis is fitted with a vertical circle with verniers reading to 20 seconds (vi) It is light and can be conveniently carried over mountains Such an instrument was used in setting out the Totley tunnel

(2) *Tape* —A 30 m steel tape is required for (i) measuring the centre line of the tunnel marked on the surface, (ii) transferring the levels underground, and also, (iii) measuring the sides of the traverse connecting the two ends of the tunnel

(3) *Tripod* —Two tripods fitted with aluminium caps are needed for supporting the tape

Surface Alignment —Tunnels are always driven from each

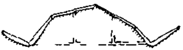


Fig 221

end if they are short, but if they are long tunnels are frequently driven from each end, and from one or more intermediate shafts It is, therefore necessary to set out the centre line of a tunnel very accurately on the surface from end to end If the whole of the centre line cannot be marked, it must at least be set out over the contiguous shafts near its ends Since the surface of the ground at the tunnel site is very steep and rough, refined instruments and refined methods of observation are required in order to avoid inaccuracies Also experience, care, and patience are necessary for the observer If there is only a single peak or ridge from which both ends of the tunnel are visible, the method of *balancing in* (page 227, Part I) is repeatedly applied until a straight line between the ends of the tunnel is obtained Permanent stations are then located at all salient points with great accuracy on this line to fix the direction of the centre line on either side of the ridge The line is then extended beyond each end of the tunnel (Fig 221), as it is advisable that two stations should be visible from each terminal station (or end of the tunnel)

to guard against any disturbance of the station marks Observatories are then erected at suitable places

Observatories —An *observatory* consists of (i) two brick, stone, or concrete pillars connected with a stone or concrete cap into which a metal plate is fixed This forms the support for the instrument (ii) an independent platform entirely surrounding the pillars and carrying a roof at the top for the observing party By this arrangement the vibration caused by the movements of the observer or by the action of the wind is not transmitted to the instrument The centre line is then marked very accurately on the metal plate

Having marked the centre line on the surface the exact horizontal distance between the two terminals (or ends) of the tunnel must be ascertained This may be done by measuring on the surface with a steel tape the tension being applied with a spring balance When the slopes are steep and the ground is rough very accurate results are obtained by the use of tripod fitted with aluminium caps the transit being used to line in the tripod heads The grades of the surface or tripod heads are determined either by levelling or with a theodolite The corrections for absolute length temperature tension grade and sag are applied in the usual way in order to obtain the true horizontal length of the centre line

In many cases it is not possible to run the centre line over the surface because of the obstacles In such cases the length and direction of the centre line (or axis) of the tunnel must be obtained by traversing or triangulation For example in tunnelling under towns the centre line cannot be set out on the surface In which case the length and direction of the centre line (or the axis) of the tunnel are ascertained by running a closed traverse between the terminals the traverse stations being marked on the metal plates let into the curbstones The co-ordinates of the stations are then calculated The chainage of the terminal and intermediate stations and also the direction of the centre line can then be computed Similarly in tunnelling under a river or a high cliff the surface alignment is impossible in which case the length and direction of the centre line are determined by

triangulation Also, the surface alignment is established by a system of triangulation tied to the ends of the tunnel and to the shafts when the tunnel is very long as in mountainous country, in which case it is not possible to measure the lengths of the sides of the traverse with the desired accuracy The setting out of the Alpine tunnels was effected by triangulation

Formation Level of a Tunnel —Tunnels are usually on a grade The highest point should be as near to the centre as possible Even when the tunnel is level it is always necessary to give a slight uniform gradient to the formation for drainage purposes If the tunnel is short it is given in one direction only But if it is long the gradient is in both directions starting from the centre towards both ends

Shafts —One or more vertical shafts (openings) are frequently sunk on the centre line to facilitate construction by providing two additional working faces the tunnel being driven in both directions from the foot of shaft They are usually lined with brick work and are useful for checking the accuracy of the alignment and levels and also for ventilation If the tunnel is very deep shafts will not be economical

Curves in Tunnels —Tunnels are generally made straight unless curves are absolutely necessary Wherever possible, curves should be avoided in tunnel work as they add greatly to the cost It is desirable though not essential to set out the curves on the surface If this is not practicable the positions of the tangent points are located exactly by making enough measurements on the surface The curves are usually set out underground in the usual way by the method of deflection angles or by the method of offsets from tangents

Setting out from the Ends —Once the centre line is established the setting out of the centre line from each end is a very simple operation To do this the theodolite is set up at the permanent terminal station outside the portal (or terminal) A back sight is then taken on two visible stations which have been already fixed on the line The instrument is then directed up the tunnel and a permanent mark made usually in the roof, or in the floor by the method of *double reversing* (i e taking

double face observations and averaging the results) The process is repeated as the work proceeds until it is found necessary for visibility to set up the transit inside the tunnel The instrument is then transferred to one of the marks and the line is prolonged in a similar manner. The centre line may be marked on nails or dogs set in wooden plugs driven into the holes drilled in the roof, the exact position being marked with a centre punch or file mark

Transferring the Alignment Underground —The operation of transferring the alignment from the surface to the bottom of a shaft is the most difficult one and requires the highest skill and greatest care, since the shaft is always small (3 to 5 m in diameter) and a short line is to be produced several thousand metres. The method usually adopted is briefly described as follows·

Two timber beams (baulks) are fixed across the top of the shaft near its edges at right angles to the direction of the tunnel and as far apart as possible A theodolite is set up over one of the stations previously established on the centre line of the tunnel and accurately sighted on the other station. The centre line is then very carefully set on the beams (preferably on the plates fixed to the beams and drilled with holes for suspending wires) by the method of lining in, taking a large number of observations on both faces and averaging the results From these points two long fine piano wires with heavy plumb bobs attached to their lower ends are then suspended down the shaft, the wires being stretched tight by the plumb-bobs weighing 10 to 15 kg as shown in Fig 222 The plumb-bobs are suspended in a pail of water or oil to damp their oscillations as much as possible. They are also fitted with projecting vanes to prevent rotation. Great care must be taken that the wires and plumb bobs are hanging free. As a check, the distance between the wires at the top and at the bottom of the shaft should be carefully measured, which should, of course, agree. The line joining the two wires gives the direction of the alignment underground.

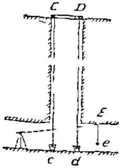


Fig. 222.

In order to prolong the line at the bottom, the theodolite is transferred to the bottom of the shaft and set up by trial *exactly* in line with both wires. In order that both wires can be observed the special device used is that the farther wire is made thicker than the nearer one and a white card placed between the two wires as in Fig. 223. This operation requires greatest care and patience in the manipulation of the instrument because of the peculiar underground conditions.

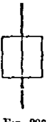


Fig. 223

Having set up the theodolite *exactly* in the plane of wires, the alignment is marked on the nails or dogs set in the wooden plugs driven into the holes drilled in the roof by taking double face observations, the exact point being marked with a centre punch or a file mark. The plumb-bobs or lamps may be suspended from these nails. The marks may be made in the brass nails set in the stout stakes driven in the floor. The stakes should be surrounded by brickwork plastered over with cement flush with the top of the stake. These marks serve as instrument stations. When the tunnels are steel lined or brick lined, the alignment is marked on nails driven into the timber wedges which are driven between the joints of the segments or in the brickwork.

Underground Sights.—The various kinds of illuminated signal used for sighting underground are: (1) A plumb line seen against a white background of a sheet of oiled paper illuminated from behind by a lamp. This is most convenient when the sights are short.

(2) Carriage candles fixed in a weighted frame. They are useful when the sights are long.

(3) An Argand oil lamp of 40 or 50 candle-power in a metal frame. It is most suitable for very long sights.

(4) A plummet lamp, or a plumb-line illuminated from behind or a vertical illuminated slit for sighting on floor stations.

Alternative Method of Connecting Surface and Underground Surveys.—In this method, also known as the method of *Weisbach Triangle*, the theodolite is set up at A very nearly in line with the two wires C and D so that the angle CAD is a few minutes (Fig. 224). The angle CAD is then measured very

accurately on both faces, and also the distances CA and DA.

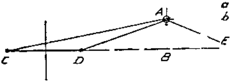


Fig 224

As a check, the distance CD is measured (all measurements must be most accurate) The angles CAE and DAE which the lines CA and DA make with any other line AE respectively, are also observed in order to check the angle CAD which is equal to the difference between the two angles The deviation BA of the station A from the vertical plane of CD is then calculated The angles DCA and CAD being very small CD may be taken equal to CA − DA The angle DCA may be calculated from the relation $\sin DCA = \dfrac{AD}{CD} \sin CAD$

Now $BA = CA \sin DCA = CA \times DCA$ (in circular measure).

A line A*b* parallel to the centre line CD is set out by taking a backsight on C, then plunging the telescope and finally turning off the deflection angle *a*A*b* (= DCA) from CA produced By shifting the mark so fixed by an amount equal to BA the centre line may be marked in the roof

Levels —Levelling along the surface alignment (or line) is done in the usual way. Before starting the work, the level should be carefully tested and adjusted Wherever possible, a longitudinal section of the whole surface alignment is taken very accurately and the bench marks are established at the ends of the tunnel and also near each shaft Care should be taken that the sights are not too long and the backsight and foresight distances are nearly equalised It is advisable to leave two bench marks near each end of the tunnel and at each shaft in order to guard against any disturbance of the bench marks and to detect it The levels of the bench marks must be carefully checked by levelling in the reverse direction In very mountainous country the usual method of levelling cannot be employed However the difference of level between the ends of the tunnel may be determined by trigonometrical levelling when executing the triangulation

Transferring Levels Underground:—The levels are transferred underground at the ends of the tunnel without any difficulty by levelling from the nearest bench mark previously established in the usual way But at the shafts the difficulty arises. However, the levels are transferred down the vertical shafts by means of steel bands, chains, or rods

Procedure —To begin with, a mark is made near the top of one of the guides in which the cage travels and its elevation is accurately determined by levelling from the bench mark previously established near the top of the shaft An assistant holds the zero end of the steel tape to the mark, and the engineer descends the shaft in the cage (or skip) and marks the position of the lower end of the tape on the guide care being taken to stretch the tape vertically The assistant then descends the shaft and holds his end on the mark made by the engineer The engineer makes another mark on the guide at the other end of the tape The process is repeated until the bottom of the shaft is reached, where the level of some temporary mark is obtained Temporary platforms must be placed in the shaft at 20 m or 30 m intervals for the use of the assistant and the engineer; otherwise they are carried on seats fixed to the winding rope. When the wire rope guides are used, measurements are made on the side of the shaft in which case care must be taken to see that the tape is held quite vertical

In the case of very deep shafts, the vertical measurement is changed to the horizontal one by the arrangement shown in Fig 225 A fine steel wire loaded with a weight of 5 to 15 kg passes over a pulley at the top of the shaft from a windlass It is then lowered down the shaft, care being taken to see that the wire is in contact with the horizontal threads AA and BB (Fig 225) stretched at the top and bottom of the shaft respectively The points of contact are then marked on the wire with a piece of chalk, the exact point being marked with a pencil The wire still loaded is wound up and stretched on the surface of the horizontal planks The distance between the marks on the wire is then accurately measured with a steel tape Since the wire is under a constant

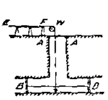


Fig 225

tension throughout the whole operation, there is no need to apply the correction for elongation The method is rapid, convenient, and very accurate

Underground Bench Marks —Having determined the elevation (R L) of the underground mark (or point) by the vertical measurement, the level is set up near the bottom of the shaft and a bench mark is established on the top of a stake driven firmly into the ground or on the side of the tunnel If the tunnel is in rock, the permanent bench marks are established on flat projecting portions of the rock the exact positions being indicated by suitable marks cut into them (a short piece of drill steel grouted in a hole drilled in the side of a wall) In the case of a brick lined tunnel, the bench marks are established on the iron spikes or wedges driven into joints of the brickwork in the side of the tunnel at a convenient height, while in the case of an iron-lined tunnel, they are established on the flanges of the segments of the lining As the work progresses the levels are carried forward by fixing new bench marks from time to time so that one or more bench marks may be kept near the working face For checking the levels a line of levels is run through the tunnel from end to end after the headings from the two ends meet As the headings are driven forward a certain distance ahead of the lining any small error that may be detected in the levels may be allowed for (i e adjusted) by putting in a junction gradient "

Accuracy of Tunnel Surveying —A very high degree of precision is necessary in tunnel surveying as there is no way to check up the work until the tunnel is driven through It is said that the headings should meet on a dime ' In actual cases the error both in alignment and level is very much less than the width of a dime The permissible error in line in (railway) tunnels is about 2 to 3 cm If the headings do not meet, and if the error is appreciable, it will be necessary to introduce a very flat curve (sometimes a reverse curve) into the alignment at their junction, and also to widen the tunnel to accommodate the curve, resulting in increased cost and permanent annoyance The error in alignment of the principal tunnels constructed in the past ranged from 1 cm to 10 cm while that in level from 0 02 cm to 5 cm

CHAPTER XIV

PHOTOGRAPHIC SURVEYING

Photographic Surveying, also called photogrammetry, is a method of surveying in which plans or maps are prepared from photographs taken at suitable camera stations. Photogrammetry may be divided into two classes (1) terrestrial or ground photogrammetry and (2) aerial photogrammetry. In the former maps are prepared from the terrestrial (or ground) photographs while in the latter they are produced by the use of aerial photographs (photographs taken from the air) The terrestrial photographic surveying is regarded as a farther development of plane table surveying Another system which is a comparatively recent development of photographic surveying is known as stereo-photogrammetry or stereo-photographic surveying This system consists in taking photographs in pairs at the two ends of a base line of known length and direction with the vertical planes of collimation of the cameras at right angles to the base line.

Photographic surveying is suitable for small scale mapping of open hilly or mountainous countries It is well adapted to topographical or preliminary surveys It is not suitable for flat or wooded country, in which case aerial surveying can be used to advantage Aerial surveying is used with great success for reconnaissance and preliminary surveys for roads, railways, transmission lines etc, for surveys of buildings, towns, harbours, etc It is also particularly suitable for inaccessible regions, forbidden properties, unhealthy malarial regions, etc.

Photo-Theodolite —The photo-theodolite is a combination of a camera and theodolite, and is used for taking photographs and measuring the angles which the vertical plane of collimation makes with the base line The photo-theodolite designed by Mr. Bridges-Lee consists essentially of—

(1) A camera of the fixed focus type The focal length of the lens should be 15 cm or more The camera is mounted on the axis in the same manner as vernier plate of the a theodolite

(2) A vertical frame inside the box It carries a pair of hair lines one vertical and the other horizontal These wires, being pressed tightly against the sensitive plate are photographed on the photographic plate The intersection of these two hair lines (or cross wires) is exactly opposite to the optical centre of the lens The line of collimation is defined by the line joining the intersection of the cross hairs to the optical centre of the lens.

(3) A horizontal transparent tangent scale attached to the frame across its rear side

(4) A circular magnetic compass mounted on the base of the frame Its needle carries a vertical cylindrical transparent scale graduated to 30 minutes The magnetic bearing of the principal vertical plane (i e the vertical plane containing the optical axis) is given by the reading of the scale at its intersection with the vertical hair on the photograph

(5) A telescope mounted on the top of the camera box and capable of rotating on an horizontal axis It is fitted with a vertical arc with verniers clamp and slow motion screw The line of sight is in the same vertical plane as the optical axis of the camera lens

(6) A graduated horizontal circle carrying verniers reading to single minutes and fitted with clamp and tangent screw below the camera

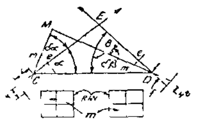
(7) A levelling head similar to that of a theodolite (i e parallel plates with three levelling screws)

(8) One or two long sensitive bubbles mounted on the top of the camera box for levelling purposes

The instrument is supported on a tripod

Principle of the Method of Terrestrial Photogrammetry —The principle underlying this method is exactly similar to that of plane table surveying It may be stated

as The position of an object with reference to the base line is given by the intersection of the rays drawn to it from each end of the base line However there is a difference between the two methods In plane tabling most of the work is executed in the field while in this method it is done in the office The principle is explained as follows



F ° 6

In Fig 226 let

C and D	=	the camera stations
CD	=	the base line of length b
CE and DE	=	the positions of the vertical plane of collimation (i e the vertical plane containing the optical axis)
α and β	=	the observed angles ECD and EDC, which the vertical plane of collimation makes with the base line at C and D respectively
M	=	the point to be located which is shown as m on both prints
x_1 and y_1	=	the distances of the point m from the vertical and horizontal hairs measured on the print at C respectively
x_2 and y_2	=	the distances of the points m from the vertical and horizontal hairs measured on the print at D respectively
f	=	the focal length of the camera lens

The point M may be plotted graphically as well as analytically

Graphical Method —(i) First plot the base line to the given scale. Draw CE making an angle of α with CD with the help of a protractor. Similarly draw DE making an angle of β with DC.

(ii) On CE mark the point e at a distance equal to f in front of C. Similarly set off a distance De equal to f along DE in front of D as shown in the figure.

(iii) Through these points e and e draw lines at right angles to CE and DE respectively. Measure em equal to x_1 and em equal to x_2 along these perpendicular lines on the same side as on the prints. (Here they are measured on the left of CD and DE.)

(iv) Join Cm and Dm and produce them so as to meet at M, which gives the required position of M on plan.

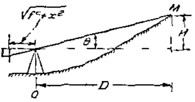


Fig 22

To determine the level of the point M (Fig. 22"),

(i) Measure y_1 which gives the height of m above the horizontal hair. Rise of the ray from m to the centre of the object glass is equal to y_1 in a horizontal distance $\sqrt{f^2 + x_1^2}$.

(ii) Measure the distance CM to scale on the plan.

(iii) The height (H) of M above the horizontal plane of collimation at C is given by

$$H = \text{CM} \frac{y_1}{\sqrt{f^2 + x_1^2}}$$

Knowing the reduced level of the horizontal plane of collimation the reduced level of M may be obtained by the relation

R L of M = R L of the horizontal plane of collimation + H

Analytical Method —Referring to Fig 22b let

α = the angle ECD $\qquad$ β = the angle CDE

$\delta\alpha$ = the angle MCE $\qquad$ $\delta\beta$ = the angle MDE

Then $\tan \delta\alpha = \frac{x_1}{f_1}$ or $\delta\alpha = \tan^{-1}\frac{x_1}{f}$

$\tan \delta\beta = \frac{x_2}{f}$ or $\delta\beta = \tan^{-1}\frac{x_2}{f}$.

$\therefore \quad \angle MCD = \alpha + \delta\alpha, \ \angle MDC = \beta - \delta\beta.$

Hence $\angle CMD = 180° - \angle MCD - \angle MDC$

$$= \{180° - (\alpha + \delta\alpha) - (\beta - \delta\beta)\}$$

The distance CM and DM may be computed by the application of the sine rule

$$\therefore \quad CM = CD\frac{\sin MDC}{\sin CMD} = b\,\frac{\sin(\beta - \delta\beta)}{\sin\{180° - (\alpha + \delta\alpha) - (\beta - \delta\beta)\}}$$

$$DM = CD\frac{\sin MCD}{\sin CMD} = b\,\frac{\sin(\alpha + \delta\alpha)}{\sin\{180° - (\alpha + \delta\alpha) - (\beta - \delta\beta)\}}$$

Height (H) of M above the horizontal plane of collimation at C (Fig 221) is given by

$$H = CM.\frac{y_1}{\sqrt{f^2 + x_1^2}}$$

R L of M = R. L of the plane of collimation at C $+ H$.

Note —If the print from D shows the point m to the right of the vertical hair, the angle MDC $= \beta + \delta\beta$.

Field Work —The field work of the terrestrial photographic surveying consists of (1) reconnaissance, (2) triangulation, and (3) camera work

Reconnaissance —The existing maps of the area to be surveyed should first be procured and a thorough, careful study should be made as it is very helpful in selecting suitable camera stations so that the area will be covered with a minimum number of photographs and the work can be done with speed and economy A careful reconnaissance of the area should next be made with a view to selecting suitable triangulation stations and camera stations The points to be considered in selecting camera stations are

(1) The stations should be so fixed that the objects to be plotted on the map can be clearly and easily recognised on at *least two* photographs taken at different stations

(2) The angle of intersection of the two direction lines locating a particular point should not be too accute or too obtuse

(3) The direction of the pointing should as far as possible be normal to the slope of the ground

(4) They should be fixed on higher points so as to command the area.

Triangulation —As in other methods of surveying the control is established by triangulation All camera stations should be connected by a triangulation system In extensive surveys two or more triangulation systems are necessary The triangulation stations may sometimes be used as camera stations The elevations of the camera stations should be determined by direct or trigonometrical levelling

Camera Work —Photographs are taken in pairs from the ends of a base line (i e a line joining the camera stations) which is carefully measured The more important points should appear on three or more photographs and each photograph should contain at least one triangulation station or a point which has been already fixed from a camera station Adjacent photographs should sufficiently overlap The number of photographs to be taken at each station depends upon the area to be mapped and the field of view of the camera (i e the ratio of the focal length of the lens to the size of the plate used)

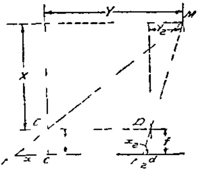
Stereo photogrammetry —This method which is the modern development of photographic surveying consists in taking stereoscopic views of the surface features in pairs at the ends of a base line The two exposures must be made with the photographic plates in the same vertical plane This can be done by taking the two photographs with the vertical planes of collimation of the cameras at *right angles* to the base line The length of the base line i e the horizontal distance between the parallel principal planes usually lies between 30 and 120 m It is measured either with a tape or obtained by the transit and stadia method

Field work —The Zeiss photo theodolite—specially designed for stereo-photogrammetry—consists of (i) a levelling head, (ii) a camera of the fixed focus type and (iii) a theodolite mounted

on the steel camera casing Horizontal and vertical angles can be observed as in the case of a transit The optical axis of the telescope and that of the camera are in the same vertical plane when the horizontal circle reads zero

The camera stations are located by a triangulation system in the same way as in terrestrial photographic surveying In this method every point is photographed twice from the parall orientations Having set up the camera at a station C—one en of the stereoscopic base line (Fig 222) and adjusted in the required position the exposure is made the base line is then set out at right angles to the principal plane by the telescope and is measured tacheometrically The instrument is then removed to D—the other end of the base line and set up with the optical axis of the camera *parallel* to its previous position (as at C) by taking a backsight on the previous station C through the telescope and the second view is then taken the two views forming a stereoscopic pair

Plotting —The two views forming a stereoscopic pair are viewed through a stereoscope so that a very bold relief is obtained The points are plotted with the help of a stereoscope The principle underlying the method of plotting is explained as follows



F 2 3

In Fig 2?8 let

C and D = the optical centres of the lens

f = the focal length of the lens

x_1 and y_1 = the distances of the point m on the plate exposed at C from the vertical and horizontal hairs respectively.

x_2 and y_2 = the distances of the same point m on the plate exposed at D from the vertical and horizontal hairs respectively.

X and Y_1 = the co ordinates of the point M (which is represented as m on the plate) from C

X and Y_2 = the co ordinates of the point M from D

b = the length of the stereoscopic base CD

h_1 = the height of the point M above the horizontal plane of collimation at C

h_2 = " " at D

Then $Y_1 = \frac{x_1}{f}X, \quad Y_2 = \frac{x_2}{f}X, \quad b = Y_1 - Y_2;$

or $$b = \frac{(x_1 - x_2)X}{f} \qquad X = \frac{f}{(x_1 - x_2)} b$$

Whence, $$Y_1 = \frac{x_1}{f} \cdot \frac{f \times b}{(x_1 - x_2)} = \frac{bx_1}{(x_1 - x_2)}, \quad Y_2 = \frac{bx_2}{(x_1 - x_2)}.$$

Now $$\frac{h_1}{y_1} = \frac{CM}{CM_1} = \frac{X}{f} \qquad h_1 = \frac{y_1}{f} X$$

Similarly, $$\frac{h_2}{y_2} = \frac{DM}{DM_2} = \frac{X}{f} \qquad h_2 = \frac{y_2}{f} X$$

Whence, the difference of level of the two collimation planes of the cameras

$$= h_1 - h_2 = \frac{(y_1 - y_2)}{f} X$$

$$\frac{h_1}{y_1} = \frac{X}{f} = \frac{Y_1}{x_1} = \frac{b}{(x_1 - x_2)} = \frac{Y_2}{x_2} = \frac{h_2}{y_2} = \frac{(h_1 - h_2)}{(y_1 - y_2)}.$$

Note —If the point M appears on the print at D to the left of the vertical hair $b = Y_1 + Y_2$

In practice, the quantities x, y, the co-ordinates X and Y and h are measured on a plotting machine known as Stereo-Comparator designed by Dr Pulfrich Plans and contours are drawn automatically by the use of the instrument called the Stereoautograph by its inventor Major Von Orel, or by means of the well known instrument known as the Zeiss stereoplanigraph which is universally used for mapping both ground and aerial surveys

Aerial Surveying —Since the First World War, the terrestrial photographic surveying has been replaced by aerial photographic surveying or briefly called aerial surveying for most of survey work due to the development of the aeroplane It is most suitable for small scale mapping especially in flat country The advantages of this method are (i) that the survey work can be carried out with great speed and (ii) it can be used with great success for other purposes such as classification of land and soil geological and archaeological investigations, etc Aerial survey is a highly technical and specialised work and must be carried out by skilled specially trained, and experienced personnel Since aerial survey is very elaborate and expensive it is mainly made by government organisations (by Survey of India Department in India) Recently, private companies have been formed to undertake this class of work Aerial surveying consists of four parts

(1) flying (2) photography (3) ground control and (4) compilation or mapping The equipment required for this class of work comprises (i) an aeroplane (ii) an aerial camera and (iii) accessories required for interpretation and plotting

(1) Flying —While the photographs are being taken it is of utmost importance that the aeroplane should fly at a *uniform* speed on a *straight* course in a given direction at a *constant* height, without tilt of the machine (the axis of the camera must be vertical i e pointed downward) If there be any variation in the flight altitude or flying height, the scale of the photographs will be changed, on the other hand any tilt of the camera will cause distortion in the photographs

(2) Aerial Photography —Photographs are taken automatically with special cameras on parallel strips or courses with generous overlap both in the direction of flight and at right angles

to it In order to ensure the required overlap between successive photographs, the photographs are taken at proper intervals along each strip, and the spacing of the parallel strips is so fixed that the necessary side overlap is obtained The overlap between successive photographs is expressed as a percentage, and the amount of overlap in the direction of flight (longitudinal overlap) lies between 50 to 60% and that in a direction at right angles to it (side overlap) varies between 25 to 30%

There are two ways of taking aerial photographs viz (1) vertical, and (2) oblique. In the former photographs are taken with the axis of the camera pointing vertically downward, while in the latter the camera axis is given a tilt (inclination) of about 30° to the forward direction The vertical photography (i e vertical photographs) is used when only a planimetric map of the area surveyed is required, while the oblique photography is used when a contoured map of the area is desired Vertical photographs give most accurate results and are, therefore, generally preferred In modern practice a multiple lens camera is preferred One vertical, and upto six oblique photographs can be taken at one exposure with this camera

(3) Ground Control —In order to produce an accurate map from aerial photographs it is absolutely necessary to furnish ground control It consists in locating the positions of a number of points all over the area to be surveyed and determining their levels These control points must be such that they can be readily identified on the photographs They are established by triangulation or traversing Another requirement is that each portion of the area must appear on at least two photographs in order to obtain stereoscopic views of the whole ground

Scale of the Photograph —The scale of the aerial photograph is expressed as a representative fraction (R F) Knowing the height of the aeroplane above the ground and the focal length of the aerial camera, the scale of a photograph may be determined as follows

Referring to Fig 229, let BC represent the level ground, ab, the picture plane, AO, the focal length (f) of the camera, A, the position of the camera lens, H the height of the camera lens above the ground, S, the representative fraction

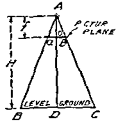


Fig 229

Then by similarity of $\triangle s$ Aab and ABC,

$$\frac{f}{\text{H}} = \frac{ab}{\text{BC}} \quad \text{or} \quad \text{H} = f\,\frac{\text{BC}}{ab} = \frac{f}{\text{S}} \quad . \tag{1}$$

since $\dfrac{ab}{\text{BC}} = \text{S} = \text{R F}$

It must be remembered that the scale of the photograph depends upon the height (H) of the camera above the ground Any variation in the value of H will change the scale It is, therefore essential that the aeroplane should fly at a *constant* height If the region is irregular the scale of the photograph will not be the same throughout the region in which case the flight altitude is kept constant and the scales are varied

By flight altitude (height of flight) is meant the true height of the aeroplane above mean sea level (M S L) It is equal to the barometric altitude as indicated by an instrument known as *altimeter* only when the atmospheric conditions are similar to those for which the instrument is calibrated

The flight altitude = height of the ground above M S L

$$+\text{H} \tag{2}$$

H being obtained by the relation $\text{H} = \dfrac{f}{\text{S}}$

It may be noted that if the terrain is irregular, the average elevation of the ground must be taken to determine the value of the flight altitude by formula (2).

Illustration.—Suppose the reduced levels of the ground surface vary from 180 to 460; the focal length of the camera is 30 cm and the scale is $\frac{1}{7200}$.

Then by equation (1), i. e. $H = \frac{f}{S}$

$$H = \frac{0\,30}{1/7200} \quad \text{or} \quad H = 2160 \text{ m.}$$

The mean reduced level of the ground $= \frac{180 + 460}{2} = 320.$

$\therefore$ Flight altitude $= 320 + 2160 = 2480$ above M. S. L

Photographs Required:—The total number of photographs required to cover the area to be surveyed may be determined, as follows

Notation :— L_p = the length of photograph in cm in the direction of flight.

W_p = the width of photograph in cm at right angles to the direction of flight

O_l = the percentage of the longitudinal overtop

O_w = the percentage of the side overlap.

L_g = the net ground distance corresponding to L_p in m or km

W_g = the net ground distance corresponding to W in m or km.

S = the scale of photograph.

N = the number of photographs required.

A_p = the net area of each photograph in sq. m or sq km

A_g = the area of the tract to be photographed in sq m or sq. km

Then

$$L_g = SL_p(1 - O_l) \text{ and } W_g = SW_p(1 - O_w)$$

Net area of each photograph $= A_p = L_q \times W_q$

$$\text{Number of photographs required} = \frac{A_g}{A_p}.$$

Theoretical spacing of flight strips = net width of a single photograph $= W_g$

$$\text{Theoretical number of strips} = \frac{\text{width of the area}}{W_q} = K$$

Actual no of strips $= K + 1$, one strip being added to cover the sides

$$\text{Theoretical no of photographs per strip} = \frac{\text{length of the area}}{L_q} = M$$

Actual no of photographs per strip $= M + 1$ one photograph being added to cover the ends of the area

Thus in practice there would be $K + 1$ strips of $M + 1$ photographs each

Actual no of photographs for complete coverage of the area
$= (K + 1)(M + 1)$

Interval between Exposures —The time interval between exposures depends upon the speed of the aeroplane and the distance it travels between exposures Time interval in seconds between exposures $= \dfrac{3600L_g}{V}$,

where V is the speed of the aeroplane in km per hour, and Lg, the distance the aeroplane travels between exposures in km

(4) Mapping —Maps may be prepared from photographs by (i) the Three point method, (ii) the Radial line method and (iii) the Slotted template method All these methods are slow, tedious, and expensive In modern practice maps are prepared by the use of the Zeiss Stereoplanigraph or Zeiss Aerocartograph

\+ + +

QUESTIONS FROM UNIVERSITY EXAMINATIONS

Q 1 Particulars of part of a traverse are as under —

Line	Length in feet	Bearing
AB	486	342° 24
BC	1724	14° 35
CD	1053	137° 20′

Calculate the distance between a point E on AB 3°6 ft from A and a point F on CD 400 ft from C. (U. B)

(Ans 1662 1 ft)

Q 2 From a point C, it is required to set out a line CD parallel to a given line AB, and such that ABD is a right angle C and D are not visible from A and B, and traversing is performed as follows —

Line	Length in ft	Bearing
BA		360°
BE	258 5	290° 5′
EF	307 0	352° 6
FC	196 5	263° 2′

Compute the required length and bearing of CD (U B)

(Ans 374 1 ft 180°)

Q 3 The following traverse is run from A to B, between which of reoccur certain obstacles

Line	Length in feet	Bearing
AC	236	78° 35′
CD	1142	10° 20
DB	455	274° 15

Calculate the length and bearing of AB (K U)

(Ans 1204 ft , 359° 10′.)

Q 4 From the initial station A of an unclosed traverse the bearing of a distant point P is observed to be 62° 18 From station E, which is 6°38 ft N. and 3016 ft W of A the bearing of P is 91°37′ Calculate the bearing of P from M of which the total co ordinates referred to A are 11273 ft N and 1419 ft E (U P.)

(Ans 114° 16′)

Q 5 Two points P and S are connected by a traverse survey PQRS. The traverse is conducted in a counter clockwise direction and the following measurements were recorded —

PQ = 583 ft.	Angle PQR = 123° 20′
QR = 795 ft	Angle QRS = 104° 40′
RS = 876 ft	

Assuming that PQ is in the meridian determine (i) the latitude and departure of S relatively to P, (ii) the length of the line PS, and (iii) the angle QPS

(G U)

(Ans (i) + 549 0 ft , — 13°3 1 ft , (ii) 1432 ft , (iii) 67° 28′)

Q 6 The following are the co ordinates of three stations A, B, and C

Station	North Co ordinate	East Co ordinate
A	0	0
B	2894	5982
C	9760	3255

The bearings to a distant point P from A and B are N 27° 32 W and N 63° 14 W respectively Compute the bearing of P from C

(Ans S 65° 21′ W)

Q 7 From an initial station A of an unclosed traverse, the bearing of a distant point P is observed to be 61° 24 From station M, which is 6423 ft N and 2815 ft W of A, the bearing to P is 91° 12 Calculate the bearing of P from K, of which the total co-ordinates referred to A are 12160 ft N and 1530 ft E (U B)

(Ans 121° 51′.)

Q. 8 Two stations A and B are fixed on either side of a wood In order to determine the length of AB, the following traverse is run

Line.	Length in feet	Bearing
AC	438	48° 24′
CD	664	110° 12
DB	582	152° 36′

Calculate the length of AB An intermediate point E is to be fixed on AB by running a line DE on a bearing of 163°6′ Determine the length of DE.

(Ans 1301 ft , 462 ft)

Q 9. The co-ordinates of two stations A and B are

Station	Coordinates	
	N	E
A	7440	5275
B	8165	6738

From A, a line is run on a bearing of 143°10 and from B, a line a is set out on a bearing of 192° 25 Find the co ordinates of the point of intersection of these lines.

(Ans 5891 N , 6237 E)

Q. 10 The co-ordinates of two stations M and N of a theodolite survey are as follows

Station.	North Co ordinate	East Co-ordinate
M	5696 ft	8725 ft
N	6470 ,,	9525 ,,

The following notes refer to the compass traverse run between M and N

Line	Length in feet	Bearing
Ma	3?4	139° 45
ab	416	70 15
bc	534	25° 45
cN	465	347° 30

The magnetic north of the compass survey is 4°24 east of the true meridian of the theodolite survey Assuming the theodolite work to be correct, find the closing error of the compass traverse

(Ans 7 865 ft.)

Q 11 A closed traverse was conducted round an obstacle and the following observations were made Work out the missing quantities

Side	Length in ft	Azimuth
AB	—	33° 45
BC	300	86° 23
CD	—	169° 36
DE	450	243° 54
EA	268	317° 30

(G U)

(Ans AB = 3?2 5 ft CD = 303 7 ft)

Q 11a Two points A and D are connected by a traverse survey ABCD The traverse is conducted in a counter clockwise direction and the following measurements were recorded

AB = 876 ft BC = 682 ft CD = 983 ft

Angle in ABC = 118° 15 Angle BCD = 108° 40

Assuming that AB is in the meridian determine (1) the latitude and departure of D relatively to A (2) the length AD and (3) the angle BAD (G U)

(Ans (1) 527 3 N 1318 75 W (2) 14 0 ft (3) 68° 12)

Q 12 A and B are two of the stations in setting out construction lines for harbour works The total latitude and departure of A referred to the origin of the system are respectively +542 7 ft and −331 2 ft and those of B are +713 0 ft and +587 8 ft A point C is fixed by measuring from A a distance of 432 ft on a bearing of 3?6° 14 and from it a line CD 115? ft in length is set out parallel to AB It is required to check the position of D by a sight from B Calculate the bearing of D from B (U B)

(Ans 13° 34 40")

Q 13 (a) Permanent adjustments of a transit theodolite have to be performed Its altitude bubble tube is attached to the vernier T frame Describe the order in which these adjustments should be carried out

(b) Give the procedure and correction of the adjustment of altitude bubble tube so as to make the line of collimation horizontal when the bubble is centred and reading of the vertical circle is zero The instrument has clamp and tangent screw on opposite side of the telescope to the vertical circle and clip screws of T frame (G U)

Q 14. Explain how a theodolite is tested, and, if necessary corrected, so that (i) line of collimation may be coincident with the longitudinal axis of the telescope, and (ii) Line of collimation may be at right angles to the transverse axis. (K U)

Q 15 (a) State the five fundamental lines which are generally embodied in the construction of a theodolite and show them by line diagrams

(b) Mention the conditions of adjustment to be examined in a pattern of a general purpose transit theodolite

(c) A transit theodolite is to be used for the measurement of horizontal angles Which of the above adjustments must be correct to ensure accurate results? Describe fully how you would check their accuracy (U P)

Q 16 Find the difference in level between two stations A and B from the following data —

Horizontal distance between points A and B		= 10275 ft
Angle of elevation from A to B		= 1° 48′ 25″
Angle of depression from B to A		= 1° 44′ 50″
Height of Instrument	at A	= 4 3 ft
Height of Instrument	at B	= 5 0 ft
Height of signal	at A	= 12 2 ft
Height of signal	at B	= 10 8 ft
Log sin 1″		= $\overline{6}$ 685575 (K U)

(Ans 319 ft 9 3)

Q 17 A vane 3 feet above the ground at B is sighted from two instrument stations A and F, 200 ft apart The angles of elevation are 45° 30 from A and 30° 20′ from F The height of the instrument axis at A above the ground = 5 10 f and at F = 4 76 ft. A staff is held upon the peg at F and a reading of 8 48 ft obtained from the instrument at A the bubble being in the centre of its run. Find the horizontal distance from A to B and the reduced level of B, that of F being 100 00 ft above datum (K U.)

(Ans °61 98 ft R L of B = 352 04)

Q 18 A target of 10 ft height is erected at a point T on the top of a building Following observations are made to this target from instrument stations A and B, 750 ft apart the situations of two stations being at considerable different elevations The angle of elevation from A to T is 47° 30 and that from B to T 32° 40 A vane 4 5 ft above the foot of the staff held on A is sighted from B and the angle of elevation observed is 1°° 10′, the height of instrument at A being 4 75 ft and that at B 4 98 ft The R L of station B is 325 67 Find (i) the horizontal distance of T from B and (ii) the R L of T the station point on the top of the hill (G U)

(Ans. (i) 1458 6 ft , (ii) R. L of T = 1255 8

Q 19 The following data were obtained from reciprocal observations for altitude of A and B —

The Reduced level of A	869 59 ft
The height of instrument at A	5 33 ft
The height of instrument at B	4 92 ft
The height of signal at A	25 31 ft
The height of signal at B	15 55 ft
Angle of elevation from A to B	3° 34′ 20″
Angle of depression from B to A	3° 31′ 20″

The horizontal distance from A to B as determined by triangulation is 18740 ft What is the reduced level of B ? Find also the mean value of the angle of refraction and coefficient of refraction

Take the radius of earth = 20890000 ft (U P)

(Ans 2036 39, 14″ 06, 0 076)

Q 20 Find the difference of level between two points A and B from the following data —

Horizontal distance between the points A and B is 16030 2 feet

Vertical angle from	A to B	= + 16 20″
Vertical angle from	B to A	= − 10 26″
Height of instrument	at A	= 4 8 ft
Height of instrument	at B	= 4 7 ft
Height of signal	at A	= 20 2 ft
Height of signal	at B	= 18 5 ft

(U B)

(Ans 63 274 ft)

Q 21 Find the difference of level between two points A and B by reciprocal vertical angle readings, given the following data —

Horizontal distance between A and B	46778 ft
Vertical angle from A to B	− 0° 16′ 35″
Height of signal at B	15 ft
Height of instrument at A	5 ft
Vertical angle from B to A	+ 0° 22′ 26″
Height of signal at A	7 ft
Height of instrument at B	6 3 ft

If the height of ground at A is 5404 ft what is the ground level height at B Assume the mean radius of the earth as 3956 miles (U B)

(Ans. 5128 94 ft)

Q 22 The following reciprocal observations were recorded at two stations A and B situated 11420 ft apart —

Height of instrument at A	= 4 68 ft
Height of signal at A	= 14 70 ft
Reduced level of A	= 421 20 ft
Elevation of signal at B	= 1° 49′ 5″

Height of instrument at B = 4 91 ft.
Height of signal at B = 12 80 ft
Depression of signal at A = 1° 45′ 18″

Find the reduced level of B, given that R sin 1″ = 101 31 ft and also the refraction correction. (U. B)

(Ans. 778 23, 8·11″.)

Q 23 A tacheometer is set up at an intermediate point on a traverse course AB, and the following observations are made on a vertically held staff —

Staff station	Vertical Angle	Staff intercept.	Axial Hair Reading
A	— 5° 24′	7 25	6 62
B	— 6° 18′	6 60	6 34

The instrument is fitted with an anallatic lens, and the constant is 100. Compute the length of AB and the reduced level of B, that of A being 325·25·

(Ans AB = 1370 7 ft, R. L of B = 321·48) (K. U.)

Q 24 (a) Explain how you would determine the constants of a tacheometer What are the advantages of an anallatic lens used in a tacheometer.

(b) Levels were carried from a bench mark to the first station A of a tacheometric survey by tacheometric observations The instrument was fitted with an anallatic lens and the value of the constant was 100 The following observations are recorded, the staff being held vertical :

Inst. station.	Height of axis	Staff station	Vertical angle	Staff readings	Remarks
O	4 40	B. M	− 2° 20′	5 00,7·11,9 22	R L of B M.
,,	,,	C. P	+ 4° 36′	4 50,6 07,7·64	= 750·75
A	3 80	C P	+ 5° 12	4 00,5 90,7·81	

Compute the elevation of station A. (K. U.)

(Ans. 236·1.)

Q 25 Determine the gradient from a point A to a point B from the following observations made with a tacheometer fitted with an anallatic lens The constant of the instrument was 100 and the staff was held vertically —

Inst station	Staff point	Bearing	Vertical angle	Staff Readings
P	A	340°	+ 14° 36	2 90, 5 63, 8 36
	B	70°	+ 9° 48′	2 60, 7·40, 12 20 (K. U)

(Ans 1 in 40 84)

Q 26 Determine the gradient from a point A to another point B from the following observations made with a fixed hair tacheometer fitted with an anallatic lens The constant of the instrument is 100

Inst station	Staff point	Bearing	Vertical angle	Hair reading
P	A	345°	+ 15°	3·00, 5·74, 8·48
P	B	75°	+ 10°	2·50, 7·35, 12·20 (U B)

(Ans. 1 in 39.3)

Q 27 Levels were carried from a bench mark to the first station A of a tacheometric survey by tacheometric observations The instrument was fitted with an anallatic lens and the value of the constant was 100 The following observations were made the staff having been held vertically —

Inst station	Ht of Axis	Staff Point	Vertical Angle	Staff readings	Elevation
I	4 60	O B M	−6° 30	2 40 5 00 7 60	560 75
I	4 60	C. P	+4° 36	3 50 5 07 6 64	
A	4 80	,	−6° 12	3 19 4 70 6 21	

Compute the elevation of station A (L B)

(Ans 676 60)

Q 28 Compare the different systems of determining distances by telescope The following readings were taken with an anallatic tacheometer the staff having been held vertically The constant of the instrument was 100 —

Inst station	Height of Axis	Staff Station	Vertical Angle	Hair readings	Remarks
L	4 50	B M	− 6° 12	3 19 4 70 6 21	R L of B M = 180 75
L	4 50	M	+ 5 1	4 00 5 91 7 82	
M	4 60	N	+10° 42	4 00 6 46 8 92	

Calculate (a) the horizontal distances LM and MN and (b) the reduced levels of L M and N (U B)

(Ans LM−378 8 ft MN=475 1 ft R L of L = 213 38 M=246 45, N 334 35)

Q 29 (a) Describe the field procedure for a tacheometric traverse survey for the preliminary location of a railway line

(b) The elevation of a point P is to be determined by observations from two adjacent stations of a tacheometric survey The staff was held vertically upon the point The instrument is fitted with an anallatic lens and the constant is 100 Compute the required elevat on from the following data taking the two observations as equally trustworthy

Inst Station	Height of axis	Elevation of station	Point	Vertical angle	Axial Hair Reading	Stadia Intercept
A	4 20	880 60	P	+2° 27	7 56	8 12
B	4 40	971 90	P	−4° 51	6 93	6 87

(U P)

(Ans 911 695)

Q 30 The staff intercept of stadia read ngs for an instrument with telescope horizontal is 3 91 ft for a distance of 400 ft and 1 46 ft for a distance of 150 ft The instr iment is then set over a station A having R L of 410 75 the height of the instrument above the station point being 4 52 ft The stadia and axial readings on a vertical s aff at station B are 1 46 4 42 and 7 38 ft when the vertical angle is — 15° Deduce the horizontal distance from A to B and the reduced level of station B

(Ans $\frac{f}{i}=10$ 1 $f+d=1$ 1 Distance AB=564 26 ft R L of B=° 9 57)

Q 31 Following observations were taken from two traverse stations by means of a techeometer fitted with an anallatic lens The constant of the instrument is 100

Inst Station	Staff station	Height of instrument	Bearing	Vertical Angle	Staff readings
A	C	4 50	295° 30′	+ 15° 40	2 15, 4 46, 6 77
B	D	4 75	64° 45	− 20° 10	3 21, 4 83, 6 45

Co ordinates of station A 675 43 N , 428 26 W
" " " B 324 62 N , 268 14 E
R L of A = 545 78, R L of B = 422 34
Compute the length and gradient of the line CD (G U
(Ans CD = 1403 ft , I in 4 03)

Q 32 (a) Explain by means of diagram the principle of the stadia
(b) The focal length of the object glass in a telescope is 10 in. and the vertical axis of the theodolite is midway between the object glass and its Principal focus When the staff is at a distance of 401 25 ft from the axis of theodolite, the intercepted height on the staff was found to be 4 ft What is the distance between the pair of webs on the diaphragm (G U)
(Ans $\frac{1}{10}$ inch.)

Q 33 Explain how you would determine the constants of a tacheometer. The stadia intercept read by means of a fixed hair instrument on a vertically held staff is 3 73 ft., the angle of elevation being 6° 27 The instrument constants are 100 and 1 1 What would be the total number of turns registered on a movable hair instrument at the same station for a 5 ft intercept on a staff held on the same point the vertical angle in this case being 6° 21′ and the constants are 1000 and 1 4 ? (G U)
(Ans Distance 369 493 ft , no of turns = 13 43)

Q 34 A tacheometer has a diaphragm with three cross hairs spaced a distances apart of $\frac{1}{40}$ inch The focal length of the object glass is 9 inches and the distance from the object glass to the trunnion axis 4½ inches A staff is held vertically at a point the level of which is 475 68 ft. above datum The telescope is inclined upwards at 8° to the horizontal and the readings taken on the staff are 6 66 ft 5 11 ft , 3 56 ft Find the distance of the point from the telescope and the level of the instrument station. The height of the trunnion axis of the telescope is 4 6′ (G U)
(Ans 548 51 ft , 399 22)

Q 35 The following notes refer to two observations in a tacheometric survey The elevation of the instrument station was 639 40, the trunnion axis of the telescope having been at 4 70 ft above the station —

Staff station	Bearing	Vertical angle	Staff readings
A	97° 45′	− 6° 12	3 19, 4 70, 6 21
B	105° 50	+10° 42′	4 00 ,6 46, 8 92

The instrument was fitted with an anallatic lens, the value of the constant being 100 The staff was held vertically Find the horizontal distance between the staff stations A and B and their elevations (K U.)

(Ans 184 5 ft 606 97, 727 40)

Q. 36 (a) Describe the main features of an auto reduction tacheometer

(b) The following observations were made with a tacheometer set up at stations A and B, the staff was held vertical

Instrument station	Staff station	Height of inst axis	Vertical Angle	Staff readings
A	B M	4 2	+4° 30	5 10, 6 10, 7 10
A	C P	4 2	+5° 30	3 50, 4 70, 5 90
B	C P	4 5	−6° 15	4 30, 6 20, 8 10

Take R L of B M 400 00 and value of the tacheometric constants as 100 and 0 Find the R L of stations A and B and also the distance from A to the B M

(Ans R L of A = 386 26, B = 451 47 distance = 198 76 ft) (U P)

Q 37 What is a transition curve? Why is it used? What is meant by "shift" of a curve?

Two straights on the centre line of a proposed railway intersect at 63 + 54 in 100-ft units, the deflection angle being 38° 24 It is proposed to put in a circular curve of 1600 ft radius with a cubic parabolic transition curve 180 ft long at each end The combined curve is to be set out by the method of deflection angles with pegs at every 50 ft of through chainage on the transition curves, and with pegs at every 100 ft of through chainage on the circular curve Tabulate the data relative to the first two stations on the first transition curve and the junctions of the transition curves with the circular arc (K U.)

(Ans Deflection angles 3 46 92" 17 25 8" 1° 4 27", 15° 58′ 48")

Q 38 Two tangents intersect at 7650 ft, the deflection angle being 63° 36 It is proposed to insert a circular curve of 800 ft radius with a cubic parabolic transition curve 142 ft in length at each end The circular curve is to be set out with pegs at every 100 ft and the transition curves with pegs at every 50 ft of through chainage The combined curve is to be set out by the method of deflection angles Tabulate the data relative to the first station on the first transition curve, and the junctions of the transition curves with the circular arc (K U)

(Ans Deflection angles 1 35", 1° 41′ 42", 26° 43)

Q 39 A railway curve is to be set out to connect two tangents having a deflection angle of 98° 30 The chainage of the intersection point is 6728 ft The maximum speed on this part of the railway is 65 miles per hour Allowing the maximum rate of change of acceleration = 1 ft /sec^3 and taking the radius of the circular curve as 1400 ft, find (a) the length of the transition curve, and (b) chainage at the beginning and end of the transition curves (G U.)

(Ans (a) 619 ft., (b) 4750·5 ft, 5399 5 ft, 7186 5 ft, 7805 5 ft.)

Q 40 Two straights AB and BC meet in an inaccessible point B and are joined by a circular curve having radius of 8 chains Points P and Q were located on the lines AB and BC respectively so that in between these two points the following measurements could be taken chainage of P from A = 35 64 chns, length of PQ = 3 5 chns, angle APQ = 140° 30 and angle CQP = 135° 40′ Calculate the tangential angles for setting out the curve by Theodolite (G U)

(Ans. chainage of T_1 = 30 917 chn. Chainage of T_2 = 42 617 chn.)

Q 41. The centre line of a railway laid in the direction of N 45° 30′ E deflects to N 75° 30 E at chainage 2650 + 00 It is proposed to introduce two transition spirals between the ends of a central 3° circular curve and the straights Work out the chainages of the beginning and end of the first transition curve and its total spiral angle The probable speed of trains on the curve is 50 miles per hour and the rate of canting is 1 inch per 80 feet (U B)

(Ans 1932 95 ft, 2341 45 ft, 6° 7 40″ 8.)

Q 42 Explain the reasons for the desirability of introducing a transition curve between a tangent and a circular curve

On a proposed railway two straights intersect at 86 + 42 in 100 ft units the deflection angle being 30° 36 A circular curve of 1200 ft radius and cubic parabolic transition curves are to be inserted the latter being 160 ft in length The combined curve is to be set out by the method of deflection angles with pegs at every 50 ft of through chainage on the transition curve, and with pegs at every 100 ft of through chainage on the circular curve Tabulate the data relative to the first two stations on the first transition curve and the junctions of the transition curves with the circular arc (U B)

(Ans Deflection angles 49″ 33 13 13″ 8 1° 16 22″ 8, 11° 28 54″)

Q 43 The bearings of two intersecting straights on the centre line of a proposed railway are respectively N 38° 15 E and N 74° 45 E the point of intersection occurring at 67 + 75 in 100 ft units It is proposed to put in a circular curve of 1200 ft radius with a cubic parabolic transition curve 145 ft in length at each end The circular curve is to be set out with pegs at every 100 ft and the transition curves with pegs at every 50 ft of through chainage (a) Calculate the chainage at the beginning and at the end of the curve (b) The combined curve is to be set out by the method of deflection angles Tabulate the data relative to the first two points on the first transition curve and the junctions of transition curves with the circular arc (U B)

(Ans (a 63 + 6 5 72 + 16 (b) 6 14″ 28 47″ 4, 1° 9 15″ 6 14° 47 10″ 2

Q 44 Write short notes on the following with sketches where necessary.—

(i) Equation time (ii) Sidereal time (iii) Right angled astronomical triangle (iv) Systems of celestial co ordinates (K U)

Q 44a Explain the various systems of specifying the position of a heavenly body on the celestial sphere A star was observed at western elongation at a place in latitude 50 40 N when its clockwise horizontal angle from a survey line was 106 48 0 Determine the azimuth of the survey line given that the star's declination was 74 12 30″ N (K U)

(Ans 297° 46 3″ 43 clockwise from north)

Q. 45 (a) State "Napier's Rules of Circular Parts."

(b) What do you understand by (i) apparent solar time, (ii) mean solar time, and (iii) sidereal time

(c) Find the local mean time of transit of a star in longitude 30° 15′ E. on November 25, 1940, given the following —
G S. T. of G M. M. on Nov 25, 1940 = 6 h 30 m 35 s
R A of star = 1h 32 m 27 s. (K U.)

(Ans 19 h 2 m 11 68 s)

Q. 46. Show with the aid of sketches, where necessary, the relationship between the following .

(a) The R. A of a star, the hour angle of the star at any instan, and the sidereal time at that instant, (ii) Local mean time, local apparent time, and the equation of time

(b) What do you understand by correction for ' parallax " "semi diameter", and " refraction" ? When are these used ?

(c) An observation is to be made on a star whose R A is 4 h 8 m, 12 s. Calculate the local mean time of upper transit of a star at a place in latitude 52° 24′ N. and longitude 16° 48′ E on a day on which the G S. T of G M N is 16 h 12 m 8 s. What is the altitude of transit if the declination of the star is 54° 10′ 36″ (U. P)

(Ans 11h 54 m 17 7 s , 88° 13′ 24″)

Q 47 (a) Give a list of corrections which must be applied to the observed altitude of the sun

(b) A star was observed at western elongation at a place in latitude 52° 30′ N. The mean observed horizontal angle between the survey line and the star was 75° 12′ 20″, the star lying between the meridian and the line. The declination of the star was 60° 24′ N. Find the azimuth of the line (K. U)

(Ans 230° 33′ 45″ 4 clockwise from north)

Q 48 Determine the G. M T. at which the star α Aurigae crossed the meridian of a station in longitude 28° 31′ E in the northern hemisphere at upper culmination on May 31st, 1926, the declination of the star being 45° 55′ 25″ N. and its Right Ascension 5h. 11 m. 6 s. with G S.T of G. M N. 4 h 32 m 55 s. If the true altitude of the star was 76° 30′ 50″, find also the latitude of the station. (U B.)

(Ans. 10 h 45 m. 23·27 s , 32° 26′ 16″N)

Q 49. The following observations of the lower limb of the Sun were taken for azimuth of a line in connection with a survey —

(i) Mean time = 16 h. 30 m.	(ii) Mean horizontal angle between the Sun and referring object = 18° 20′ 30″
(iii) The sun is west of R. O.	(iv) Sun's semi diameter = 16′5″.
(v) Sun's parallax = 8″.	(vi) Refraction correction = 1′ 25″).

(vii) Mean observed altitude = 33° 20′ 22″ (viii) Declination of Sun from N A.
(ix) Latitude of the place = +52° 30′ 20″ = + 22° 5 36″

Determine the azimuth of the line (K. U)

(Ans 281° 13 42″ clockwise from north)

Q 50 Find L. S T at a station in longitude 76° 20 E at 9 30 A M (Indian Zone time) on August 10 On that date at G M M, the R A of mean sun is 9 h 13 m 30 9 s. (G U)

(Ans 6 h 19 m 30 3 s.)

Q 51 (a) An observation was made at zone time 9 h 10 m 30 s on 8th August 1939, the Zone time being that of the standard meridian of 75° E Find out the hour angle of the sun at a place in longitude 73°E corresponding to the observation time given above Equation of time at G M N on the date of observation is 2 m 15 18 s. subtractive and decreasing at 0 70″5 s per hour

(b) From nautical almanac it is found that on the date of observation G S T of G M N is 3 h. 14 m 28 s Taking Retardation as 9 8565 s. per hour of longitude in problem (a), find the local sidereal time (G U)

(Ans. (a) 135° 2′ 19″·25 easterly (b) 12 h 17 m. 35 15 s)

Q 52 An observation for latitude was made on Nov 15 1948 in longitude 72°57 24″E the meridian altitude of the Sun s lower limb being then 48° 35 13″ What was the approximate latitude of the place? (a) The sun being south of the observer's zenith (b) The sun being north of the observer's zenith For computation the following data is taken from the Nautical Almanac

Sun s apparent declination at G M T 0 hours Nov 15 1948 = 18° 24 , 42″ S increasing 38″ 2 per hour Refraction 0 8, Sun s semi diameter 16 12″, Horizontal paralax 8″ 84 Equation of time + 15 m 25 5 s. (variation 0 4 s decreasing per hour) (G U)

(Ans. (a) 22° 40′ 12″ 2 N (b) 59° 38 22″ 1 S)

Q 53 A circumpolar star declination + 80° 17, right ascension 9 h 49 m 11s. is observed at western elongation in the evening in latitude 60°4 N longitude 127° 30′ W when its whole circle bearing from a reference line OA is found to be 207° 47 Find the bearing of OA from the meridian and also the local mean time at which elongation is to be expected if the G S T of 24 h. G M T is 16 h 54 m 13 s. The difference between sidereal and mean time intervals may be taken as 10 seconds per hour (G U)

(Ans 13.° 26 50″ 5 clockwise from north 21h 40m 41 s)

Q 54 (a Explain the following terms —

(i) Equation of time (ii) Declination, (iii) Longitude
(iv) Azimuth (v) Semi diameter and (vi) Local sidereal time

(b) Find the L. S T. corresponding to 5 p m. at Bombay in longitude 72° 48′ 46″.8 E. on march 2, 1954, the G. S T. of G M. M. being 10 h. 37 m. 56.97 s. (G U.)

(Ans (b) 3 h. 39 m 56 69 s)

Q 55. A star was observed at western elongation at a place in latitude 54° 30′ N. and longitude 52° 20′ E, when its clockwise horizontal angle from a surveyline was 116° 18′ 36″. The declination of the star was 62° 12 21″ N and its right ascension 11 h 58 m. 36 s. The G S.T of G M N was 4 h 38 m 32 6 s. Determine (a) the azimuth of the survey line and (b) the local mean time of elongotion. (K U)

(Ans. (a) 190° 16′ 25″ 67 clockwise from north, (b) 9h 8 m 32 94 p m)

Q 56. A star was observed at its eastern elongation in latitude 53° 32 N and the mean angle between a line and star was found to be 75° 18 20″, the star and the line being on opposite sides of the meridian Find (a) the azimuth of the line, (b) the altitude of the star at observation, (c) the L.M T of observation, with the following data —Declination of the star 56° 42′ 53″ 2 N, longitude of the place 5 h. 40 m. 18 s. W, R A. of the star 10 h. 58 m 3 9 s, S T at G. M. M. 4 h. 58 m. 23·84 s. (U P.)

(Ans. (a) 352° 7′ 3″·7, (b) 74° 9′ 32″·9, (c) 4 h 8 m 41 8 s)

Q. 57. A star was observed at western elongation at a place in latitude 52° 20′ N. and longitude 52° 20′ E, when its clockwise horizontal angle from a survey line was 105° 49′ 55″. Find the azimuth of the survey line, and the local mean time of elongation, given that the star's declination was 74° 27′ 30″ N. and its right ascension 14 h 50 m. 54 s, the G. S T of G M N being 5 h. 16 m 54 s. (U B)

(Ans 228° 9′ 40″·64, 14 h 7 m 47 3s.)

Q. 58. (a) Explain the following terms.—

(i) Equation of time, (ii) Celestial sphere, (iii) Parallax, and (iv) Sidereal time

(b) An observation was made on December 30, 1919 in longitude 82° 17′ 30″ E, the meridian altitude of the sun's lower limb was 40° 15′ 13″. The sun was on the south of the observer's zenith. Calculate the approximate latitude of the place. Correction for refraction = 1′ 10″, correction for parallax = 6″ 9, correction for semi diameter 16′ 17″·5 Declination of the sun at G A N. = 23° 13′ 15″ decreasing at the rate of 9″·17 per hour. (U B.)

(Ans N 26° 17′ 7 91″).

Q. 59. Discuss the relative merits of Triangulation and Precise Traversing and enumerate the conditions under which the latter is more suitable. Which type of angle measuring instrument is suitable for Precise traversing (G. U)

Q. 60. (a) Give details with sketches of the special features in the modern theodolites used for taking observations in Geodetic surveying.

(b) Describe the errors that can be eliminated by repetition method with a transit instrument out of adjustment and mention such errors as cannot be eliminated by this method. (G U)

Q 61 During reconnaissance of the hilly part of a country for Geodetic surveying following information was obtained regarding the profile of intervening ground between stations P and Q distance 85 miles Elevations above mean sea level of P = 845 ft Q – 4128 ft Peak L = 1250 ft Peak M = 2430 ft Peaks L and M are situated in between line PQ Distance PL = 38 miles and PM = 62 miles If P is to be ground station find the minimum station height above ground at Q to get 10 ft clearance of the line of sight PQ above the intervening peaks (G U)

(Ans 25 23 ft)

Q 62 (a) Describe various methods of extending a base line and explain its necessity

(b) The proposed elevations of two stations A and B 70 miles apart are respectively 516 and 1428 feet above mean sea level The only likely obstruction is situated at C, 20 miles from B and has an elevation of 598 feet Ascertain by how much if any B should be raised so that line of sight may clear C by 10 feet (K U)

(Ans 20 6 ft.)

Q 63 State the various kinds of signals used in triangulation surveys. The elevation of an instrument at A is 310 3 ft Find the minimum height of signal required at B, 28 3 miles distant, where the elevation of the ground is 396 0 ft. The intervening ground may be assumed to have a uniform elevation of 250 ft and the line of sight must nowhere be less than 6 ft above the surface (K U)

(Ans. 57 9 ft)

Q 64 Two stations A and B are 60 miles apart, the top of the scaffold at A is 75 feet above mean sea level and height of the ground at B is 2080 ft above the same datum The highest intervening point is at C 25 miles from B at a height of 750 feet above M S L. Find the height of the scaffolding at B in order that the line of sight may clear the point C by 10 ft (U B)

(Ans 27 1 ft)

Q 65 The altitude of two proposed stations A and B 80 miles apart are 2066 ft and 3487 ft respectively The highest intervening point is at C, 30 miles from A at an altitude 1803 ft Ascertain if A and B are intervisible (G U)

(Ans The line of sight fails to clear C by 64 87 ft)

Q 66 Two proposed stations A and B 75 miles apart are at altitude of 1850 and 2300 ft above M S L Two intervening points C and D are at altitude of 965 and 1760 ft. respectively above the same datum, their distances from A being 20 and 60 miles respectively If the height of instrument at A is 25ft) above ground at the station find the height of signal at B so that the ray from A to B may clear the ground at D by 10 ft What will be the height of ray AB above point C? (Ans 86 71 ft, 417 49 ft) (U P)

Q. 67. Directions were observed from a satellite station S, 6·54 ft. from station A, and the following results were obtained —

Station	Observed direction	Distance in ft from A
A	0° 0′	—
B	39 15	7021
C	92 46	6122
D	169 28	5556
E	264 44	7335

Correct the observed directions to those which would have been measured if the transit had been set up at station A (U. B.)

{true direction of AB = 39° 15′ 1″ 4 | True direction of AD = 169° 28′ 44″ 39
" " of AC = 92 49 40 · 2 | " " of AE = 264 40 56 9}

Q. 68. Explain how you would prolong a "Base line" A tape of 100 ft nominal length was standardised on the flat and its length at 65° F under a pull of 20 lb, was found to be 99·973 ft It was used in catenary at the same pull and at a temperature of 57° F. to measure a short span, the measured length of which was 76 41 ft What was the true length of the span between supports if the tape weighed 2 lb per 100 ft. and the coefficient of thermal expansion was 0 0000062 per F.1° ? (K U)

(Ans. 76 37 ft)

Q. 69 (a) Give a list of corrections to be applied in Base line measurements, indicating whether they are additive or subtractive

(b) A part of Base line was measured 300 ft by a tape of 300 ft. length standardised on the flat under a pull of 22 lbs It was used in catenary with tripod supports in between, dividing the length into three equal parts. The R L S of tops of tripods from start were 100, 101, 102 5, and 103 5 and the pull applied was 22 lbs Find the correct length of the base line measured, given, width of tape = 0 2 inch, Thickness of tape = 0 02 inch, weight of 1 cub. inch of steel = 0 28 lb (K U.)

(Ans 299 932 ft)

Q. 70. Give a list of corrections to be applied in base line measurements You measure a base of six bays with 100 ft steel tape in catenary at a temperature of 94°F. The supports of the tape are 1 65, 2 43, 2 64, 4 08, 3 21 and 3 94 ft above the first support The steel tape in catenary with supports at the same level measures 100 073 ft at 70° F. Coefficient of expansion 0 0000065 per 1°F. What is the true length of the base ? (U. B)

(Ans 600 5 ft)

Q 71. A tape of 300 ft nominal length was standardised on the flat and its true length found to be 300 014 at 72° F It was then used in catenary, in three equal spans of 100 ft each, to measure a level base line, the apparent length of which was found to be 2699 5 ft The weight of the tape was 12 oz. per 100 ft. length and the pull used, both during standardization and during the field measurements was 16 lbs Assume that the mean temperature during the

fie d measurements was 62° F and the coefficient of thermal expansion of the tape = 0 0000062 per 1° F What was the true length of the base line ? (U P)

(Ans. 2699 187 ft.)

Q 72 A steel tape is 100 ft long at a temperature of 60° F when placed horizontally on the ground. If its sectional area is 0 012 sq in. and weight 4 lb and coefficient of expansion 65 × 10⁻⁷ per 1° F, calculate the actual length of the tape at 90° F and pull 40 lbs and sag the tape is to be used between three supports (G U)

(Ans 100 009 ft.)

Q 73 Describe the equipment required for measuring a base line accurately in Triangulation survey

A field steel tape is 300 ft. long at a temperature of 60° F when lying horizontal on the ground at the time of standardization Its cross-sectional area is 0 0075 sq in. and its total weight 8 lbs. In the field, the tape is supported at three points the supports being equidistant and at the same level Calculate the actual length of the tape at a temperature of 71° F with a pull of 25 lbs Take coefficient of expansion = 0 0000065 and E = 30 × 10⁶ lbs. per sq in. (G U)

(Ans 299 725 ft)

Q 74 While measuring a base line the field tape was standardized at 60° F with a pull of 20 lbs stretched in three equal spans of 100 ft each. It was apparently 0 054 ft longer than the standard distance of 300 ft between scratches. Find the correction per tape length if at the time of measurement in the field, the temperature was 76° F and the pull exerted was 25 lbs. Weight of the tape = ? 25 lbs. Section of tape = ¼" × 1/50" E = 27 × 10⁶ lbs per sq in. coefficient of expansion per 1° F = 5 × 10⁻ (G U)

(Ans. 300 058 ft.)

Q 75 What is meant by Base net' ? Describe, with sketches the methods of prolonging a given base line A tape of 300 ft nominal length was standardised on the flat and its true length found to be 300 013 ft. at 70° F It was then used in catenary in three equal spans of 100 ft. each, to measure a base line the apparent length of which was found to be 2698 67 ft The weight of the tape was 1? oz per 100 ft length and the pull used, both during standardization and during the field measurements was 15 lb Assume that the mean temperature during the field measurements was 60° F and the coefficient of thermal expansion of the tape was 0 0000062 per 1° F What was the length of the line (U B)

(Ans. 2698 34 ft.)

Q 76 (a) Discuss the relative merits of triangulation and precise traversing for carrying out the survey of an extensive area

(b) Calculate the corrections for temperature, pull, and sag from the following data for measurement of a base line (i) Measured length of base line 11,200 ft (ii) Length of steel tape 200 ft under a pull of 15 lbs at a standard temperature of 55° F (iii) Sum of thermometer readings 7280° F Number of

readings on each tape length 2 (iv) Pull on tape in the field 20 lbs tape supported at every 100 ft (v) Section of tape 0 ?0×0 0? inch Weight of tape per cubic inch = 0 28 lbs Assume coefficient o expansion = 0 0000063 per 1° F E = 30 × 10⁶ lbs per sq inch Find the correct length of the line (U P)
(Ans C_t − +0 7056 ft , C_p − +0 4648 ft C_s = −2 1050 ft length = 11199 06 ft)

Q 77 (a) Describe the field operations necessary for measuring a base line

(b) A steel tape was exactly 100 ft in length on a plane surface under a pull of 20 lbs at a temperature 60° F It weighed 2 8lbs A base line was measured with this tape suspended in ten equal spans of 100 ft each under the same pull of 20 lbs the temperature being 70° F The first five spans were in level and the remaining were in uniform slope of 1 in 100 Compute the true length of the base line Coefficient of expansion = 0 0000063 per 1° F (K U)
(Ans 999 196 ft)

Q 78 (a) What is meant by Convergence of Meridians ? Determine the convergence in a traverse having a departure of 21 593 miles in a mean latitude of 56° 3 Take R = 3916 miles and og tan 1 = 4 4637

(b) The angles of a triangle ABC were recorded as follows —
A = 7?° 14 20″ weight 4
B = 49 40 35 ,, 3
C − 53 4 52 ,, 2
Give the corrected values of the angles (K U)
(Ans (a) 28 9″ (b) Corrections C_A = +3″ C_B = +4″ C = 6″)

Q 79 (a) Describe the various grades of triangulation What are the limits of (i) errors in angular measurements and computed distances, (ii) the lengths of base lines and sides of triangles

(b) Find the most probable values of the angles A B and C of a triangle ABC from the following observed values —
A = 50° 20 29″ B = 60° 14 40″ C = 69° 24 46″ (K U)
(Ans Corrections to the angles A B and C are each equal to + 1″ 67)

Q 80 (a) What is meant by (i) Convergence of Meridians and (ii) Spherical Excess ?

(b) Two points A and B have the following co ordinates —

Point	Latitude	Longitude
A	52° 22 12″ N	92° 38 40″ E
B	52° 25 16″ N	92° 32′ 10″ E

Find the convergence of the meridians through A and B (K U.)
(Ans 5 8″ 98)

Q 81 The observations closing the horizon at a station are —

A = 24° 22 18 2″ weight 1	C=30o° 25 13 9″ weight 2	
B = 30° 12 24 4″ 2	B+C=335° 37 38 0″ „ 3	
A+B = 54° 34 48 6″ „ 3		

Find the most probable values of the angle A, B and C (U P)

(Ans Corrections $C_A = +3\ 192''$, $C_B = +0\ 685''$)

Q 82 What is meant by side equation ? State the equations of condition which must be satisfied in the adjustment of (i) a triangle with a central station and (ii) geodatic quadrilateral Explain clearly how you would adjust these figures. (U P)

Q 82 (a) What are the effects of the curvature of the earth on surveys?

The following results were obtained in running a traverse survey for a proposed railway

Station	A		B		C		D
Deflection Angle			8°R		10°L		
Length in miles		12		15		20	

The latitude of A was 48° N the azimuth of AB (by astronomical observation) 46° 30′ State what correction must be applied to the bearing of CD at D as obtained from the traverse to allow for the convergence of the meridians Take the mean radius of the earth = 3916 miles (U P)

(Ans 32 15 78″)

Q 83 (a) Enumerate the principle of least squares Show how this principle is used for determining two unknowns in linear equations

(b) Measurement of the angles of two triangles having a common side BC gives

A = 54° 17 28″ 2	C_2 = 71° 25 14″ 1
B_1 = 6o 2 36 0	B = 111 14 38 2
C_1 = 60 39 57 2	C = 132 5 13 4
B_2 = 46 12 3 8	D = 62 22 44 0

Adjust the angles (U B)

Q 84 (a) What is meant by Convergence of Meridians ? Derive an expression for the same

(b) Determine the approximate increase in azimuth in a traverse which has total northings and eastings each of 42500 ft. from a station in latitude 59° 10 N , given that the radius of the earth = 20890000 ft and log tan 1 = 4 4637 (U B)

(Ans 11 42″ 6)

Q 85 (a) Show that the change in Azimuth in a long survey line is the product of the difference of longitude at its ends and the sine of the average latitude of its ends Assume the shape of the earth to be a sphere

(b) From a station A of latitude 56° N the following traverse was run —

Line	Length	Bearing
AB	8 miles	N "6° E
BC	6 miles	N 71° E
CD	9 miles	N 65° E

The bearing of AB was deduced from Azimuth observation and those of BC and CD from deflection angles Find what correction must be applied to the reduced bearing of CD at D to allow for convergence of the meridians Assume the mean radius of the earth as 3916 miles (U B)

(Ans 28 9 6")

Q 86 Adjust the following station observations —

A = 34° 18 20" 4 weight 1 A + B − 62 50 ?9 6 weight 2
B − 28 32 12 8 , 2 A + B + C = 85 39 8 6 , 1
C = 22 48 32 6 , 2 (U B)

(Ans Corrections $C_A = -1"$ 07 $C_B - -0"$ 53 $C_C = +1"$ 47)

Q 87 Find the most probable values of the following station observations closing the horizon

α − 54° 13 38" 5 weight 1 | γ = 257° 13 4" 3 weight 2
β = 48 33 13 8 „ 2 | $\beta + \gamma$ = 305° 46 17 8 2
$\alpha + \beta$ = 102 46 57 3 „ 1 | (G U)

(Ans α − 54° 13 41" 54 β = 48° 33 14" 33 γ = 25?° 13 4" 13)

Q 88 The angles in a quadrilateral ABCD resulting from the station adjustments are as follows

∠CAD = 38° 44 6"	∠BCA = 69° 4 21"
∠CAB = 23 44 38	∠ACD = 39 37 48
∠ABD = 42 19 9	∠CDB = 26 25 51
∠DBC = 44 52 1	∠BDA = 75 12 14

Compute the adjustment for Geoemtric condition and for Trigonometric condition (U B)

Q 89 Explain the method of adjusting observations by the method of Least Squares

Precise levelling was carried out to establish heights of three stations B C D above a datum A The following are the record of observations —

End of lines	Rise	Fall	Weight
AB	4 71		1
BC	3 59		2
CD	1 48		2
DA		9 72	1
BD	5 10		2

Compute the most probable errors in levelling and the Reduced Levels of B C, and D, if R L of datum is 100 00 (U B)

Q 90 The following angles were measured at a station O so as to close the horizon —

∠AOB = 83° 4? 28" 75 weight 3 ∠COD = 94° 38 27" 22 weight 4
∠BOC = 102 15 43 26 2 ∠DOA = 79 23 23 77 „ 2

Adjust the angles (K U.)

(Ans − 0 63", −0" 95 −0 47", — 0 95")

Q 91 Solve the triangle ABC in a triangulation survey, by Legendre s method from the following data —

Length of side AC = 97375 ft ∠A = 88° 34 0", ∠B = 40° 15' 30", ∠C = 51° 10 34", observations being of equal weight (U P.)

(*Ans* a = 124949 2 ft , b = 80771 2 ft)

Q 92 In order to locate the position O of a boat, observations were made to three points A, B, C on shore The angles AOB and BOC were found to be 48° 35′ and 30° 29 respectively From the map AB was scaled as 1200 ft. and BC as 700 ft, while the angle ABC measured 158° 39 What were the distances of O from A B, and C respectively ? (G U)

(Ans OA = 1564 ft, OB = 1986 ft, OC = 1361 ft)

Q 93 (a) Give a list of the various methods of locating soundings in Hydrographic surveying

(b) Write a short note on Tide gauge and its use (K U.)

94 In order to locate the position P of a boat observations were made to three points A, B and C on shore, the points B and P being on opposite sides of AC The angles APB and BPC were found to be 20° 6 and 35°6 respectively The lengths of AB and BC were 6670 ft and 12480 ft respectively, and the angle ABC was 152° 24 Determine the distances PA PB and PC (K U)

(Ans PA = 18339 3 ft, PB = 19405 8 ft, PC = 21466 3 ft

Q 95 The co ordinates of three stations are —

Station	Co ordinates	
	South	East
A	0	0
B	0	800
C	600	1250

With a sextant at a point P, the angles to A, B and C are found to be APB = 52° 12′, BPC = 70° 36 Find the co ordinates of P (U P)

(Ans 714 2 S, 705 2 E)

Q 96 A, B and C are three visible and charted points in a hydrographical survey The angles APB and BPC are observed with a sextant between A and B and B and C respectively from a sounding boat at P, and found to be 27° 44 and 25° 40 respectively, the points B and P being on opposite sides of AC. The lengths of AB and BC are 3080 ft and 3586 ft respectively, and the angle ABC is 61° 52 Determine the distances PA, PB, and PC (U B.)

(Ans 4243 ft, 6119 ft, 3101 ft)

+ + +

INDEX

+ + +